Reg. No. :

Question Paper Code : 30243

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Computer Science and Engineering

MA 3354 — DISCRETE MATHEMATICS

(Common to : Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the truth table for $(\sim p) \vee (\sim q)$.

2. Symbolise the following statement, "All the world loves a lover".

3. How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

4. State the Pigeonhole principle.

5. What is meant by simple graph? Give an example.

6. Draw a graph with the adjacency matrix $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$.

7. What is meant by commutative semi group?

8. Define a field.

9. Draw the Hasse diagram for $(D_{24}, /)$, where $D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$.

10. State De Morgan's law in any Boolean Algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. (8)

(ii) Show that the following argument is valid, "Every, micro computer has a serial interface port. Some micro computers have a parallel port. Therefore some micro computers have both serial interface port and parallel port". (8)

Or

(b) (i) Find a principal disjunctive normal form and a principal conjunctive normal form of $p \vee (\sim p \rightarrow (q \vee (\sim q \rightarrow r)))$. (8)

(ii) Using the indirect method, Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (8)

12. (a) (i) Use mathematical induction to show that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$, whenever n is a positive integer. (8)

(ii) Solve : $T(k) - 7T(k-1) + 10T(k-2) = 6 + 8k$, $T(0) = 1$ $T(1) = 2$. (8)

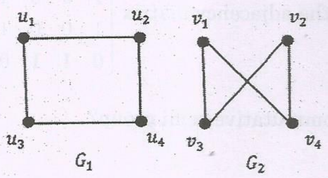
Or

(b) (i) Solve the recurrence relation $y_{n+2} - 4y_{n+1} + 3y_n = 0$, $y_0 = 2$, $y_1 = 4$, using the generating function. (8)

(ii) In a survey of 100 students, it was found that 40 studied Mathematics, 64 studied Physics, 35 studied Chemistry, 1 studied all the three subjects, 25 studied Mathematics and Physics, 3 studied Mathematics and Chemistry, 20 studied Physics and Chemistry. Use the principle of inclusion and exclusion, find the number of students who studied Chemistry only and the number who studied none of these subjects? (8)

13. (a) (i) Prove that the number of vertices of odd degree in a graph G is always even. (8)

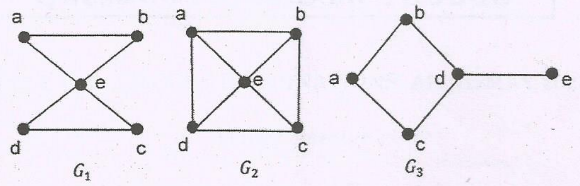
(ii) Determine whether the following pairs of graphs G_1 and G_2 are isomorphic. (8)



Or

(b) (i) Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph. (8)

(ii) Find an Euler path or an Euler circuit, if it exists in each of the following three graphs. If it does not exist, explain why? (8)



14. (a) (i) Prove that $[Z_5, +_5]$ is an abelian group. (8)

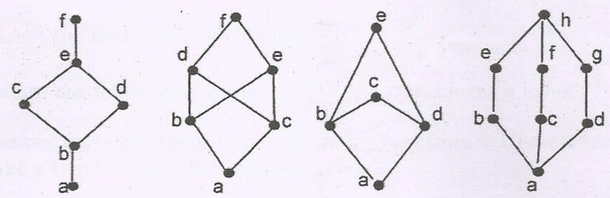
(ii) Show that any two right (left) cosets of H in G are either disjoint or identical. (8)

Or

(b) (i) State and prove Lagrange's Theorem. (8)

(ii) Show that the intersection of two normal subgroups of a group of $(G, *)$ is a normal subgroup of $(G, *)$. (8)

15. (a) (i) Determine whether the following partial ordering sets with the Hasse diagrams are lattices. (8)



(ii) Let $(L, \leq)$ be a lattice in which $\wedge$ and $\vee$ denote the operations of meet and join respectively. For any $a, b \in L$, prove that $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b$. (8)

Or

(b) (i) If L is a distributive lattice, prove $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a)$. (8)

(ii) In a Boolean algebra, prove

(1) $b \leq c \Rightarrow a \cdot b \leq a \cdot c$ and

(2) $b \leq c \Rightarrow a + b \leq a + c$. (8)

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