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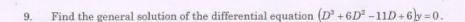
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	Reg. No. :
	Question Paper Code: 30250
	B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.
	* Fourth Semester
	Aeronautical Engineering
	MA 3452 — VECTOR CALCULUS AND COMPLEX FUNCTIONS
	(Common to : Aerospace Engineering)
	(Regulations 2021)
Tin	ne: Three hours Maximum: 100 marks
	Answer ALL questions.
1.	PART A — $(10 \times 2 = 20 \text{ marks})$ The force field $\vec{F} = y\vec{i} + x\vec{j} + cz\vec{k}$ is solenoidal. Find the value of c . Let $\vec{v} = e^x \cos y\vec{i} + e^x \sin y\vec{j} + v_3\vec{k}$ be the velocity of the fluid flow. For what value of v_3 the fluid flow will be incompressible?
3.	Find the fixed points of the mapping $w = \frac{z-1}{z+1}$.
4.	Find the points at which the mapping $w = z(z^4 - 5)$ is not conformal.
5.	. Is $\oint \overline{z} dz$ over the unit circle is zero? Why?
6.	Write the singularities of the following function and classify it
	(a) $f(z) = e^{\frac{z}{z}}$ (b) $\frac{\sin z}{z}$
7.	If $L\{f(t)\} = \frac{1}{s(s+a)}$, find the $\lim_{t\to\infty} f(t)$.
8.	Derive Laplace Transform of Unit step function $u(t-a)$.

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10. Reduce the differential equation $((1+x)^3D^3 + 2(1+x)^2D^2 - (1+x)D + I)y = (1+x)^{-2} \quad \text{into} \quad \text{a linear differential equation with constant coefficients.}$

PART B — $(5 \times 16 = 80 \text{ marks})$

- 11. (a) (i) Use Stoke's theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F} = xz\vec{i} + xy\vec{j} + 3xz\vec{k}$ where C is the boundary of the portions of the plane 2x + y + z = 2 in the first octant traversed counterclockwise as viewed from above. (8)
 - (ii) Show that the function $\vec{F} = (x^2 + y)\vec{k} + (y^2 + z)\vec{j} + ze^z\vec{k}$ is conservative. Also find the corresponding potential function f such that $\vec{F} = \nabla f$.

Or

(b) (i) Verify Divergence theorem for $\vec{F}=4xy\vec{i}-y^2\vec{j}+yz\vec{k}$, taken over the cube bounded by the planes x=0, y=0, z=0, x=1, y=1, z=1.

(ii) Use Green's theorem to find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (4)

12. (a) (i) Prove that an analytic function with constant modulus is constant.

(ii) Verify $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a harmonic conjugate function v of u. (8)

Or

- (b) Find the linear fractional transformation that maps $z_1=-1$, $z_2=i$, $z_3=1$ onto $w_1=0$, $w_2=i$, $w_3=\infty$ respectively. Also show that unit disk is mapped onto right half plane by this transformation. (16)
- 13. (a) (i) Find all Taylor and Laurent series expansion of the function $f(z) = \frac{-2z+3}{z^2-3z+2} \quad \text{with center} \quad 0 \quad \text{over the regions} \quad (1) \quad |z| < 1$ (2) 1 < |z| < 2 (3) |z| > 2.
 - (ii) Integrate $\frac{\tan z}{z^2 1}$ counterclockwise around the circle |z| = 3/2. (8)

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(b) (i) Evaluate
$$\int_{0}^{2\pi} \frac{d\theta}{\sqrt{2 - \cos \theta}}.$$
 (8)

(ii) Using contour integration method, show that
$$\int_{0}^{\infty} \frac{dx}{1+x^{4}}.$$
 (8)

14. (a) (i) Find the Laplace Transform of the half wave rectifier

$$f(t) = \begin{cases} \sin \omega t, & 0 \le t \le \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$$
 (8)

(ii) Find the inverse Laplace transform of the function $\ln\left(1+\frac{\omega^2}{s^2}\right)$. (8)

Or

- (b) (i) Using Laplace transform solve the differential equation $y'' 3y' + 2y = e^{-t}$, y(0) = 1, y'(0) = 0. (8)
 - (ii) Find the Laplace inverse of the function $\frac{se^{-s}}{s^2 + \omega^2}$. (8)
- (a) (i) Solve by method of variation of parameters the following differential equation y'+ y = sec x. (8)
 (ii) Find the general solution of differential equation x²y² + y = 3x². (8)
 - (b) (i) Solve the initial value problem y'' + 5y' + 6y = 2x + 1 with initial conditions y(0) = 0 and y'(0) = 1/3. (8)
 - (ii) Solve the simultaneous equations $\frac{dx}{dt} 7x + y = 0$, $\frac{dy}{dt} 2x 5y = 0$.

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