Reg. No. :

Question Paper Code : 20810

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Third Semester

Computer Science and Engineering

MA 8351 — DISCRETE MATHEMATICS

(Common to : Artificial Intelligence and Data Science/ Computer Science and Business Systems/Information Technology)

(Regulations 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Let P : He is rich : Q : He is happy. Write the following in symbolic form. He is neither rich nor happy.

2. Write the truth value of $(\exists x)P(x)$ where $P(x): x^2 > 10$ with universe $\{2,3,4\}$.

3. How many integers between 1 and 10^4 contains exactly one 8 and one 9?

4. Find the coefficient of x^5y^8 in $(x+y)^{13}$

5. What is the largest possible number of vertices in a graph with 35 edges and all vertices of degree at least three.

6. Draw a graph with the adjacency matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

7. Define Semigroup and Monoid.

8. Find all the left co-sets of $H = \{1,-1\}$ in the group $(G,.)$ where $G = \{1,-1,i,-i\}$.

9. Let $L = \{2,3,6,12,24,36\}$ and R be a relation on R such that aRb if and only if 'a divides b'. Construct its Hasse diagram.

10. Define Boolean Algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let : $M(x)$: x is a man. $R(x)$: x is mortal.

Produce the suitable English statement for the following :

(1) $(\forall x)(M(x) \to 7R(x))$

(2) $(\exists x)(M(x) \wedge R(x))$.

(ii) Show that the following set of premises is inconsistent :

If war is near then the army would be mobilized. If the army has mobilized then the labor costs are high. However the war is near and yet the labor costs are not high.

Or

(b) (i) Verify the validity of the argument. "Every living thing is a plant or an animal. John's gold fish is alive, and it is not a plant. All animals has hearts therefore, John's gold fish has a heart.

(ii) Using CP rule, prove the following argument $(\forall x)P(x) \to Q(x), (\forall x)(R(x) \to \neg P(x)) \Rightarrow (\forall x)(R(x) \to \neg P(x))$.

12. (a) (i) Determine the number of integers between 1 and 250 that are divisible by any of the integers 2,3,5 and 7

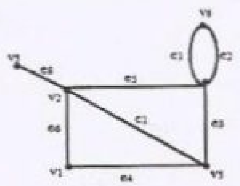
(ii) A magnetic tape contains a collection of 5 lakh strings made up of four or fewer number of English letters. Can all the strings in the collection be distinct?

Or

(b) Use the method of generating function to solve recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; n \geq 2$.

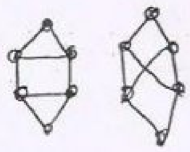
13. (a) (i) If a connected graph G with n vertices, where $n \geq 3$ with $d(v) \geq n/2$, for all $v \in v(G)$, then prove that the graph G must be Hamiltonian.

(ii) Represent the graph with an incidence matrix.



Or

(b) (i) Define isomorphism of graphs. Check whether the following graphs are isomorphic or not.



(ii) Prove that C_5 is the only cycle graph isomorphic to its complement.

14. (a) (i) Prove that a group of prime order is cyclic.

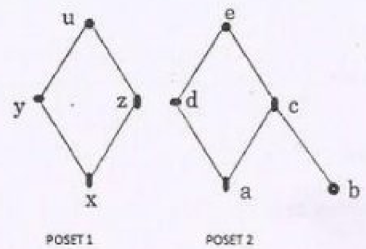
(ii) Prove that the Kernal of a homomorphisms f from $<G,*>$ to $<H,A>$ is a normal subgroup of $<G,*>$.

Or

(b) (i) Show that the group $<G,*>$ is abelian if and only if $(a*b)^2 = a^2 * b^2, \forall a,b \in G$

(ii) Prove that the subgroup of a cyclic group is cyclic.

15. (a) (i) Identify the maximal elements, minimal elements, least and greatest (if they exist) of the POSETs given by the following Hasse diagrams.



(ii) Show that the De Margon laws are valid in a distributive complemented lattice.

Or

(b) (i) Give an example of a lattice which is a modular but not distributive.

(ii) Show that in any Boolean Algebra (a+b) (a'+c) =ac+a'b+bc.
