Reg. No. :

Question Paper Code : 20807

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Second Semester

Civil Engineering

MA 8251 – ENGINEERING MATHEMATICS – II

[Common to : All branches (Except B.E. Marine Engineering, Artificial Intelligence and Data Science, Computer Science and Business Systems]

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define Eigenvalues and Eigenvectors of a matrix A.

2. Find the nature of the following quadratic form $5x^2 + 2y^2 + 4xy$.

3. What is directional derivative of $\phi = xy^2 + yz^3$ at the point (2, –1, 1) in the direction of vector $\vec{i} + 2\vec{j} + 2\vec{k}$?

4. State Green's theorem.

5. Examine whether $f(z) = e^z$ is analytic.

6. Find the invariant points of the transformation $w = \dfrac{2z+6}{z+7}$.

7. Evaluate $\oint_C z\,dz$, where C is $|z| = 1$.

8. Define removable singularity with an example.

9. Is Laplace transform of the function $f(t) = e^{t^2}$ exist? Give the reason.

10. Find $f(t)$, if $L\{f(t)\} = \dfrac{1}{(s-1)^2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$.

(ii) Verify that the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ satisfies its characteristic equation.

Or

(b) Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation and hence show that it is positive semi-definite.

12. (a) (i) Show that $F = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential.

(ii) Use Green's theorem, find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Or

(b) Verify Gauss divergence theorem for $F = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, where S is the surface of the cuboid formed by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$ and $z = 2$.

13. (a) (i) Determine the analytic function, whose real part is $e^x(x\cos y - y\sin y)$.

(ii) Show that an analytic function with constant real part is a constant.

Or

(b) (i) Find the image of the region the half plane $x > c$, when $c > 0$ under the transformation $w = 1/z$.

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$.

14. (a) (i) Use Cauchy's integral formula, evaluate $\int_C \frac{z+1}{z^3-2z^2}dz$ where C is the circle $|z-2-i|=2$.

(ii) Find the Laurent's series of $f(z)=\frac{1}{z(1-z)}$ in $1<|z+1|<2$.

Or

(b) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+9)(x^2+4)}dx$, using contour integration.

15. (a) (i) Apply convolution theorem to evaluate the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

(ii) Find the Laplace transform of the periodic function $f(t)=\begin{cases} \sin\omega t & 0<t<\pi/\omega \\ 0, & \pi/\omega<t<2\pi/\omega \end{cases}$, with period $2\pi/\omega$.

Or

(b) Using Laplace transform, solve $t\frac{d^2y}{dt^2}+2\frac{dy}{dt}+ty=\cos t$ given that $y(0)=1$.

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