Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 20812

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Third Semester

Aeronautical Engineering

MA 8353 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aerospace Engineering/Agriculture Engineering/Automobile Engineering/Civil Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Production Engineering/Robotics and Automation/Bio Technology/Biotechnology and Biochemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Pharmaceutical Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from the equation $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$.

2. Find the particular integral for the partial differential equation $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\,\partial y}=\cos x\,\cos 2y$.

3. Sketch the graph of even and odd expansion of $f(x)=x^2$ in $[0,2]$.

4. State giving reasons, whether the function $f(x)=\sin\left(\frac{1}{x}\right)+\tan x$ can be expanded in Fourier series in the interval of $(-\pi,\pi)$.

5. Find the value of c, for which $u=e^{-t}\cos 4x$ is the solution of the equation $c^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$.

6. What are the classifications of partial differential equation?

7. Does the Fourier cosine transform of $f(x)=5, (0<x<\infty)$ exist? Give your reason.

8. If $F(s)$ is the complex Fourier transform of $f(x)$, then Show that $F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$, $a \neq 0$ (a constant).

9. Find $Z[n \sin n\theta]$.

10. If Raju invests Rs. 1,000 at 6% interest compounded quarterly. (Note that Raju can not withdraw the money before the quarter is up). Model this as a difference equation (no need to solve).

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x+2y)$

(ii) Solve : $2(z + xp + yp) = yp^2$. (8+8)

Or

(b) (i) Solve : $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$.

(ii) Solve : $z = p^2 + q^2$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. (8+8)

12. (a) Obtain the Fourier series expansion of $f(x)=1+x+x^2$ in $(-\pi, \pi)$ and further evaluate the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ (16)

Or

(b) The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel. Expand y in terms of a Fourier sine series upto third harmonic defined in $(0, \pi)$. (16)

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
y	2.34	2.2	1.6	0.83	0.51	0.88	1.19

13. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y=y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$ of any point x of the string at any time $t>0$. (16)

Or

(b) An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π; this end is maintained at a temperature u_0 at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in the steady-state. (16)

14. (a) (i) Find the Fourier cosine transform of $\frac{e^{-ax}}{x}$.

(ii) Find Fourier sine transform of $x e^{-ax}$. (8+8)

Or

(b) Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1, & |x|<a \\ 0, & |x|>a>0\end{cases}$, Using Parseval's identify deduct that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. (16)

15. (a) (i) Find $Z^{-1}\left\{(z-5)^{-3}\right\}$ when $|z|>5$. Determine the region of convergence.

(ii) Using Convolution theorem, find the inverse Z-transform of $\left(\frac{z}{z-a}\right)^3$ and deduce the value of Z-transform of $\left(\frac{z}{z-1}\right)^3$. (8+8)

Or

(b) Solve the difference equation using Z- transform

(i) $y(n)-0.5\, y(n-1)=5(0.2)^n u(n), y(-1)=1$

(ii) $y_{n+2}+6y_{n+1}+9y_n=2^n$ with $y_0=0, y_1=0$. (8+8)