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Reg. No. :

Question Paper Code : 70137

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Electrical and Electronics Engineering

MA 3303 – PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours   Maximum : 100 marks

Statistical Z table should be given.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A pair of dice is tossed twice. Find the probability of scoring 7 points at least once.

2. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that exactly two will be defective.

3. If the joint probability density function of the random variable (X,Y) is given by $f(x,y)=kxye^{-(x^2+y^2)}$, $x>0$, $y>0$. Find the value of K.

4. State central limit theorem.

5. Show that the function $f(z)=z\bar{z}$ is nowhere analytic.

6. Under the transformation $W=\frac{1}{Z}$, find the images of $2x+y=2$.

7. Evaluate $\int_c \frac{e^z}{z(1-z)^3}\,dz$ if 0 lies inside c and 1 lies outside c.

8. Expand $f(z)=e^z$ as a Taylor's series about $z=0$.

9. Solve $(D^2-3D+2)y=e^x$.

10. Solve $(xD^2+D)y=0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has the following probability function: (8)

x :	0	1	2	3	4	5	6	7
p(x) :	0	k	2k	2k	3k	k²	2k²	7k²+k

(1) Find the value of k?

(2) Evaluate $P(X < 6)$ and $P(0 < X < 5)$.

(ii) In 256 sets of 12 tosses of a coin in how many cases one can except 8 heads and 4 tails. (8)

Or

(b) (i) In attest on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (8)

(1) More than 2150 hours,

(2) Less than 1950 hours.

(ii) Find the moment generating function of the exponential distribution $f(x)=\frac{1}{c}e^{-\frac{x}{c}}, 0 \le x \le 8, c > 0$. Hence find its mean and standard deviation. (8)

12. (a) (i) The joint probability mass function of (X,Y) is given by $f(x,y)=k(2x+3y), x=0,1,2;\ y=1,2,3$. Find all the marginal distribution of X given $Y=2, Y=3$. (8)

(ii) Find the covariance for the following heights of fathers X and their sons Y: (8)

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

Or

(b) (i) A study of prices of rice of Chennai and Madurai gave the following data: (8)

	Chennai	Madurai
Mean	19.5	17.75
S.D.	1.75	2.5

Also the coefficient of correlation between the two is 0.8. Estimate the most likely price at Chennai corresponding to the price of 18 at Madurai.

(ii) If X and Y follow exponential distribution with parameter 1 and are independent, find the probability distribution function of $U = X - Y$. (8)

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13. (a) (i) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

(ii) If $f(z)=u+iv$, is analytic find $f(z)$ and v if $u=\frac{\sin 2x}{\cos 2x+\cos h2y}$. (8)

Or

(b) (i) Find the bilinear map which maps the points $z=1, i, 0$ onto $w=1, i, -1$. (8)

(ii) Show that the map $w=\frac{1}{z}$ maps the totality of circles and lines as circles or lines. (8)

14. (a) (i) Evaluate, using Cauchy's integral formula $\int_c \frac{z+1}{z^2+2z+4}\,dz$, where c is the circle $|z+1+i|=2$. (8)

(ii) Expand $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in a Laurent's series if $|z|<2$. (8)

Or

(b) (i) Evaluate $\oint_C \frac{z\sec z}{(1-z^2)}dz$, where c is the ellipse $4x^2+9y^2=9$. (8)

(ii) Evaluate $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$. (8)

15. (a) (i) Solve the equation $(D^2-4D+3)y=\sin 3x+x^2$. (8)

(ii) Solve $(x^2D^2-xD+1)y=\left(\frac{\log x}{x}\right)^2$. (8)

Or

(b) (i) Solve the equation $\frac{d^2y}{dx^2}+a^2y=\tan ax$, by the method of variation of parameters. (8)

(ii) Solve the simultaneous equation $\frac{dx}{dt}+2x-3y=t$, $\frac{dy}{dt}-3y+2y=2e^{2t}$. (8)

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