POLYTECHNIC, B.E/B.TECH, M.E/M.TECH, MBA, MCA & SCHOOL

*Notes*           *Available @*
*Syllabus*
*Question Papers*      www.binils.com
*Results and Many more…*

Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐☐☐

## Question Paper Code : 90815

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Aeronautical Engineering

MA 8353 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aerospace Engineering/Agriculture Engineering/Automobile Engineering/Civil Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (sandwich) Mechanical and Automation Engineering/Mechatronics Engineering/ Production Engineering/Robotics and Automation/ Bio Technology/Biotechnology and Biochemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Pharamceutical Technology)

(Regulations 2017)

Time : Three hours          Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating a and b from $z = a^2 x + a y^2 + b$

2. Find the complete integral of $\dfrac{z}{pq} + \dfrac{x}{q} + \dfrac{y}{p} + \sqrt{pq}$

3. State Dirichlet's conditions for a given function to be expanded in Fourier series.

4. State Parseval's identity for the function f(x) as a fourier series in $(0, 2l)$

5. The ends A and B of a rod of length 10cm long have their temperature Kept 20°C and 70°C. Find the steedy state temperature distribution on the rod.

6. In the diffusion equation $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$ what does $\alpha^2$ stand for?

7. Find the Fourier transform of f(ax), a>0

8. State the convolution theorem of Fourier transform.

9. State the initial value theorem of $Z$ – transforms.

10. Form a difference equation by eliminating arbitrary constant from $u_n = a2^{n+1}$

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve $z = p^2 + q^2$.

    (ii) Solve $x^2 p^2 + y^2 q^2 = z^2$.

Or

    (b) (i) Solve $x(y - z)p + y(z - x)q = z(x - y)$.

    (ii) Solve $\dfrac{\partial^3 z}{\partial x^3} - 2\dfrac{\partial^3 z}{\partial x^2 \partial y} = e^{x+2y} + 4\sin(x+y)$

12. (a) (i) Find the Fourier series for $f(x) = |x|$ in $(-\pi, \pi)$ and hence find $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots$

    (ii) Find the half range sine series for $f(x) = x^2$, $0 \le x \le \pi$.

Or

    (b) (i) Expand $f(x) = x(2l - x)$ in $(0, 2l)$ as a Fourier series of period $2l$. Deduce the sum $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \cdots$.

    (ii) Find the Fourier series of $f(x) = x^2$ in $-\pi < x < \pi$. Hence prove that $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n^4} = \dfrac{\pi^4}{90}$.

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = 50$ is initially at rest in its equilibrium position. If it is set to vibrate by giving each point a velocity $V_0 \sin\dfrac{\pi x}{50}\cos\dfrac{3\pi x}{50}$, find the displacement of any point of the string at any subsequent time.

Or

POLYTECHNIC, B.E/B.TECH, M.E/M.TECH, MBA, MCA & SCHOOL

*Notes*            *Available @*
*Syllabus*
*Question Papers*       www.binils.com
*Results and Many more…*

21

(b) A bar 10 cm long with insulated sides, has its ends A and B kept at 20°C and 40°C respectively until steady state conditions prevail. The temperature at $A$ is then suddenly raised to 50°C and at the same instant at $B$ is lowered to 10°C. Find the subsequent temperature at any point of the bar at any time.

14. (a) (i) Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence

deduce that $\int\limits_0^\infty \left(\dfrac{\sin t}{t}\right)^4 dt = \dfrac{\pi}{3}$.

(ii) Find the Fourier sine transform of $f(x) = \dfrac{1}{x(a^2 + x^2)}$.

Or

(b) (i) Find the Fourier cosine transform of $e^{-ax}$, $a > 0$ and hence find the Fourier transform of $e^{-a|x|} \cos bx$.

(ii) Using parseval's identity of Fourier transforms evaluate $\int\limits_0^\infty \dfrac{x^2}{(a^2 + x^2)^2} dx$.

15. (a) (i) Find $Z\left[\dfrac{2n+3}{(n+1)(n+2)}\right]$.

(ii) Find the inverse $Z$ transform of $\dfrac{z(z+1)}{(z-1)^3}$.

Or

(b) (i) Find the inverse $Z$ – transform using convolution theorem $Z^{-1}\left[\dfrac{z^2}{(z-a)(z-b)}\right]$.

(ii) Form the difference equation from $y_n = A \cdot 3^n + B \cdot 5^n$.

—————————