UNIT: I - CONDUCTION

Fourier's Law of conduction.

The rate of heat conduction is proportional to the area measured – normal to the direction of heat flow and to the temperature gradient in that direction.

$$Q \propto -A\frac{dt}{dx}$$

$$Q = -KA\frac{dt}{dx}$$

Where, A are in m^2

$\frac{dt}{dx}$ is temperature gradient in K/m

K is Thermal Conductivity W/mk

Newton's law of cooling or convection law.

Heat transfer by convection is given by Newton's law of cooling

$Q = hA\,(Ts - T_\infty)$

Where

A – Area exposed to heat transfer in m^2 , h - heat transfer coefficient in W/m^2K

Ts – Temperature of the surface in K, T_∞ - Temperature of the fluid in K.

Overall heat transfer co-efficient.

The overall heat transfer by combined modes is usually expressed in terms of an overall conductance or overall heat transfer co-efficient 'U'.

Heat transfer $Q = UA\,\Delta T$.

Equation for heat transfer through composite pipes or cylinder.

Heat transfer $Q = \frac{\Delta Toverall}{R}$ where $\Delta T = Ta - Tb$

$$R = \frac{1}{2\pi L}.\frac{1}{h_a r_1} + \frac{ln\left[\frac{r_2}{r_1}\right]}{K_1} + \frac{ln\left[\frac{r_1}{r_2}\right]}{K_1}L_2 + \frac{1}{h_a r_{13}}$$

critical radius of insulation (or) critical thickness?

Critical radius = r_c Critical thickness = $r_c - r_1$

Addition of insulating material on a surface does not reduce the amount of heat transfer rate always. In fact under certain circumstances it actually increases the heat loss up to certain thickness of insulation. The radius of insulation for which the heat transfer is maximum is called critical radius of insulation, and the corresponding thickness is called critical thickness.

Fin efficiency and Fin effectiveness.

The efficiency of a fin is defined as the ratio of actual heat transfer by the fin to the maximum possible heat transferred by the fin.

$$\eta = \frac{Q_{fin}}{Q_{max}}$$

Fin effectiveness is the ratio of heat transfer with fin to that without fin

$$fin\ effectiveness = \frac{Q_{withfin}}{Q_{withoutfin}}$$

critical thickness of insulation with its significance.

Addition of insulating material on a surface does not reduce the amount of heat transfer rate always. In fact under certain circumstances it actually increases the heat loss up to certain thickness of insulation. The radius of insulation for which the heat transfer is maximum is called critical radius of insulation, and the corresponding thickness is called critical thickness. For cylinder, Critical radius = rc = k/h, Where k- Thermal conductivity of insulating material, h- heat transfer coefficient of surrounding fluid. Significance: electric wire insulation may be smaller than critical radius. Therefore the plastic insulation may actually enhance the heat transfer from wires and thus keep their steady operating temperature at safer levels.

lumped system analysis? When is it applicable?

In heat transfer analysis, some bodies are observed to behave like a "lump" whose entire body temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only. Heat transfer analysis which utilizes this idealization is known as the lumped system analysis. It is applicable when the Biot number (the ratio of conduction resistance within the body to convection resistance at the surface of the body) is less than or equal to 0.1.

three dimensional heat transfer poisson and laplace equation in Cartesian co-ordinates

Poisson equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g}{k} = 0$$

Laplace equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

10. A 3 mm wire of thermal conductivity 19 W/mK at a steady heat generation of 500 MW/m³ .Determine the center temperature if the outside temperature is maintained at 25⁰C

Critical temperature $T_c = T_\infty + \frac{qr^2}{4K}$

$$= 298 + \left[\frac{500 \times 10^6 \times 0.0015^2}{4 \times 19}\right]$$

$$T_c = 312.8K$$

the three types of boundary conditions.

1. Prescribed temperature
2. Prescribed heat flux
3. Convection Boundary Conditions.

fins (or) extended surfaces.

It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surfaces used for increasing heat transfer are called extended surfaces or sometimes known as fins.

thermodynamics differ from heat transfer?

- Thermodynamics doesn't deals with rate of heat transfer
- Thermodynamics doesn't tell how long it will occur
- Thermodynamics doesn't tell about the method of heat transfer

1. **Derive the General Differential Equation of Heat Conduction in Cartesian coordianates.(NOV/DEC 2014)**

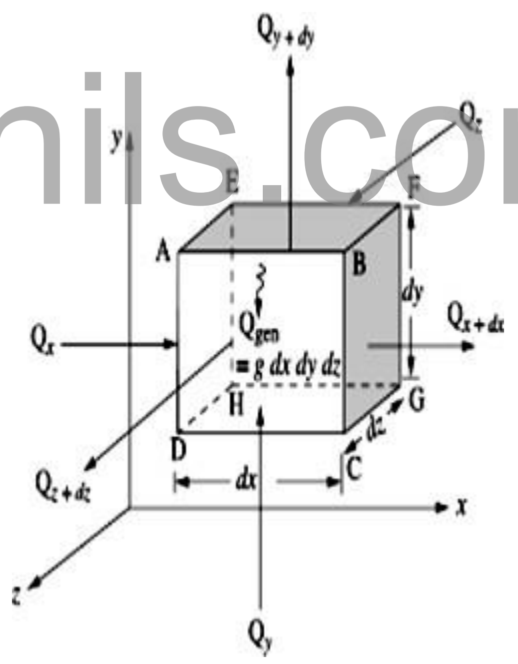


Fig 2.1

Consider a small volume element in Cartesian coordinates having sides dx, dy and dz as shown in Fig. 2.1 the energy balance for this little element is obtained from the first law of thermodynamics as:

$$\begin{Bmatrix} Net heat conducted into element \\ dx dy dz per unit time \\ (I) \end{Bmatrix} + \begin{Bmatrix} Internal heat generated \\ per unit time \\ (II) \end{Bmatrix}$$

$$= \begin{Bmatrix} Increaseininternal \\ perunittime \\ (III) \end{Bmatrix} + \begin{Bmatrix} Workdonebyelement \\ perunittime \\ (IV) \end{Bmatrix} \quad (2.2)$$

The last term of Eqn. (2.2) is very small because the flow work done by solids due to temperature changes is negligible.

The three terms, I, II and III of this equation are evaluated as follows:

Let q_x be the heat flux in x-direction at x, face ABCD and q_{x+dx} the heat flux at x + dx, face A'B'C'D'. Then rate of heat flow into the element in x-direction through face ABCD is:

$$Q_x = q_x dydz = -k_x \frac{\partial T}{\partial x} dydz \quad (2.3)$$

Where k_x is the thermal conductivity of material in x-direction and $\frac{\partial T}{\partial x}$ is the temperature gradient in

x-direction. The rate of heat flow out of the element in x-direction through the face at x+dx. A'B'C'D' is:

$$Q_x = -k_x \frac{\partial T}{\partial x} dydz - \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) dxdydz \quad (2.4)$$

Then, the net rate of heat entering the element in x-direction is the difference between the entering and leaving heat flow rates, and is given by:

$$Q_x - Q_{x+dx} = \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) dxdydz \quad (2.5)$$

$$Q_y - Q_{y+dy} = \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial x}\right) dxdydz$$

$$Q_z - Q_{z+dz} = \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) dxdydz$$

The net heat conducted into the element dx dydz per unit time, term I in Eqn. (2.2) is:

$$I = \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)\right] dxdydz \quad (2.6)$$

Let q be the internal heat generation per unit time and per unit volume (W/m^3), the rate of energy generation in the element, term II in Eqn. (2.2), is

$$II = q \; dx \; dydz \quad (2.7)$$

The change in internal energy for the element over a period of time dt is: (mass of element) (specific heat) (change in temperature of the element in time dt)

$$(\rho dxdydz)\,(c_p)\, dT = (\rho c_p dT) dxdydz \quad (2.8)$$

Where ρ and c_p are the density and specific heat of the material of the element.

Then, the change in internal energy per unit time, term III of Eqn. (2.2) is:

$$\text{III} = \rho c_p \frac{\partial T}{\partial t} dxdydz \tag{2.9}$$

Substitution of Eqns. (2.6),(2.7) and (2.9) into Eqn. (2.2) leads to the general three-dimensional equation for heat conduction:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + q = \rho c_p \frac{\partial T}{\partial t} \tag{2.10}$$

Since for most engineering problems the materials can be considered isotropic for which $K_x = K_y = K_z = k =$ Constant, the general three-dimensional heat conduction equation becomes:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The quantity is known as the thermal diffusivity, α of the material. It has got $\frac{k}{\rho C_P}$ the units m^2/s.

2. Derive the Heat conduction equation in cylindrical coordinates.

The heat conduction equation derived in the previous section can be used for solids with rectangular boundaries like slabs, cubes, etc. but then there are bodies like cylinders, tubes, cones, spheres to which Cartesian coordinates system is not applicable.

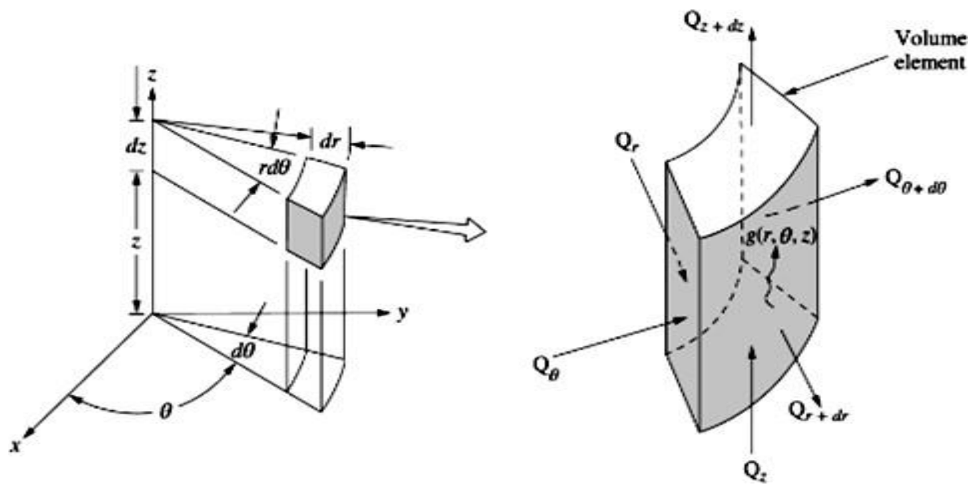


Fig 2.2

A more suitable system will be one in which the coordinate surfaces coincide with the boundary surfaces of the region. For cylindrical bodies, a cylindrical coordinate system should be used. The heat conduction equation in cylindrical coordinates can be obtained by an energy balance over a differential element, a procedure similar to that described previously. The equation could also be obtained by doing a coordinate transformation from Fig. 2.2.

Consider a small volume element having sidesdr, dz and r $d\emptyset$ as shown in Fig. 2.2. Assuming the material to be isotropic, the rate of heat flow into the element in r-direction is:

$$Q_r = -k\frac{\partial T}{\partial r} rd\emptyset dz$$

The rate of heat flow out of the element in r-direction at r+dr is:

$$Q_{r+dr} = Q_r + \frac{\partial Q_r}{\partial r} dr$$

Then, the net rate of heat entering the element in r-direction is given by

$$Q_r - Q_{r+dr} = k\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) drd\emptyset dz$$

$$= k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) drd\emptyset dz$$

Similarly,

$$Q_\emptyset - Q_{\emptyset+d\emptyset} = -k\frac{\partial T}{rd\emptyset} drdz - \left[-k\frac{\partial T}{rd\emptyset} drdz - \frac{k\partial}{rd\emptyset}\left(\frac{\partial T}{r\partial\emptyset}\right).rd\emptyset drdz\right]$$

$$= k\left(\frac{1}{r^2}\frac{\partial^2 T}{\partial\emptyset}\right) rdrd\emptyset dz$$

$$Q_z - Q_{z+dz} = -k\frac{\partial T}{\partial z}.rd\emptyset dz - \left[-k\frac{\partial T}{\partial z} rd\emptyset dr - k\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right).rd\emptyset drdz\right]$$

$$= -k\left(\frac{\partial^2 T}{\partial z^2}\right) rdrd\emptyset dz$$

The net heat conducted into the element dr.r dzper unit time, term I of Eqn.d
(2.2)

$$I = k\left(\frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) rdrd\emptyset dz$$

Taking q s the internal heat generation per unit time and per unit volume, term II of Eqn (2.2) is

$$II = q\ r\ dr\ d\emptyset dz$$

The change in internal energy per unit time, term III of Eqn. (2.2) is:

$$III = \rho c_p \frac{\partial T}{\partial t} rdrd\emptyset dz$$

Substitution of terms I, II and III into the energy balance Eqn. (2.2) leads to three-dimensional equation for an isentropic material in cylindrical coordinate system as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\left(\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \emptyset^2} + \frac{\partial^2}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

3. A furnace wall is made up of three layer of thickness 25 cm, 10 cm and 15 cm with thermal conductivities of 1.65 W/mK and 9.2 W/mK respectively. The inside is exposed to gases at 1250^{0}c with a convection coefficient of 25 W/m^2 K and the inside surface is at 1100^{0}c , the outside surface is exposed to air at 25^{0}C with convection coefficient of 12 W/m^{2}K .Determine (i) the unknown thermal conductivity (ii)the overall heat transfer coefficient (iii) All the surface temperature.(May/June 2012)

Given:

Thickness L_1 =25 cm =0.25m

L_2 =10 cm =0.1m

L_3 =15 cm =0.15m

Thermal conductivity , k_1 =1.65W/mK,

k_{2}=9.2 W/mK

Inside Gas Temperature , T_a=1250^{0}c=1523K

T_b=25^{0}c=298K

Inner surface temperature ,T_1=1100^{0}C=1373K

Inside heat transfer coefficient ,h_a=25 W/m^{2}K

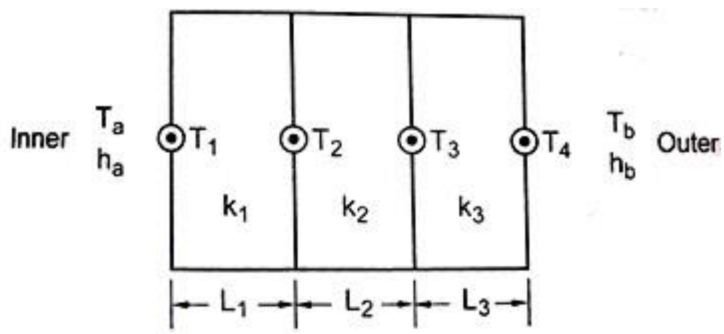
Outside Heat Transfer Coefficient ,h_b=12 W/m^{2}K

To find:

i) The Unknown Thermal Conductivity ,

ii) The Overall Heat Transfer Coefficient

iii) All The Surface Temperature



Solution:

STEP-1

Heat transfer $Q = h_a A(T_a - T_1)$

$=25(1523-1373)=3750\ W/m^2$

From HMT data book P.No 45

Heat Flow , $Q=\Delta T_{overall}/R$

$$R = \frac{1}{H_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{H_b A}$$

$$Q = \frac{T_a - T_b}{\frac{1}{H_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{H_b A}}$$

$$\frac{Q}{A} = \frac{1523 - 298}{\frac{1}{25} + \frac{0.25}{1.65} + \frac{0.10}{k_2} + \frac{0.15}{9.2} + \frac{1}{12}}$$

$k_2=2.816 W/mk$

STEP-2

From HMT data book P.No 45

Overall Thermal resistance (R)

$$R = \frac{1}{H_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{H_b A}$$

[Take $A=1\ m^2$]

$R_{total}=0.3267\ W/m^2$

$U=1/R_{total}=1/0.3267=3.06 W/m^2K$

STEP-3

$$Q = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_b}{R_b}$$

$$Q = \frac{T_a - T_1}{R_a},$$

$$Q = \frac{T_1 - T_2}{R_1},$$

$$R_1 = \frac{L_1}{K_1} = 0.1515$$

$$3750 = \frac{1373 - T_2}{0.1515}$$

T_2=804.8K

$$Q = \frac{T_2 - T_3}{R_2}\left[\because R_2 = \frac{L_2}{K_2}\right]$$

$$3750 = \frac{804.8 - T_3}{\frac{0.10}{2.816}}$$

T_3=671.45K

$$Q = \frac{T_3 - T_4}{R_3}\left[\because R_3 = \frac{L_3}{K_3}\right]$$

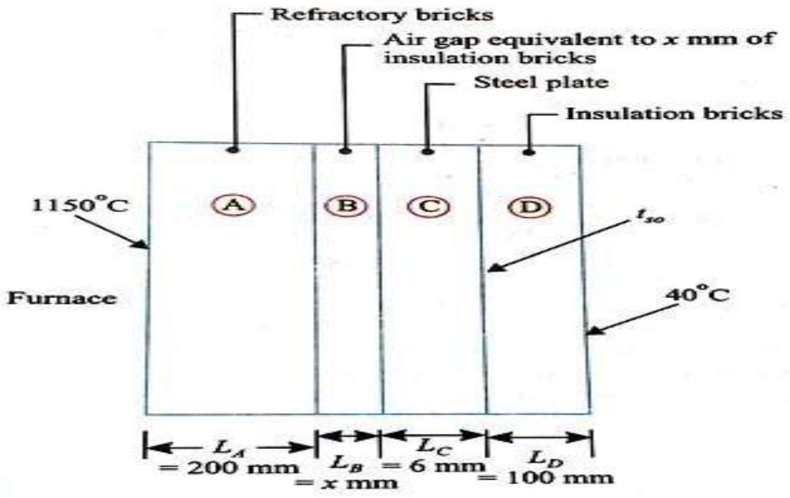
$$3750 = \frac{671.45 - T_4}{\frac{0.15}{9.2}}$$

T_4=610.30K

4. A furnace wall consists of 200mm layer of refractory bricks, 6 mm layer of steel plate and a 100mm layer of insulation bricks. The maximum temperature of the wall is 1150°C on the furnace side and the minimum temperature is 40°C on the outermost side of the wall. An accurate energy balance over the furnace shows that the heat loss from the wall is 400W/m². It is known that there is a thin layer of air between the layers of refractory bricks and steel plate. Thermal conductivities for the three layers are 1.52, 45 and 0.138 W/m°C respectively. Find

i) To how many millimeters of insulation bricks is the air layer equivalent?

ii) What is the temperature of the outer surface of the steel plate? (Nov/Dec 2014)



Given

Thickness of refractory bricks, Thickness of steel plate, Thickness of insulation bricks,

$L_A = 200mm = 0.2m L_C = 6mm = 0.006m$
$L_D = 100mm = 0.1m$

Difference of temperature between the innermost and outermost sides of the wall,

$\Delta t = 1150 - 40 = 1110°C$

$K_A = 1.52\ W/m°C$

$K_B = K_D = 0.138\ W/m°C$

$K_C = 45\ W/m°C$

Heat loss from the wall, $q = 400\ W/m^2$

i) The value of x(=L_c)

From HMT data book P.No 45

Heat Flow , Q=ΔT $_{overall}$/R

$$R = \frac{1}{H_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{H_b A}$$

$$400 = \frac{1110}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C} + \frac{L_D}{K_D}}$$

$$400 = \frac{1110}{\frac{0.2}{1.52} + \frac{(x/1000)}{0.138} + \frac{0.006}{45} + \frac{0.1}{0.138}}$$

$$= \frac{1110}{0.1316+0.0072x+0.00013+0.7246}$$

$$= \frac{1110}{0.8563+0.0072x}$$

$$0.8563 + 0.0072x = \frac{1110}{400} = 2.775$$

$$x = \frac{2.775-0.8563}{0.0072} = 266.5\ mm$$

$$x = 266.5\ mm$$

ii)Temperature of the outer surface of the steel plate t_{so}:

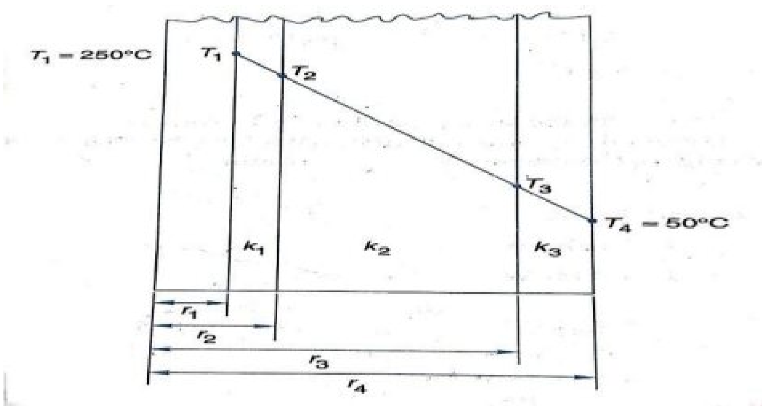
$$q = 400 = \frac{(t_{so} - 40)}{L_D/K_D}$$

$$400 = \frac{(t_{so} - 40)}{0.1/0.1.38}$$

$$t_{so} = \frac{400}{1.38} + 40 = 329.8°C$$

$$t_{so} = 329.8°C$$

5. A steel pipe line(K=50W/mk) of I.D 110mm is to be covered with two layers of insulation each having a thickness of 50mm. The thermal conductivity of the first insulation material is 0.06W/mk and that of the second is 0.12W/mk. Calculate the loss of heat per metre length of pipe and the interface temperature between the two layers of insulation when the temperature of the inside tube surface is 250°C and that of the outside surface of the insulation is 50 °C. (April/ may 2015)



Given :

$r_1 = 50mm$

$r_2 = 55mm$

$r_3 = 105mm$

$r_4 = 155mm$

$K_1 = 50\ \frac{W}{mk}$

$K_2 = 0.06\ \frac{W}{mk}$

$K_3 = 0.12\ \frac{W}{mk}$

$T_1 = 250^oC$

$T_4 = 50^oC$

To find

$T_3 = ?$

Solution:

step-1

From HMT data book P.No 46

Heat Flow , Q=ΔT $_{overall}$/R

$$R = \frac{1}{2\pi L}\left[\frac{1}{H_a r_1} + \frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left(\frac{r_3}{r_2}\right) + \frac{1}{k_3} ln\left(\frac{r_4}{r_3}\right) + \frac{1}{H_b r_4}\right]$$

$$\frac{Q}{L} = \frac{2\pi(T_1 - T_4)}{\frac{ln\left(\frac{r_2}{r_1}\right)}{K_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{K_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{K_3}}$$

$$\frac{Q}{L} = \frac{2 \times 3.14(250 - 50)}{\frac{ln\left(\frac{55}{50}\right)}{50} + \frac{ln\left(\frac{105}{55}\right)}{0.06} + \frac{ln\left(\frac{155}{105}\right)}{0.12}}$$

$$\boxed{\frac{Q}{L} = 89.6\ W/m}$$

step-2

The interface temperature,T_3 is obtained from the equation

$$\frac{Q}{L} = \frac{2\pi(T_3 - T_4)}{\frac{ln\left(\frac{r_4}{r_3}\right)}{K_3}}$$

$$T_3 = \frac{\frac{Q}{L} \times ln\left(\frac{r_4}{r_3}\right)}{2\pi K_3} + T_4$$

$$= \frac{89.6 \times ln\left(\frac{155}{105}\right)}{0.12 \times 6.28} + 50$$

$$\boxed{T_3 = 96.3^0C}$$

6. A plane wall 10cm thick generates heat at a rate of 4×10⁴ W/m³ when an electric current is passed through it. The convective heat transfer co-efficient between each face of the wall and the ambient air is 50W/m²K. Determine a) the surface temperature b) the maximum air temperature on the wall, Assume the ambient air temperature to be 20⁰c and the thermal conductivity of the wall material to be 15 W/mK. (May/June 2016)

<u>Given:</u>

Thickness L = 10cm =0.10m

Heat generation = 4×10^4 W/m³ $\dot{q}$

Convective heat transfer co-efficient =50 W/m²K.

Ambient air temperature T_∞=20⁰c+273=293K

Thermal conductivity k=15 W/mK.

<u>Solution:</u>

<u>Step 1</u>

From HMT data book P.No 48

Surface temperature $T_W = T_\infty + \frac{\dot{q}L}{2h}$

$$= 293 + \frac{4 \times 10^4 \times 0.10}{2 \times 50}$$

$$\boxed{T_w = 333K}$$

<u>Step2</u>

Maximum temperature $T_{max} = T_w + \frac{\dot{q}L^2}{8k}$

$$= 333 + \frac{4\times10^4\times(0.10)^2}{8\times15}$$

T_{max}=336.3K

7. A cylinder 1m long and 5 cm in diameter is placed in an atmosphere at 45^0c . It is provided with 10 longitudinal straight finsof material having k=120W/mk.The height of 0.76mm thick fins is 1.27cm from the cylinder surface.The heat transfer co-efficient between cylinder and the atmospheric air is 17W/ m^2K.Calculate the rate of heat transfer and the temperature at the end of fins if the surface temperature of cylinder is 150^0c.(Nov/Dec 2015)

Given:

Length of cylinder W =1 m

Length of the fin L =1.27cm=1.27m. Thickness of

the fin t = 0.76mm=0.76m.$\times 10^{-2}\times 10^{-3}$

Thermal conductivity k=120W/mk

heat transfer co-efficient h=17W/ m^2K

Base temperature of the cylinder T_b =150^0c+273=423k

Ambient temperature =45^0c+273 =318 KT_∞

Diameter of the cylinder d =5cm=5m.$\times 10^{-2}$

To find

i) Heat transfer rate, Q_{total}

ii) Temperature at the end of the fin , T

Solution:

Step-1

Perimeter = 2W =2×1=2m

Area =Wt =1× 0.76$\times 10^{-3}$= 0.76$\times 10^{-3}$$m^2$

From HMT data book P.No 50

$$m = \sqrt{\frac{hp}{kA}}$$

$$=\sqrt{\frac{17\times2}{120\times0.76\times10^{-3}}}$$

m=19.31

Step-2

$$\tan h\,(mL) = tanh(19.81\times1.27\times10^{-2}) = 0.241$$

$$\frac{h}{mk} = \frac{17}{19.31\times120} = 0.00734$$

From HMT data book P.No 50

$$Q_{fin} = \sqrt{hpkA}\,(T_b - T_\infty)\left[\frac{tanh(ml) + \left(\frac{h}{mk}\right)}{1 + \left(\frac{h}{mk}\right)tanh(ml)}\right]$$

$$= \sqrt{17\times2\times120\times0.76\times10^{-3}}(423-318)\left[\frac{0.241+(0.00734)}{1+(0.00734)0.241}\right]$$

$$\boxed{Q_{fin} = 45.65\ KWperfin}$$

From HMT data book P.No 44

$$Q_b = h[\pi D - [10\times0.76\times10^{-3}]L(T_b - T_\infty)]$$

$$= 17[\pi\times0.05 - [10\times0.76\times10^{-3}]1(423-318)]$$

$$Q_b = 266.82W$$

Step-3

$$Q_{total} = 10Q_{fin} + Q_b$$

$$= (10\times45.7) + 266.82$$

$$Q_{total} = 723.82W$$

Step-4

From HMT data book P.No 50

The temperature at the end of the fin

$$T - T_\infty = \frac{T_b - T_\infty}{Cosh(ml) + \left(\frac{h}{mk}\right)sinh(ml)}$$

$$T - 318 = \frac{423-318}{Cosh(19.81\times1.27\times10^{-2}) + (0.00734)sinh(19.81\times1.27\times10^{-2})}$$

$$\boxed{T = 419.74K}$$

8. A circumferential rectangular fins of 140mm wide and 5mm thick are fitted on a 200mm diameter tube. The fin base temperature is 170°C and the ambient temperature is 25°C Estimate fin Efficiency and heat loss per fin. Take Thermal conductivity K = 220W/mk Heat transfer co-efficient h= 140W/m²k.

Given:

Wide L = 140mm=0.140m

Thickness t =5mm = 0.005m

Diameter d =200mm $\Rightarrow r = 100mm = 0.100m$

Fin base temperature $T_b = 170°C + 273 = 443K$

Ambient temperature $T_\infty = 25°C + 273 = 298K$

Thermal conductivity k= 220W/mk

Heat transfer co-efficient h= 140W/m²k

To find:

Fin Efficiency,η

Heat loss Q

Solution:

A rectangular fin is long and wide.So heat loss is calculated by fin efficiency curves

From HMT data book P.No 52

Step1

Corrected length $L_c = L + {}^{t}/_{2}$

$$= 0.140 + {}^{0.005}/_{2}$$

$$\boxed{L_c = 0.1425\ m}$$

Step2

$$r_{2c} = r_1 + L_c$$

$$= 0.100 + 0.1425$$

$$\boxed{r_{2c} = 0.2425m}$$

Step 3

$$A_s = 2\pi\,[r_{2c}^2 - r_1^2]$$

$$= 2\pi[(0.2425)^2 - (0.100)^2]$$

$$A_s = 0.30650 m^2$$

Step4

$$A_m = t[r_{2c} - r_1]$$

$$A_m = 0.005[0.2425 - 0.100]$$

$$A_m = 7.125 \times 10^{-4} m^2$$

From the graph, we know that, [HMT data book page no.51]

$$X_{axis} = (L_C)^{1.5}\left[\frac{h}{KA_m}\right]^{.05}$$

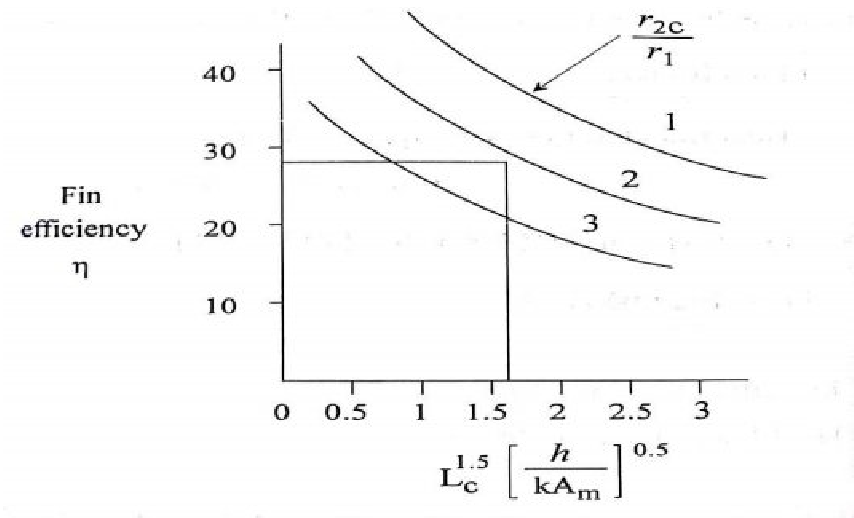
$$= (0.1425)^{1.5}\left[\frac{140}{220 \times 7.125 \times 10^{-4}}\right]^{.05}$$

$$X_{axis} = 1.60$$

Curve $\rightarrow \frac{r_{2c}}{r_1} = \frac{0.2425}{0.1} = 2.425$

X_{axis} *value is* 1.60

Curve value is 2.425



By using these values we can find fin efficiency, η from graph

Fin Efficiency $\eta = 28$ %

Heat transfer $= \eta\, A_s\, h\, (T_b - T_\infty)$ from HMT data book P.No 50

$=0.28 \times 0.30650 \times 140 \times [443 - 298]$

Q =1742.99W

9. A metallic sphere of radius 10mm is initially at a uniform temperature of 400^{0}C. It is heat treated by first cooling it in air (h=10 W/m²k) at 20^{0}C until its central temperature reaches 335^{0}C . It is then quenched in a water bath at 20^{0}C with h=6000 W/m²K until the centre of the sphere cools from 335^{0}C to 50^{0}C. compute the time required for cooling in air and water for the following physical properties of the sphere.

$Density, \rho = 3000 kg/m^3$

$c = 1000 J/kgK$

K= 20 W/mK

$\alpha = 6.66 \times 10^{-6} m^2/s$

Given

$Density, \rho = 3000 kg/m^3$

$c = 1000 J/kgK$ K= 20 W/mK

$\alpha = 6.66 \times 10^{-6} m^2/s$

To find

Surface temperature at end of cooling in water.

Solution

Step-1

i) Cooling in air .

Let us check whether lumped capacity method can be used here

$$B_i = \frac{hr_0}{3k} = \frac{10 \times 0.01}{3 \times 20} = 16.66 \times 10^{-4} \ll 0.1$$

From HMT data book P.No 58

$$\therefore \frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left\{\frac{hA}{\rho c V}\right\}.t\right]$$

$$t = \frac{\rho c V}{hA} \ln \frac{T_0 - T_\infty}{T - T_\infty} = \frac{\rho r_0 c}{3h} \ln \frac{T_0 - T_\infty}{T - T_\infty}$$

$$\boxed{t = 188s}$$

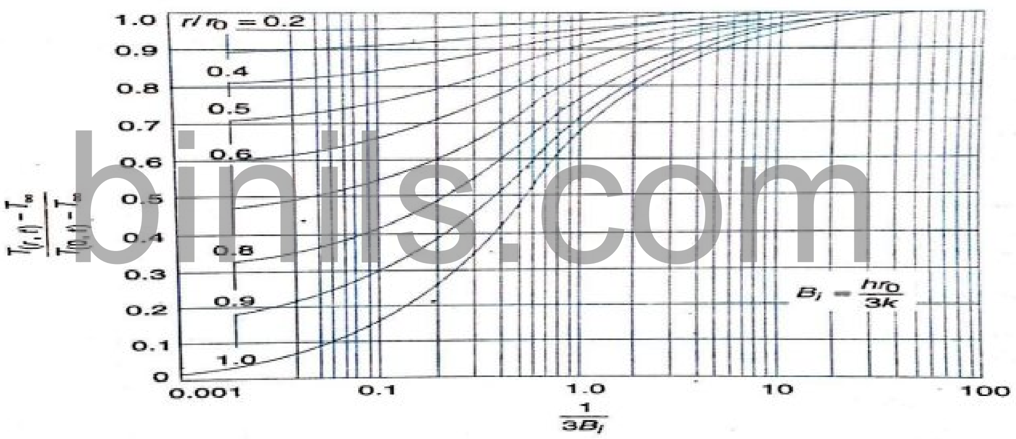
$$= \frac{3000 \times 0.01 \times 1000}{3 \times 10} \ln \frac{400 - 20}{335 - 20}$$

Step-2

ii) Cooling in water

$$B_i(\text{for lumped capacity method}) = \frac{hr_0}{3k} = \frac{6000 \times 0.01}{3 \times 20} = 1.0 > 0.1$$

So the lumped capacity method cannot be employed, but heisler charts can be used



$$\frac{1}{B_i} = \frac{k}{hr_0} = \frac{20}{6000 \times 0.01} = 0.33$$

$$\frac{T_{(0,t)} - T_\infty}{T_0 - T_\infty} = \frac{50 - 20}{335 - 20} = 0.095$$

$$F_o = \frac{\alpha t}{r_0^2} = 0.5$$

$$t = \frac{F_o r_0^2}{\alpha} = \frac{0.5 \times 0.01^2}{6.66 \times 10^{-6}} = 7.5s$$

The surface temperature at the end of quenching in water may be obtained from fig with

$$\frac{1}{3B_i} = 0.33$$

$$\frac{r}{r_0} = 1$$

$$\frac{T_{(r_0)} - T_\infty}{T_{(0,t)} - T_\infty} = 0.33$$

$$T(r_0) = [0.33 \times (50 - 20)] + 20 = 30°C$$

$$\boxed{T(r_0) = 30°C}$$

10. A thermocouple junction is in the form of 8 mm diameter sphere. Properties of material are c=420 J/kg°C, ρ=8000 kg/m³, k=40 W/m°C and h=40 W/m²C. The junction is initially at 40°C and inserted in a stream of hot air at 300°C. Find

i) Time constant of the thermocouple

ii) The thermocouple is taken out from the hot air after 10 seconds and kept in still air at 30°C. Assuming the heat transfer coefficient in air 10W/m²C, find the temperature attained by the junction 20 seconds after removing from hot air.(Nov/Dec 2008)

Given

R=4 mm= 0.004m

C= 420 J/kg°C

ρ=8000 kg/m³

k=40 W/m°C

h=40 W/m²C (gas stream)

h=10 W/m²C (gas air)

To Find

- **i)** Time constant of the thermocouple τ*
- **ii)** The temperature attained by the junction (t)

Solution

Step-1

$$\tau^* = \frac{\rho V C}{h A_s} = \frac{\rho \times \left[\frac{4}{3}\pi R^3\right] \times c}{h \times 4\pi R^2} = \frac{\rho R c}{3h}$$

$$\tau^* = \frac{8000 \times 0.004 \times 420}{3 \times 40} = 112\ s$$

$$\boxed{\tau^* = 112\ s}$$

Step-2

$t_i = 40°C, t_a = 300°C, \tau = 10s$

The temperature variation with respect to time during heating (when dipped in gas stream) is given by

From HMT data book P.No 58

$$\frac{t - t_a}{t_i - T_a} = \exp\left[-\left\{\frac{hA}{\rho cV}\right\}.t\right]$$

$$\frac{t - 300}{40 - 300} = \exp\left[-\left\{\frac{\tau}{\tau^*}\right\}\right] = e^{(10/112)}$$

$$\frac{1}{e^{\left(\frac{10}{112}\right)}} = 0.9146$$

$$t = 300 + 0.9146(40 - 300) = 62.2°C$$

$$\boxed{t = 62.2°C}$$

The temperature variation with respect to time during cooling (when exposed to air) is given by

$$\frac{t - t_a}{t_i - T_a} = e^{\frac{\tau}{\tau^*}}$$

Where

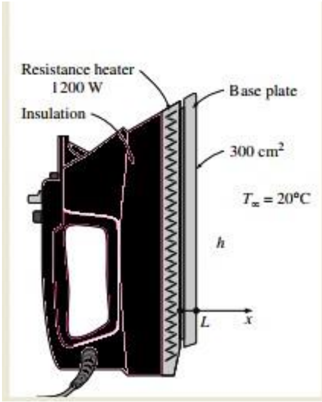
$$\tau^* = \frac{\rho Rc}{3h} = \frac{8000 \times 0.004 \times 420}{3 \times 10} = 448s$$

$$\frac{t - 30}{62.2 - 30} = e^{-\left(\frac{20}{448}\right)}$$

$$t = 30 + 0.9563(62.2 - 30) = 60.79°C$$

$$\boxed{t = 60.79°C}$$

1. Heat Conduction in the Base Plate of an Iron Consider the base plate of a 1200-W household iron that has a thickness of L 0.5 cm, base area of A 300 cm2, and thermal conductivity of k 15 W/m · °C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside, and the outer surface loses heat to the surroundings at T 20°C by convection, as shown in Figure



Taking the convection heat transfer coefficient to be h 80 W/m2 · °C and disregarding heat loss by radiation, obtain an expression for the variation of temperature in the base plate, and evaluate the temperatures at the inner and the outer surfaces.

SOLUTION

The base plate of an iron is considered. The variation of temperature in the plate and the surface temperatures are to be determined.

Assumptions

1 Heat transfer is steady since there is no change with time.

2 Heat transfer is one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides are uniform.

3 Thermal conductivity is constant.

4 There is no heat generation in the medium.

5 Heat transfer by radiation is negligible.

6 The upper part of the iron is well insulated so that the entire heat generated in the resistance wires is transferred to the base plate through its inner surface.

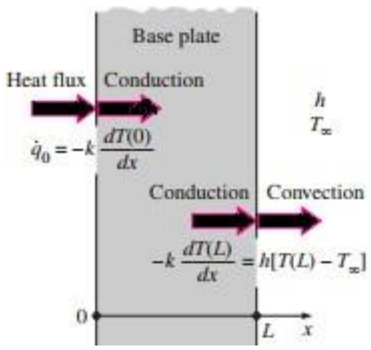
Properties

The thermal conductivity is given to be k 15 W/m · °C.

Analysis The inner surface of the base plate is subjected to uniform heat flux at a rate of

$$q_0 = \frac{Q_o}{A_{base}} = \frac{1200}{0.03} = 40{,}000 \text{ W/m}^2$$

The outer side of the plate is subjected to the convection condition. Taking the direction normal to the surface of the wall as the x-direction with its origin on the inner surface, the differential equation for this problem can be expressed as fig



$$\frac{d^2T}{dx^2} = 0$$

With the boundary conditions

$$-k\frac{dT(0)}{dx} = q_0 = 40000\ W/m^2$$

$$-k\frac{dT(L)}{dx} = h[\,T(L) - T_\infty]$$

The general solution of the differential equation is again obtained by two successive integrations to be

$$\frac{dT}{dx} = C_1$$

And

$T(x) = C_1x + C_2$ ---------- (1)

Where C_1 and C_2 are arbitrary constants. Applying the first boundary condition,

$$-k\frac{dT(0)}{dx} = q_0 \longrightarrow -KC_1 = q_0 \longrightarrow C_1 = -\frac{q_0}{k}$$

$$-k\frac{dT(L)}{dx} = h[\,T(L) - T_\infty] \longrightarrow -KC_1 = h[(C_1L + C_2) - T_\infty$$

Substituting $C_1 = -\frac{q_0}{k}$ and solving for C_2 We obtain

$$C_2 = T_\infty + \frac{q_0}{h} + \frac{q_0}{k} L$$

Now substituting C_1 and C_2 into the general solution (1) gives

$$T(x) = T_\infty + q_0\left(\frac{L-x}{k} + \frac{1}{h}\right) \text{-------(2)}$$

Which is the solution for the variation of the temperature in the plate. The temperatures at the inner and outer surfaces of the plate are determined by substituting x=0 and x=L, respectively, into the relation (2)

$$T(0) = T_\infty + q_0\left(\frac{L}{k} + \frac{1}{h}\right)$$

$$= 20^\circ C + (40000\ W/m^2)\left(\frac{0.005m}{15} + \frac{1}{80}\right) = 533^\circ C$$

And

$$T(L) = +T_\infty + q_0\left(0 + \frac{1}{h}\right) = 20^0 C \frac{40000}{80} = 520^0 C$$

Discussion Note that the temperature of the inner surface of the base plate will be 13° C higher than the temperature of the outer surface when steady operating conditions are reached. Also note that this heat transfer analysis enabels us to calculate the temperatures of surfaces that we cannot even reach. This example demonstrates how the heat flux and convection boundary conditions are applied to heat transfer problems.

2. A person is found dead at 5 PM in a room whose temperature is 20°C. The temperature of the body is measured to be 25°C when found, and the heat transfer coefficient is estimated to be *h* _ 8 W/m2 · °C. Modeling the body as a 30-cm-diameter, 1.70-m-long cylinder, estimate the time of death of that person

SOLUTION A body is found while still warm. The time of death is to be estimated.

Assumptions **1** The body can be modeled as a 30-cm-diameter, 1.70-m-long cylinder. **2** The thermal properties of the body and the heat transfer coefficient are constant. **3** The radiation effects are negligible. **4** The person was healthy(!) when he or she died with a body temperature of 37°C.

Properties The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of (37 + 25)/2 = 31°C; k = 0.617 W/m · °C, ρ = 996 kg/m3, and Cp =4178 J/kg · °C

Analysis The characteristic length of the body is

$$L_C = \frac{V}{A_S} = \frac{\pi r_0^2 L}{2\pi r_0 L + 2\pi r_0^2} = \frac{\pi (0.15)^2 (1.7)}{2\pi(0.15)(1.7) + 2\pi(0.15)^2} = 0.0689$$

Then the biot number becomes

$$B_i = \frac{hL_c}{k} = \frac{8\times0.0689}{0.617} = 0.89 > 0.1$$

Therefore lumped system analysis is not applicable. However, we can still use it to get a rough estimate of the time of death.

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \text{--------(1)}$$

The exponent b in this case is

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{8}{996\times4178\times0.0689} = 2.79\times10^{-5}$$

now substitute these values into equation (1)

$$\frac{25-20}{37-20} = e^{-2.79\times10^{-5}t}$$

t= 43860 s = 12.2 h

The person died about 12 h before the body was found and thus the time of death is 5 AM.