Civil - R

Reg. No. :

Question Paper Code : 50832

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Civil Engineering

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aerospace Engineering/Agriculture Engineering/Automobile Engineering/ Aeronautical Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Production Engineering/Robotics and Automation/ Bio Technology/Biotechnology and Biochemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Pharmaceutical Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation from $z = f(x^2 + y^2) + x + y$.

2. Solve the first order PDE $(z-y)p + (x-z)q = y - x$.

3. State Dirichlet's conditions for Fourier series expansion.

4. Determine the Fourier coefficient a_0 of $f(x) = e^{-x}$, in the interval $-l < x < l$.

5. State D'Alembert's solutions of the one – dimensional wave equation.

6. Classify the PDE : $x^2 \frac{\partial^2 u}{\partial x^2} + (1 - y^2)\frac{\partial^2 u}{\partial y^2} = 0, -1 < y < 1, -\infty, x < \infty$.

7. Write formula of Fourier cosine and sine integral.

8. State Parseval's identities for Fourier cosine and transform.

9. Find the Z transform of $a^n \cos n\theta$.

10. Find the inverse Z transform of $\frac{2z^2+3z}{(z+2)(z-4)}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the complete solution of $z^2(p^2+q^2)=x^2+y^2$. (4)

(ii) Solve : $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-6\frac{\partial^2 z}{\partial y^2}=\cos(2x+y)$. (12)

Or

(b) (i) Solve : $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial z}{\partial y}=x^2+y^2$. (8)

(ii) $\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x \partial y}+4\frac{\partial^2 z}{\partial y^2}=e^{2x+y}$. (8)

12. (a) (i) Find the Fourier series of the function $f(x)=\begin{cases} x & \text{for } 0 \le x \le \pi \\ 2\pi - x & \text{for } \pi \le x \le 2\pi \end{cases}$ and deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}\ldots\ldots\ldots=\frac{\pi^2}{8}$. (8)

(ii) If $f(x)=|\cos x|$, expand $f(x)$ as a Fourier series expansion in the interval $(-\pi,\pi)$. (8)

Or

(b) (i) Obtain the Fourier for $y=x^2$, in $-\pi<x<\pi$. Using the two values of y show that $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{n^4}{90}$. (8)

(ii) Find the complex form of Fourier series of $f(x)=\cos ax$ for $-\pi<x<\pi$. (8)

13. (a) (i) The end A and B of a rod 20cm long have the temperature at 30°C and 80°C until the steady state. The temperature of the ends are changed to 40°C end 60°C respectively. Find the temperature distribution in the rod at time t. (8)

(ii) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions $u(0, y) = u(l, y) = 0, u(x, 0) = 0$ and $u(x, a) = \frac{\sin n\pi x}{l}$. (8)

Or

(b) The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the mid-point of string always remains at rest. (16)

14. (a) (i) Show that $\int_0^\infty \frac{dt}{(t^2+1)^2} = \frac{\pi}{4}$ and $\int_0^\infty \frac{t^2\, dt}{(t^2+9)(t^2+4)} = \frac{\pi}{10}$, using Parseval's identity. (8)

(ii) State the prove convolution theorem for Fourier transform and verify the convolution theorem for $f(x) = g(x) = e^{-x^2}$. (8)

Or

(b) (i) Find the Fourier transform of $e^{-a^2x^2}$. (6)

(ii) Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx = \frac{\pi e^{-m}}{2}$. (10)

15. (a) (i) Using the Z-transform to solve $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$. (10)

(ii) Solve the difference equation using Z-transform $y(n+2) + 2y(n+1) + y(n) = 0$, given that $y(0) = y(1) = 0$. (6)

Or

(b) (i) Find the inverse Z-transform of $\frac{2z}{(z-1)(z^2+1)}$. (8)

(ii) Find the Z-transform of $n^2 e^{n\theta}$ and $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. (8)
