DEPARTMENT OF

MECHANICAL ENGINEERING

ME8594 **DYNAMICS OF MACHINES** **L T P C**
4 0 0 4

OBJECTIVES:

- To understand the force-motion relationship in components subjected to external forces and analysis of standard mechanisms.
- To understand the undesirable effects of unbalances resulting from prescribed motions in mechanism.
- To understand the effect of Dynamics of undesirable vibrations.
- To understand the principles in mechanisms used for speed control and stability control.

UNIT I FORCE ANALYSIS 12

Dynamic force analysis – Inertia force and Inertia torque– D Alembert's principle –Dynamic Analysis in reciprocating engines – Gas forces – Inertia effect of connecting rod– Bearing loads – Crank shaft torque – Turning moment diagrams –Fly Wheels – Flywheels of punching presses- Dynamics of Cam- follower mechanism.

UNIT II BALANCING 12

Static and dynamic balancing – Balancing of rotating masses – Balancing a single cylinder engine – Balancing of Multi-cylinder inline, V-engines – Partial balancing in engines – Balancing of linkages – Balancing machines-Field balancing of discs and rotors.

UNIT III FREE VIBRATION 12

Basic features of vibratory systems – Degrees of freedom – single degree of freedom – Free vibration– Equations of motion – Natural frequency – Types of Damping – Damped vibration– Torsional vibration of shaft – Critical speeds of shafts – Torsional vibration – Two and three rotor torsional systems.

UNIT IV FORCED VIBRATION 12

Response of one degree freedom systems to periodic forcing – Harmonic disturbances – Disturbance caused by unbalance – Support motion –transmissibility – Vibration isolation vibration measurement.

UNIT V MECHANISM FOR CONTROL 12

Governors – Types – Centrifugal governors – Gravity controlled and spring controlled centrifugal governors – Characteristics – Effect of friction – Controlling force curves. Gyroscopes –Gyroscopic forces and torques – Gyroscopic stabilization – Gyroscopic effects in Automobiles, ships and airplanes.

TOTAL : 60 PERIODS

OUTCOMES:

Upon the completion of this course the students will be able to

CO1 Calculate static and dynamic forces of mechanisms.
CO2 Calculate the balancing masses and their locations of reciprocating and rotating masses.
CO3 Compute the frequency of free vibration.
CO4 Compute the frequency of forced vibration and damping coefficient.
CO5 Calculate the speed and lift of the governor and estimate the gyroscopic effect on automobiles, ships and airplanes.

TEXT BOOKS:

1. F. B. Sayyad, "Dynamics of Machinery", McMillan Publishers India Ltd., Tech-Max Educational resources, 2011.
2. Rattan, S.S, "Theory of Machines", 4th Edition, Tata McGraw-Hill, 2014.
3. Uicker, J.J., Pennock G.R and Shigley, J.E., "Theory of Machines and Mechanisms", 4th Edition, Oxford University Press, 2014.

REFERENCES:

1. Cleghorn. W. L, "Mechanisms of Machines", Oxford University Press, 2014
2. Ghosh. A and Mallick, A.K., "Theory of Mechanisms and Machines", 3rd Edit ion Affiliated East-West Pvt. Ltd., New Delhi, 2006.
3. Khurmi, R.S.,"Theory of Machines", 14th Edition, S Chand Publications, 2005.
4. Rao.J.S. and Dukkipati.R.V. "Mechanisms and Machine Theory", Wiley-Eastern Ltd., New Delhi, 1992.
5. Robert L. Norton, "Kinematics and Dynamics of Machinery", Tata McGraw-Hill, 2009.
6. V.Ramamurthi, "Mechanics of Machines", Narosa Publishing House, 2002.

CONTENTS

TOPIC

UNIT-1 FORCE ANALYSIS

UNIT – 2 BALANCING

UNIT – 3 FREE VIBRATION

UNIT – 4 FORCED VIBRATION

UNIT -5 MECHANISMS FOR CONTROL

UNIT – I - FORCE ANALYSIS

1.1 INTRODUCTION

The subject Dynamics of Machines may be defined as that branch of Engineering-science, which deals with the study of relative motion between the various parts of a machine, and forces which act on them. The knowledge of this subject is very essential for an engineer in designing the various parts of a machine.

A machine is a device which receives energy in some available form and utilises it to do some particular type of work.

If the acceleration of moving links in a mechanism is running with considerable amount of linear and/or angular accelerations, inertia forces are generated and these inertia forces also must be overcome by the driving motor as an addition to the forces exerted by the external load or work the mechanism does.

1.2 NEWTON' S LAW :

First Law

Everybody will persist in its state of rest or of uniform motion (constant velocity) in a straight line unless it is compelled to change that state by forces impressed on it. This means that in the absence of a non-zero net force, the center of mass of a body either is at rest or moves at a constant velocity.

Second Law

A body of mass *m* subject to a force **F** undergoes an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force and inversely proportional to the mass, i.e., $\mathbf{F} = m\mathbf{a}$. Alternatively, the total force applied on a body is equal to the time derivative of linear momentum of the body.

Third Law

The mutual forces of action and reaction between two bodies are equal, opposite and collinear. This means that whenever a first body exerts a force **F** on a second body, the second body exerts a force −**F** on the first body. **F** and −**F** are equal in magnitude and opposite in direction. This law is sometimes referred to as the *action-reaction law*, with **F** called the "action" and −**F** the "reaction"

1.3 TYPES OF FORCE ANALYSIS:

- Equilibrium of members with two forces
- Equilibrium of members with three forces
- Equilibrium of members with two forces and torque
- Equilibrium of members with two couples.
- Equilibrium of members with four forces.

1.3.1 Principle of Super Position:

Sometimes the number of external forces and inertial forces acting on a mechanism are too much for graphical solution. In this case we apply the method of superposition. Using superposition the entire system is broken up into (n) problems, where n is the number of forces, by considering the external and inertial forces of each link individually. Response of a linear system to several forces acting simultaneously is equal to the sum of responses of the system to the forces individually. This approach is useful because it can be performed by graphically.

1.3.2 Free Body Diagram:

A free body diagram is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest. A free body diagram shows all forces of all types acting on this body. Drawing such a diagram can aid in solving for the unknown forces or the equations of motion of the body. Creating a free body diagram can make it easier to understand the forces, and torques or moments, in relation to one another and suggest the proper concepts to apply in order to find the solution to a problem. The diagrams are also used as a conceptual device to help identify the internal forces—for example, shear forces and bending moments in beams—which are developed within structures.

1.4 DYNAMIC ANALYSIS OF FOUR BAR MECHANISM:

A **four-bar linkage** or simply a **4-bar** or **four-bar** is the simplest movable linkage. It consists of four rigid bodies (called bars or links), each attached to two others by single joints or pivots to form closed loop. Fourbars are simple mechanisms common in mechanical engineering machine design and fall under the study of kinematics.

- Dynamic Analysis of Reciprocating engines.
- Inertia force and torque analysis by neglecting weight of connecting rod.
- Velocity and acceleration of piston.
- Angular velocity and Angular acceleration of connecting rod.
- Force and Torque Analysis in reciprocating engine neglecting the weight of connecting rod.
- Equivalent Dynamical System
- Determination of two masses of equivalent dynamical system

The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position. It is numerically equal to the accelerating force in magnitude, but ***opposite*** in direction. Mathematically,

Inertia force = – Accelerating force = $-m.a$

where m = Mass of the body, and

a = Linear acceleration of the centre of gravity of the body.

Similarly, the inertia torque is an imaginary torque, which when applied upon the rigid body, brings it in equilib-rium position. It is equal to the accelerating couple in magni-tude but ***opposite*** in direction.

1.4.1 D-Alembert's Principle

Consider a rigid body acted upon by a system of forces. The system may be reduced to a single resultant force acting on the body whose magnitude is given by the product of the mass of the body and the linear acceleration of the centre of mass of the body. According to Newton's second law of motion,

$F = m.a$

F = Resultant force acting on the body, m

= Mass of the body, and

= Linear acceleration of the centre of mass of the

a body.

The equation **(*i*)** may also be written as:

$F - m.a = 0$

A little consideration will show, that if the quantity $- m.a$ be treated as a force, equal, opposite and with the same line of action as the resultant force F, and include this force with the system of forces of which F is the resultant, then the complete system of forces will be in equilibrium. This principle is known as ***DAlembert's principle***. The equal and opposite force $- m.a$ is known as ***reversed effective force*** or the ***inertia force*** (briefly written as F_I). The equation **(*ii*)** may be written as

$F + F_I = 0$...**(*iii*)**

Thus, D-Alembert's principle states that ***the resultant force acting on a body together with the reversed effective force (or inertia force), are in equilibrium.***

This principle is used to reduce a dynamic problem into an equivalent static problem.

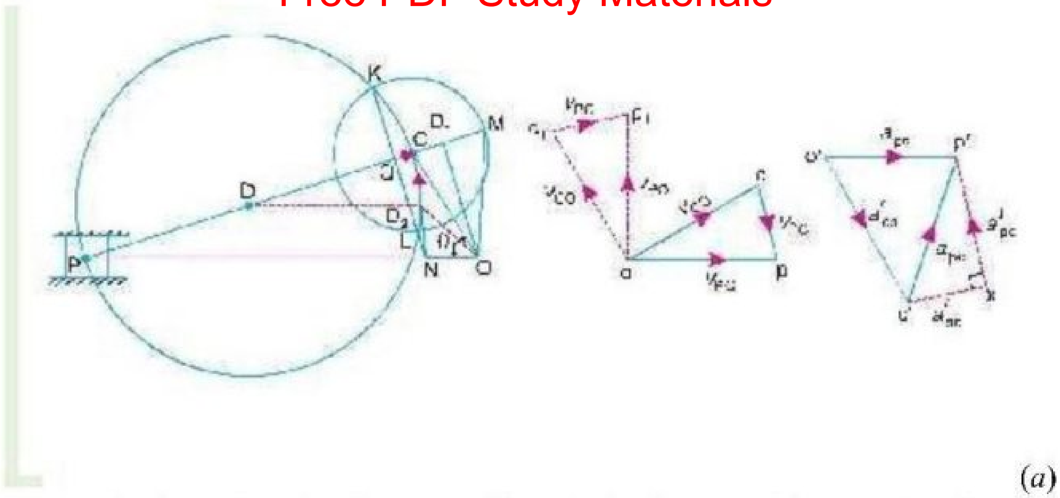
1.4.2 Velocity and Acceleration of the Reciprocating Parts in Engines

The velocity and acceleration of the reciprocating parts of the steam engine or internal combustion engine (briefly called as I.C. engine) may be determined by graphical method or analytical method. The velocity and acceleration, by graphical method, may be determined by one of the following constructions: **1.** Klien's construction, **2.** Ritterhaus's construction, and **3.** Bennett's construction.

We shall now discuss these constructions, in detail, in the following pages.

1.5 KLIEN'S CONSTRUCTION

Let OC be the crank and PC the connecting rod of a reciprocating steam engine, as shown in Fig. 15.2 (*a*). Let the crank makes an angle θ with the line of stroke PO and rotates with uniform angular velocity ω rad/s in a clockwise direction. The Klien's velocity and acceleration diagrams are drawn as discussed below:



(*a*) Klien's acceleration diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 15.2. Klien's construction.

1.5.1 Klien's velocity diagram

First of all, draw *OM* perpendicular to *OP*; such that it intersects the line *PC* produced at *M*. The triangle *OCM* is known as ***Klien's velocity diagram***. In this triangle *OCM*,

OM may be regarded as a line perpendicular to *PO*,

CM may be regarded as a line parallel to *PC*, and ...(Q It is the same line.)

CO may be regarded as a line parallel to *CO*.

We have already discussed that the velocity diagram for given configuration is a triangle *ocp* as shown in Fig. 15.2 (*b*). If this triangle is revolved through 90°, it will be a triangle $oc_1 p_1$, in which oc_1 represents v_{CO} (*i.e.* velocity of *C* with respect to *O* or velocity of crank pin *C*) and is paralel to *OC*, op_1 represents v_{PO} (*i.e.* velocity of *P* with respect to *O* or velocity of cross-head or piston *P*) and is perpendicular to *OP*, and c_1p_1 represents v_{PC} (*i.e.* velocity of *P* with respect to *C*) and is parallel to *CP*.

Thus, we see that by drawing the Klien's velocity diagram, the velocities of various points may be obtained without drawing a separate velocity diagram.

1.5.2 Klien's acceleration diagram

The Klien's acceleration dia-gram is drawn as discussed below:

1. First of all, draw a circle with *C* as centre and *CM* as radius.
2. Draw another circle with *PC* as diameter. Let this circle intersect the previous circle at *K* and *L*.
3. Join *KL* and produce it to intersect *PO* at *N*. Let *KL* intersect *PC* at *Q*.

This forms the quadrilateral *CQNO*, which is known as ***Klien's acceleration diagram.***

We have already discussed that the acceleration diagram for the given configuration is as shown in Fig. 15. 2 (*c*). We know that (*i*) $o'c'$ represents a^r_{CO} (*i.e.* radial component of the acceleration of crank pin C with respect to O) and is parallel to CO; (*ii*) $c'x$ represents a^r_{PC} (*i.e.* radial component of the acceleration of crosshead or piston P with respect to crank pin C) and is parallel to CP or CQ;

(iii) xp' represents a^t_{PC} (*i.e.* tangential component of the acceleration of P with respect to C) and is parallel to QN (because QN is perpendicular to CQ); and

(iv) $o'p'$ represents a_{PO} (*i.e.* acceleration of P with respect to O or the acceleration of piston P) and is parallel to PO or NO.

A little consideration will show that the quadrilateral $o'c'x\,p'$ [Fig. 15.2 (*c*)] is similar to quadrilateral $CQNO$ [Fig. 15.2 (*a*)]. Therefore,

$$\frac{o'c'}{OC} = \frac{c'x}{CQ} = \frac{xp'}{QN} = \frac{o'p'}{NO} = \omega^2 \text{(a constant)}$$

or

$$\frac{a^r_{CO}}{OC} = \frac{a^r_{PC}}{CQ} = \frac{a^t_{PC}}{QN} = \frac{a_{PO}}{NO} = \omega^2$$

$$\therefore \quad a^r_{CO} = \omega^2 \times OC;\ a^r_{PC} = \omega^2 \times CQ$$

$$a^t_{PC} = \omega^2 \times QN;\ \text{and}\ a_{PO} = \omega^2 \times NO$$

Thus we see that by drawing the Klien's acceleration diagram, the acceleration of various points may be obtained without drawing the separate acceleration diagram.

1.6 SOLVED PROBLEMS

1. *The crank and connecting rod of a reciprocating engine are 200 mm and 700 mm respectively. The crank is rotating in clockwise direction at 120 rad/s. Find with the help of Klein's construction:* **1.** *Velocity and acceleration of the piston,* **2.** *Velocity and acceleration of the mid point of the connecting rod, and* **3.** *Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to I.D.C. (inner dead centre).*

Solution. Given: $OC = 200$ mm $= 0.2$ m ; $PC = 700$ mm $= 0.7$ m ; $\omega = 120$ rad/s

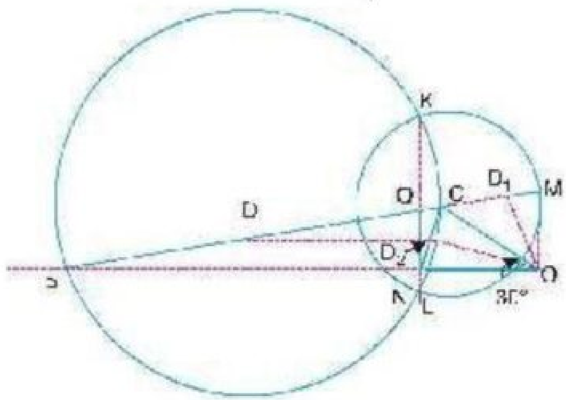


Fig. 15.5

The Klein's velocity diagram *OCM* and Klein's acceleration diagram *CQNO* as shown in Fig. 15.5 is drawn to some suitable scale, in the similar way as discussed in Art. 15.5. By measurement, we find that

$OM = 127$ mm $= 0.127$ m ; $CM = 173$ mm $= 0.173$ m ; $QN = 93$ mm $= 0.093$ m ; $NO = 200$ mm $= 0.2$ m

1. *Velocity and acceleration of the piston*

We know that the velocity of the piston *P*,

$$v_P = \omega \times OM = 120 \times 0.127 = 15.24 \text{ m/s}$$ **Ans.** and

acceleration of the piston *P*,

$$a_P = \omega^2 \times NO = (120)^2 \times 0.2 = 2880 \text{ m/s}^2$$ **Ans.**

2. *Velocity and acceleration of the mid-point of the connecting rod*

In order to find the velocity of the mid-point *D* of the connecting rod, divide *CM* at D_1 in the same ratio as *D* divides *CP*. Since *D* is the mid-point of *CP*, therefore D_1 is the mid-point of *CM*, *i.e.* $CD_1 = D_1M$. Join OD_1. By measurement,

$$OD_1 = 140 \text{ mm} = 0.14 \text{ m}$$

∴ Velocity of *D*, $v_D = \omega \times OD_1 = 120 \times 0.14 = 16.8$ m/s **Ans.**

In order to find the acceleration of the mid-point of the connecting rod, draw a line DD_2 parallel to the line of stroke *PO* which intersects *CN* at D_2. By measurement,

$$OD_2 = 193 \text{ mm} = 0.193 \text{ m}$$

∴ Acceleration of *D*,

$$a_D = \omega^2 \times OD_2 = (120)^2 \times 0.193 = 2779.2 \text{ m/s}^2$$ **Ans.**

3. *Angular velocity and angular acceleration of the connecting rod*

We know that the velocity of the connecting rod *PC* (*i.e.* velocity of *P* with respect to *C*), $v_{PC} = \omega \times CM = 120 \times 0.173 = 20.76$ m/s

∴ Angular acceleration of the connecting rod PC,

$$\omega_{PC} = \frac{v_{PC}}{PC} = \frac{20.76}{0.7} = 29.66 \text{ rad/s} \quad \textbf{Ans.}$$

We know that the tangential component of the acceleration of P with respect to C,

$$a^t_{PC} = \omega^2 \times QN = (120)^2 \times 0.093 = 1339.2 \text{ m/s}^2$$

∴ Angular acceleration of the connecting rod PC,

$$\alpha_{PC} = \frac{a^t_{PC}}{PC} = \frac{1339.2}{0.7} = 1913.14 \text{ rad/s}^2 \quad \textbf{Ans.}$$

1.7 APPROXIMATE ANALYTICAL METHOD FOR VELOCITY AND ACCELERATION OF THE PISTON

Consider the motion of a crank and connecting rod of a reciprocating steam engine as shown in Fig. 15.7. Let OC be the crank and PC the connecting rod. Let the crank rotates with angular velocity of ω rad/s and the crank turns through an angle θ from the inner dead centre (briefly written as I.D.C). Let x be the displacement of a reciprocating body P from I.D.C. after time t seconds, during which the crank has turned through an angle θ .

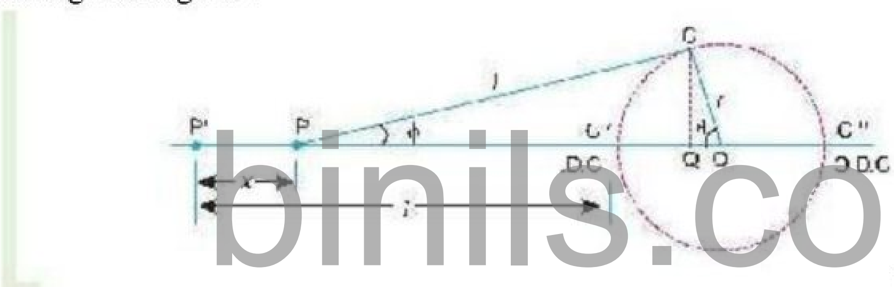


Fig. 15.7. Motion of a crank and connecting rod of a reciprocating steam engine.

Let l = Length of connecting rod between the centres,

r = Radius of crank or crank pin circle,

φ = Inclination of connecting rod to the line of stroke PO, and n = Ratio of length of connecting rod to the radius of crank = l/r.

Velocity of the piston:

From the geometry of Fig. 15.7,

$$x = P'P = OP' - OP = (P'C' + C'O) - (PQ + QO)$$

$$= (l + r) - (l\cos\phi + r\cos\theta) \qquad \left(\because \begin{array}{l} PQ = l\cos\phi, \\ \text{and } QO = r\cos\theta \end{array}\right)$$

$$= r(1 - \cos\theta) + l(1 - \cos\phi) = r\left[(1 - \cos\theta) + \frac{l}{r}(1 - \cos\phi)\right]$$

$$= r\,[(1 - \cos\theta) + n(1 - \cos\phi)] \qquad ...(i)$$

From triangles CPQ and CQO,

$$CQ = l\sin\phi = r\sin\theta \quad \text{or} \quad l/r = \sin\theta/\sin\phi$$

$$\therefore \quad n = \sin\theta/\sin\phi \quad \text{or} \quad \sin\phi = \sin\theta/n \qquad ...(ii)$$

We know that, $\cos\phi = (1 - \sin^2\phi)^{\frac{1}{2}} = \left(1 - \dfrac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}}$

Expanding the above expression by binomial theorem, we get

$$\cos\phi = 1 - \frac{1}{2} \times \frac{\sin^2\theta}{n^2} + ... \qquad \text{...(Neglecting higher terms)}$$

or

$$1 - \cos\phi = \frac{\sin^2\theta}{2n^2} \qquad ...(iii)$$

Substituting the value of $(1 - \cos\phi)$ in equation (i), we have

$$x = r\left[(1 - \cos\theta) + n \times \frac{\sin^2\theta}{2n^2}\right] = r\left[(1 - \cos\theta) + \frac{\sin^2\theta}{2n}\right] \qquad ...(iv)$$

Differentiating equation (iv) with respect to θ,

$$\frac{dx}{d\theta} = r\left[\sin\theta + \frac{1}{2n} \times 2\sin\theta.\cos\theta\right] = r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \qquad ...(v)$$

$$(\because 2\sin\theta.\cos\theta = \sin 2\theta)$$

∴ Velocity of P with respect to O or velocity of the piston P,

$$v_{PO} = v_P = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = \frac{dx}{d\theta} \times \omega$$

...(∵ Ratio of change of angular velocity $= d\theta/dt = \omega$)

Substituting the value of $dx/d\theta$ from equation (v), we have

$$v_{PO} = v_P = \omega r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \qquad ...(vi)$$

Acceleration of the piston:

Since the acceleration is the rate of change of velocity, therefore acceleration of the piston P,

$$a_P = \frac{dv_P}{dt} = \frac{dv_P}{d\theta} \times \frac{d\theta}{dt} = \frac{dv_P}{d\theta} \times \omega$$

Differentiating equation (*vi*) with respect to θ,

$$\frac{dv_P}{d\theta} = \omega . r \left[\cos\theta + \frac{\cos 2\theta \times 2}{2n} \right] = \omega . r \left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

Substituting the value of $\frac{dv_P}{d\theta}$ in the above equation, we have

$$a_P = \omega r \left[\cos\theta + \frac{\cos 2\theta}{n} \right] \times \omega = \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n} \right] \quad ...(vii)$$

1.8 ANGULAR VELOCITY AND ACCELERATION OF THE CONNECTING ROD

Consider the motion of a connecting rod and a crank as shown in Fig. 15.7.From the geometry of the figure, we find that

$$CQ = l \sin\varphi = r \sin\theta$$

$$\therefore \quad \sin\phi = \frac{r}{l} \times \sin\theta = \frac{\sin\theta}{n} \quad ...\left(\because n = \frac{l}{r} \right)$$

Differentiating both sides with respect to time t,

$$\cos\phi \times \frac{d\phi}{dt} = \frac{\cos\theta}{n} \times \frac{d\theta}{dt} = \frac{\cos\theta}{n} \times \omega \quad \left(\because \frac{d\theta}{dt} = \omega \right)$$

Since the angular velocity of the connecting rod PC is same as the angular velocity of point P with respect to C and is equal to $d\phi/dt$, therefore angular velocity of the connecting rod

$$\omega_{PC} = \frac{d\phi}{dt} = \frac{\cos\theta}{n} \times \frac{\omega}{\cos\phi} = \frac{\omega}{n} \times \frac{\cos\theta}{\cos\phi}$$

We know that, $\cos\phi = (1 - \sin^2\phi)^{\frac{1}{2}} = \left(1 - \frac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}} \quad ...\left(\because \sin\phi = \frac{\sin\theta}{n} \right)$

$$\therefore \quad \omega_{PC} = \frac{\omega}{n} \times \frac{\cos\theta}{\left(1 - \frac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}}} = \frac{\omega}{n} \times \frac{\cos\theta}{\frac{1}{n}(n^2 - \sin^2\theta)^{1/2}}$$

$$= \frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{1/2}} \quad ...(i)$$

Angular acceleration of the connecting rod PC,

$$\alpha_{PC} = \text{Angular acceleration of } P \text{ with respect to } C = \frac{d(\omega_{PC})}{dt}$$

We know that

$$\frac{d(\omega_{PC})}{dt} = \frac{d(\omega_{PC})}{d\theta} \times \frac{d\theta}{dt} = \frac{d(\omega_{PC})}{d\theta} \times \omega \qquad \text{...(ii)}$$

$$\text{...}(\because d\theta/dt = \omega)$$

Now differentiating equation (*i*), we get

$$\frac{d(\omega_{PC})}{d\theta} = \frac{d}{d\theta}\left[\frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{1/2}}\right]$$

$$= \omega\left[\frac{(n^2 - \sin^2\theta)^{1/2}(-\sin\theta)] - [(\cos\theta) \times \frac{1}{2}(n^2 - \sin^2\theta)^{-1/2} \times -2\sin\theta\cos\theta}{n^2 - \sin^2 0}\right]$$

$$= \omega\left[\frac{(n^2 - \sin^2\theta)^{1/2}(-\sin\theta) + (n^2 - \sin^2\theta)^{-1/2}\sin\theta\cos^2\theta}{n^2 - \sin^2\theta}\right]$$

$$= -\omega\sin\theta\left[\frac{(n^2 - \sin^2\theta)^{1/2} - (n^2 - \sin^2\theta)^{-1/2}\cos^2\theta}{n^2 - \sin^2\theta}\right]$$

$$= -\omega\sin\theta\left[\frac{(n^2 - \sin^2\theta) - \cos^2\theta}{(n^2 - \sin^2\theta)^{3/2}}\right] \qquad \text{...[Dividing and multiplying by } (n^2 - \sin^2 0)^{1/2}]$$

$$= \frac{-\omega\sin\theta}{(n^2 - \sin^2\theta)^{3/2}}\left[n^2 - (\sin^2\theta + \cos^2\theta)\right] = \frac{-\omega\sin\theta\,(n^2 - 1)}{(n^2 - \sin^2\theta)^{3/2}}$$

$$\text{...}(\because \sin^2\theta + \cos^2\theta = 1)$$

$$\therefore \alpha_{PC} = \frac{d(\omega_{PC})}{d\theta} \times \omega = \frac{-\omega^2 \sin\theta\,(n^2 - 1)}{(n^2 - \sin^2\theta)^{3/2}} \qquad \text{...[From equation (ii)]} \qquad \text{...(iii)}$$

The negative sign shows that the sense of the acceleration of the connecting rod is such that it tends to reduce the angle φ..

2.In a slider crank mechanism, the length of the crank and connecting rod are 150 mm and 600 mm respectively. The crank position is 60° from inner dead centre. The crank shaft speed is 450 r.p.m. (clockwise). Using analytical method, determine: 1. Velocity and acceleration of the slider, and 2. Angular velocity and angular acceleration of the connecting rod.

Solution. Given : $r = 150$ mm $= 0.15$ m ; $l = 600$ mm $= 0.6$ m ; $\theta = 60°$; $N = 400$ r.p.m or $\omega = \pi \times 450/60 = 47.13$ rad/s

1. ***Velocity and acceleration of the slider***

We know that ratio of the length of connecting rod and crank, $n =$

$l / r = 0.6 / 0.15 = 4$

∴ Velocity of the slider,

$$v_P = \omega r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) = 47.13 \times 0.15\left(\sin 60^\circ + \frac{\sin 120^\circ}{2\times 4}\right) \text{m/s}$$

$$= 6.9 \text{ m/s} \quad \textbf{Ans.}$$

and acceleration of the slider

$$a_P = \omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = (47.13)^2 \times 0.15\left(\cos 60^\circ - \frac{\cos 120^\circ}{4}\right) \text{m/s}^2$$

$$= 124.94 \text{ m/s}^2 \quad \textbf{Ans.}$$

2. ***Angular velocity and angular acceleration of the connecting rod***

We know that angular velocity of the connecting rod,

$$\omega_{PC} = \frac{\omega \cos\theta}{n} = \frac{47.13 \times \cos 60^\circ}{4} = 5.9 \text{ rad/s} \quad \textbf{Ans.}$$

and angular acceleration of the connecting rod,

$$\alpha_{PC} = \frac{\omega^2 \sin\theta}{n} = \frac{(47.13)^2 \times \sin 60^\circ}{4} = 481 \text{ rad/s}^2 \quad \textbf{Ans.}$$

1.9 FORCES ON THE RECIPROCATING PARTS OF AN ENGINE, NEGLECTING THE WEIGHT OF THE CONNECTING ROD

The various forces acting on the reciprocating parts of a horizontal engine are shown in Fig. 15.8. The expressions for these forces, neglecting the weight of the connecting rod, may be derived as discussed below :

1. ***Piston effort***. It is the net force acting on the piston or crosshead pin, along the line of stroke. It is denoted by F_P in Fig. 15.8.

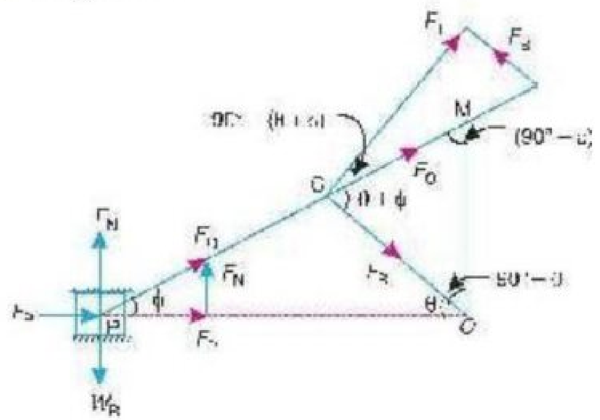


Fig. 15.8. Forces on the reciprocating parts of an engine.

Let m_R = Mass of the reciprocating parts, *e.g.* piston, crosshead pin or gudgeon pin etc., in kg, and

W_R = Weight of the reciprocating parts in newtons = $m_R.g$ We know that acceleration of the reciprocating parts,

$$a_R = a_P = \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

Accelerating force or inertia force of the reciprocating parts,

$$F_I = m_R . a_R = m_R . \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

It may be noted that in a horizontal engine, the reciprocating parts are accelerated from rest, during the latter half of the stroke (*i.e.* when the piston moves from inner dead centre to outer dead centre). It is, then, retarded during the latter half of the stroke (*i.e.* when the piston moves from outer dead centre to inner dead centre). The inertia force due to the acceleration of the reciprocating parts, opposes the force on the piston due to the difference of pressures in the cylinder on the two sides of the piston. On the other hand, the inertia force due to retardation of the reciprocating parts, helps the force on the piston.

Therefore,

Piston effort, F_P = Net load on the piston $\mp$ Inertia force

$= F_L \mp F_I$ ...(Neglecting frictional resistance)

$= F_L \mp F_I - R_F$ ...(Considering frictional resistance)

where R_F = Frictional resistance.

The –ve sign is used when the piston is accelerated, and +ve sign is used when the piston is retarded.

In a double acting reciprocating steam engine, net load on the piston,

$$F_L = p_1 A_1 - p_2 A_2 = p_1 A_1 - p_2 (A_1 - a)$$

where p_1, A_1 = Pressure and cross-sectional area on the back end side of the piston,

p_2, A_2 = Pressure and cross-sectional area on the crank end side of the piston,

a = Cross-sectional area of the piston rod.

2. ***Force acting along the connecting rod.*** It is denoted by F_Q in Fig. 15.8. From the geom-etry of the figure, we find that

$$F_Q = \frac{F_P}{\cos\phi}$$

We know that $\cos\phi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$

$$\therefore \quad F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

3. ***Thrust on the sides of the cylinder walls or normal reaction on the guide bars.*** It is denoted by F_N in Fig. 15.8. From the figure, we find that

$$F_N = F_Q \sin\phi = \frac{F_P}{\cos\phi} \times \sin\phi = F_P \tan\phi \qquad \ldots\left[\because F_Q = \frac{F_P}{\cos\phi}\right]$$

4. ***Crank-pin effort and thrust on crank shaft bearings.*** The force acting on the connecting rod F_Q may be resolved into two components, one perpendicular to the crank and the other along the crank. The component of F_Q perpendicular to the crank is known as ***crank-pin effort*** and it is ***denoted by*** F_T in Fig. 15.8. The component of F_Q along the crank produces a thrust on the crank shaft bearings and it is denoted by F_B in Fig. 15.8. Resolving F_Q perpendicular to the crank,

$$F_T = F_Q \sin(\theta + \phi) = \frac{F_P}{\cos\phi} \times \sin(\theta + \phi)$$

and resolving F_Q along the crank,

$$F_B = F_Q \cos(\theta + \phi) = \frac{F_P}{\cos\phi} \times \cos(\theta + \phi)$$

5. ***Crank effort*** or ***turning moment or torque on the crank shaft.*** The product of the crank-pin effort (F_T) and the crank pin radius (r) is known as ***crank effort*** or ***turning moment*** or ***torque on the crank shaft***. Mathematically, Crank effort $T = F_T \times r = \frac{F_P \sin(\theta + \phi)}{\cos \phi} \times r$

$$= \frac{F_P (\sin\theta \cos\phi + \cos\theta \sin\phi)}{\cos\phi} \times r$$

$$= F_P \left(\sin\theta + \cos\theta \times \frac{\sin\phi}{\cos\phi} \right) \times r$$

$$= F_P (\sin\theta + \cos\theta \tan\phi) \times r \quad ...(i)$$

We know that $l \sin\phi = r \sin\theta$

$$\sin\phi = \frac{r}{l} \sin\theta = \frac{\sin\theta}{n} \quad ...\left(\because n = \frac{l}{r}\right)$$

and $$\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$\therefore$ $$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{n} \times \frac{n}{\sqrt{n^2 - \sin^2\theta}} = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Substituting the value of $\tan\phi$ in equation (*i*), we have crank effort,

$$T = F_P \left(\sin\theta + \frac{\cos\theta \sin\theta}{\sqrt{n^2 - \sin^2\theta}} \right) \times r$$

$$= F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right) \quad ...(ii)$$

$$...(\because 2\cos\theta \sin\theta = \sin 2\theta)$$

3. *The crank-pin circle radius of a horizontal engine is 300 mm. The mass of the reciprocating parts is 250 kg. When the crank has travelled 60° from I.D.C., the difference between the driving and the back pressures is 0.35 N/mm² length between centre the cylinder bore is 0.5 m. If the engine runs at 250 r.p.m. and if the effect of piston rc neglected, calculate : **1.** bars, **2.** thrust in the connecting rod, **3.** tangential force on the crc the crank shaft.*

*The connecting rod pressure on slide pin, and **4.** Turning moment of the shaft*

Solution. Given: $r = 300$ mm $= 0.3$ m ; $m_R = 250$ kg; $\theta = 60°$; $p_1 - p_2 = 0.35$ N/mm^2; $l = 1.2$ m ; $D = 0.5$ m $= 500$ mm ; $N = 250$ r.p.m. or $\omega = 2\pi \times 250/60 = 26.2$ rad/s

First of all, let us find out the piston effort (F_P).

We know that net load on the piston,

$$F_L = (p_1 - p_2)\frac{\pi}{4} \times D^2 = 0.35 \times \frac{\pi}{4}(500)^2 = 68730 \text{ N}$$

$$...(\because \text{Force} = \text{Pressure} \times \text{Area})$$

Ratio of length of connecting rod and crank,

$$n = l / r = 1.2 / 0.3 = 4$$

and accelerating or inertia force on reciprocating parts,

$$F_I = m_R \cdot \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$= 250\,(26.2)^2\ 0.3 \left(\cos 60^\circ + \frac{\cos 120^\circ}{4} \right) = 19\,306\ \text{N}$$

$\therefore$ Piston effort, $F_P = F_L - F_I = 68\,730 - 19\,306 = 49\,424\ \text{N} = 49.424\ \text{kN}$

1. ***Pressure on slide bars***

Let φ = Angle of inclination of the connecting rod to the line of stroke.

We know that, $\sin\varphi = \frac{\sin\theta}{4} = \frac{\sin 60^\circ}{4} = \frac{0.866}{4} = 0.2165\ n$

$\therefore\ \varphi = 12.5^\circ$

We know that pressure on the slide bars,

$F_N = F_P \tan\varphi = 49.424 \times \tan 12.5^\circ = 10.96$ kN **Ans.**

2. ***Thrust in the connecting rod***

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\varphi} = \frac{49.424}{\cos 12.5^\circ} = 50.62\ \text{kN}$$ **Ans.**

3. ***Tangential force on the crank-pin***

We know that tangential force on the crank pin,

$F_T = F_Q \sin(\theta + \varphi) = 50.62 \sin(60^\circ + 12.5^\circ) = 48.28$ kN **Ans.**

4. ***Turning moment on the crank shaft***

We know that turning moment on the crank shaft,

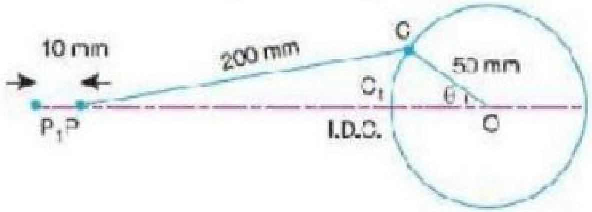
$T = F_T \times r = 48.28 \times 0.3 = 14.484$ kN-m **Ans.**

*4. The crank and connecting rod of a petrol engine, running at 1800 r.p.m.are 50 mm and 200 mm respectively. The diameter of the piston is 80 mm and the mass of the recipr-ing is 1 kg. At a point during the power stroke, the pressure on the piston is 0.7 N/mm², when it 10 mm from the inner dead centre. Determine : **1.** Net load on the gudgeon pin, **2.** Thrust in the connecting rod, **3.** Reaction between the piston and cylinder, and **4.** The engine speed at which the above values become zero.*

Solution. Given : $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.52$ rad/s ; $r = 50$ mm $= 0.05$ m; $l = 200$ mm ; $D = 80$ mm ; $m_R = 1$ kg ; $p = 0.7$ N/mm² ; $x = 10$ mm

1. ***Net load on the gudgeon pin*** We know that load on the piston,

$$F_L = \frac{\pi}{4} D^2 \times p = \frac{\pi}{4} \times (80)^2 \times 0.7 = 3520 \text{ N}$$



When the piston has moved 10 mm from the inner dead centre, *i.e.* when $P_1P = 10$ mm, the crank rotates from OC_1 to OC through an angle θ as shown in Fig. 15.10. By measurement, we find that *θ = 33°.

We know that ratio of lengths of connecting rod and crank,

$$n = l/r = 200/50 = 4$$

and inertia force on the reciprocating parts,

$$F_I = m_R . a_R = m_R . \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$= 1 \times (188.52)^2 \times 0.05 \left(\cos 33° + \frac{\cos 66°}{4} \right) = 1671 \text{ N}$$

We know that net load on the gudgeon pin,

$$F_P = F_L - F_I = 3520 - 1671 = 1849 \text{ N Ans.}$$

2. *Thrust in the connecting rod*

Let φ = Angle of inclination of the connecting rod to the line of stroke.

We know that, $\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 33°}{4} = \frac{0.5446}{4} = 0.1361$

∴ φ = 7.82°

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{1849}{\cos 7.82°} = 1866.3 \text{ N}$$ **Ans.**

3. *Reaction between the piston and cylinder*

We know that reaction between the piston and cylinder,

$$F_N = F_P \tan\phi = 1849 \tan 7.82° = 254 \text{ N}$$ **Ans.**

4. *Engine speed at which the above values will become zero*

A little consideration will show that the above values will become zero, if the inertia force on the reciprocating parts (F_I) is equal to the load on the piston (F_L). Let ω_1 be the speed in rad/s, at which $F_I = F_L$.

$$\therefore \quad m_R (\omega_1)^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right) = \frac{\pi}{4} D^2 \times p$$

$$1 (\omega_1)^2 \times 0.05 \left(\cos 33^\circ + \frac{\cos 66^\circ}{4}\right) = \frac{\pi}{4} \times (80)^2 \times 0.7 \quad \text{or} \quad 0.047 (\omega_1)^2 = 3520$$

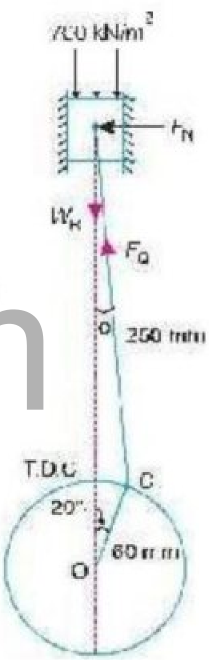
$$\therefore \quad (\omega_1)^2 = 3520 / 0.047 = 74\,894 \text{ or } \omega_1 = 273.6 \text{ rad/s}$$

∴ Corresponding speed in r.p.m.,

$$N_1 = 273.6 \times 60 / 2\pi = 2612 \text{ r.p.m.} \quad \textbf{Ans.}$$

5. *A vertical petrol engine 100 mm diameter and 120 mm stroke has a connecting rod 250 mm long. The mass of the piston is 1.1 kg. The speed is 2000 r.p.m. On the expansion stroke with a crank 20° from top dead centre, the gas pressure is 700 kN/m². Determine:*

***1.** Net force on the piston, **2.** Resultant load on the gudgeon pin, **3.** Thrust on the cylinder walls, and **4.** Speed above which, other things remaining same, the gudgeon pin load would be reversed in direction.*



Solution. Given: $D = 100$ mm $= 0.1$ m ; $L = 120$ mm $= 0.12$ m or $r = L/2 = 0.06$ m ; $l = 250$ mm $= 0.25$ m ; $m_R = 1.1$ kg ; $N = 2000$ r.p.m. or $\omega = 2\pi \times 2000/60 = 209.5$ rad/s ; $\theta = 20°$; $p = 700$ kN/m²

1. ***Net force on the piston***

The configuration diagram of a vertical engine is shown in Fig. 15.11.

We know that force due to gas pressure,

$$F_L = p \times \frac{\pi}{4} \times D^2 = 700 \times \frac{\pi}{4} \times (0.1)^2 = 5.5 \text{ kN}$$
$$= 5500 \text{ N}$$

and ratio of lengths of the connecting rod and crank,

$$n = l/r = 0.25/0.06 = 4.17$$

∴ Inertia force on the piston,

We know that for a vertical engine, net force on the piston,

$$F_P = F_L - F_I + W_R = F_L - F_I + m_R.g$$
$$= 5500 - 3254 + 1.1 \times 9.81 = 2256.8 \text{ N} \quad \textbf{Ans.}$$

2. ***Resultant load on the gudgeon pin***

Let φ = Angle of inclination of the connecting rod to the line of stroke.

We know that,

$$\sin\varphi = \sin\theta / n = \sin 20°/4.17 = 0.082$$

$\therefore \varphi = 4.7°$

We know that resultant load on the gudgeon pin,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{2256.8}{\cos 4.7^\circ} = 2265 \text{ N}$$

3. ***Thrust on the cylinder walls***

We know that thrust on the cylinder walls,

$$F_N = F_P \tan\varphi = 2256.8 \times \tan 4.7^\circ = 185.5 \text{ N} \textbf{ Ans.}$$

4. ***Speed, above which, the gudgeon pin load would be reversed in direction***

Let N_1 = Required speed, in r.p.m.

The gudgeon pin load *i.e.* F_Q will be reversed in direction, if F_Q becomes negative. This is only possible when F_P is negative. Therefore, for F_P to be negative, F_I must be greater than ($F_L + W_R$),

i.e. $$m_R\,(\omega_1)^2\,r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) > 5500 + 1.1 \times 9.81$$

$$1.1 \times (\omega_1)^2 \times 0.06\left(\cos 20^\circ + \frac{\cos 40^\circ}{4.17}\right) > 5510.8$$

$$0.074\,(\omega_1)^2 > 5510.8 \quad \text{or} \quad (\omega_1)^2 > 5510.8/0.074 \text{ or } 74\,470$$

or $$\omega_1 > 273 \text{ rad/s}$$

$\therefore$ Corresponding speed in r.p.m.,

$$N_1 > 273 \times 60/2\pi \quad \text{or} \quad 2606 \text{ r.p.m. } \textbf{Ans.}$$

6. *A horizontal steam engine running at 120 r.p.m. has a bore of 250 mm and a stroke of 400 mm. The connecting rod is 0.6 m and mass of the reciprocating parts is 60 kg. When the crank has turned through an angle of 45° from the inner dead centre, the steam pressure on the cover end side is 550 kN/m² and that on the crank end side is 70 kN/m². Considering the diameter of the piston rod equal to 50 mm, determine:*

1. *turning moment on the crank shaft,* **2.** *thrust on the bearings, and* **3.** *acceleration of the flywheel, if the power of the engine is 20 kW, mass of the flywheel 60 kg and radius of gyration 0.6 m.*

Solution. Given : N = 120 r.p.m. or $\omega = 2\pi \times 120/60 = 12.57$ rad/s ; D = 250 mm = 0.25 m ;
L = 400 mm = 0.4 m or $r = L/2 = 0.2$ m ; l = 0.6 m ; m_R = 60 kg ; $\theta = 45°$; d = 50 mm = 0.05 m ;
$p_1 = 550$ kN/m² $= 550 \times 10^3$ N/m² ; $p_2 = 70$ kN/m² $= 70 \times 10^3$ N/m²

1. ***Turning moment on the crankshaft***

First of all, let us find the net load on the piston (F_P).

We know that area of the piston on the cover end side,

$$A_1 = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times (0.25)^2 = 0.049 \text{ m}^2$$

and area of piston rod, $a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.05)^2 = 0.00196 \text{ m}^2$

∴ Net load on the piston,

$$F_L = p_1.A_1 - p_2.A_2 = p_1.A_1 - p_2 (A_1 - a)$$
$$= 550 \times 10^3 \times 0.049 - 70 \times 10^3 (0.049 - 0.001\,96) = 23\,657 \text{ N}$$

We know that ratio of lengths of the connecting rod and crank,

$$n = l/r = 0.6/0.2 = 3$$

and inertia force on the reciprocating parts,

$$F_I = m_R.\omega^2.r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$
$$= 60 \times (12.57)^2 \times 0.2\left(\cos 45° + \frac{\cos 90°}{3}\right) = 1340 \text{ N}$$

∴ Net force on the piston or piston effort,

$$F_P = F_L - F_I = 23657 - 1340 = 22\,317 \text{ N} = 22.317 \text{ kN}$$

Let ϕ = Angle of inclination of the connecting rod to the line of stroke.

We know that, $\sin\phi = \sin\theta/n = \sin 45°/3 = 0.2357$

∴ $\phi = 13.6°$

We know that turning moment on the crankshaft,

$$T = \frac{F_P \sin(\theta + \phi)}{\cos\phi} \times r = \frac{22.317 \times \sin(45° + 13.6°)}{\cos 13.6°} \times 0.2 \text{ kN-m}$$
$$= 3.92 \text{ kN-m} = 3920 \text{ N-m}$$ **Ans.**

2. ***Thrust on the bearings***

We know that thrust on the bearings,

$$F_B = \frac{F_P \cos(\theta + \phi)}{\cos\phi} = \frac{22.317 \times \cos(45° + 13.6°)}{\cos 13.6°} = 11.96 \text{ kN}$$ **Ans.**

3. ***Acceleration of the flywheel***

Given: $P = 20$ kW $= 20 \times 10^3$ W; $m = 60$ kg ; $k = 0.6$ m Let α = Acceleration of the flywheel in rad/s^2.

We know that mass moment of inertia of the flywheel,

$$I = m.k^2 = 60 \times (0.6)^2 = 21.6 \text{ kg-m}^2$$

∴ Accelerating torque, $T_A = I.\alpha = 21.6\,\alpha$ N-m ...(*i*) and resisting torque, $T_R = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1591$ N-m $\quad \left(\because P = \frac{2\pi NT}{60}\right)$

Since the accelerating torque is equal to the difference of torques on the crankshaft or turning moment (T) and the resisting torque (T_R), therefore, accelerating torque,

$$T_A = T - T_R = 3920 - 1591 = 2329 \text{ N-m} \quad ...(ii)$$

From equation (*i*) and (*ii*),

$$\alpha = 2329/21.6 = 107.8 \text{ rad/s}^2 \quad \textbf{Ans.}$$

1.10 EQUIVALENT DYNAMICAL SYSTEM

In order to determine the motion of a rigid body, under the action of external forces, it is usually convenient to replace the rigid body by two masses placed at a fixed distance apart, in such a way that,

1. the sum of their masses is equal to the total mass of the body ;
2. the centre of gravity of the two masses coincides with that of the body ; and
3. the sum of mass moment of inertia of the masses about their centre of gravity is equal to the mass moment of inertia of the body.

When these three conditions are satisfied, then it is said to be an ***equivalent dynamical system.*** Consider a rigid body, having its centre of gravity at G, as shown in Fig. 15.14.

Let m = Mass of the body,

kG = Radius of gyration about its centre of gravity G, $m1$ and $m2$ =

Two masses which form a dynamical equivalent system, $l1$ =

Distance of mass $m1$ from G, $l2$ = Distance of mass $m2$ from G,

$$m_1 + m_2 = m \quad ...(i)$$

$$m_1.l_1 = m_2.l_2 \quad ...(ii)$$

$$m_1 (l_1)^2 + m_2 (l_2)^2 = m (k_G)^2 \quad ...(iii)$$

From equations (*i*) and (*ii*),

$$m_1 = \frac{l_2.m}{l_1 + l_2} \quad ...(iv)$$

and

$$m_2 = \frac{l_1.m}{l_1 + l_2} \quad ...(v)$$

Substituting the value of m_1 and m_2 in equation (*iii*), we have

$$\frac{l_2.m}{l_1 + l_2}(l_1)^2 + \frac{l_1.m}{l_1 + l_2}(l_2)^2 = m (k_G)^2 \quad \text{or} \quad \frac{l_1.l_2(l_1 + l_2)}{l_1 + l_2} = (k_G)^2$$

$$\therefore \quad l_1.l_2 = (k_G)^2 \quad ...(vi)$$

This equation gives the essential condition of placing the two masses, so that the system becomes dynamical equivalent. The distance of one of the masses (*i.e.* either $l1$ or $l2$) is arbitrary chosen and the other distance is obtained from equation **(*vi*).**

7 *A connecting rod is suspended from a point 25 mm above the centre of small end, and 650 mm above its centre of gravity, its mass being 37.5 kg. When permitted to oscil- late, the time period is found to be 1.87 seconds. Find the dynamical equivalent system constituted of two masses, one of which is located at the small end centre.*

Solution. Given : $h = 650$ mm $= 0.65$ m ; $l_1 = 650 - 25 = 625$ mm

$= 0.625$ m ; $m = 37.5$ kg ; $t_p = 1.87$ s

First of all, let us find the radius of gyration (k_G) of the connect- ing rod (considering it is a compound pendulum), about an axis passing through its centre of gravity, *G*.

We know that for a compound pendulum, time period of oscillation (t_p),

$$1.87 = 2\pi\sqrt{\frac{(k_G)^2 + h^2}{g.h}} \quad \text{or} \quad \frac{1.87}{2\pi} = \sqrt{\frac{(k_G)^2 + (0.65)^2}{9.81 \times 0.65}}$$

Squaring both sides, we have

$$0.0885 = \frac{(k_G)^2 + 0.4225}{6.38}$$

$$(k_G)^2 = 0.0885 \times 6.38 - 0.4225 = 0.1425 \text{ m}^2$$

$$\therefore \quad k_G = 0.377 \text{ m}$$

It is given that one of the masses is located at the small end centre. Let the other mass is located at a distance l_2 from the centre of gravity *G*, as shown in Fig. 15.19. We know that, for adynamicallyequivalentsystem,

$$l_1.l_2 = (k_G)^2$$

$$\therefore \quad l_2 = \frac{(k_G)^2}{l_1} = \frac{0.1425}{0.625} = 0.228 \text{ m}$$

Let m_1 = Mass placed at the small end centre *A*, and

m_2 = Mass placed at a distance l_2 from *G*, *i.e.* at *B*.

We know that, for a dynamically equivalent system,

$$m_1 = \frac{l_2.m}{l_1 + l_2} = \frac{0.228 \times 37.5}{0.625 + 0.228} = 10 \text{ kg} \quad \textbf{Ans.}$$

and

$$m_2 = \frac{l_1.m}{l_1 + l_2} = \frac{0.625 \times 37.5}{0.625 + 0.228} = 27.5 \text{ kg} \quad \textbf{Ans.}$$

1.11 CORRECTION COUPLE TO BE APPLIED TO MAKE TWO MASS SYSTEM DYNAMICALLY EQUIVALENT

In Art. 15.11, we have discussed the conditions for equivalent dynamical system of two bodies. A little consideration will show that when two masses are placed arbitrarily*, then the conditions (*i*) and (*ii*) as given in Art. 15.11 will only be satisfied. But the condition (*iii*) is not possible to satisfy. This means that the mass moment of inertia of these two masses placed arbitrarily, will differ than that of mass moment of inertia of the rigid body.

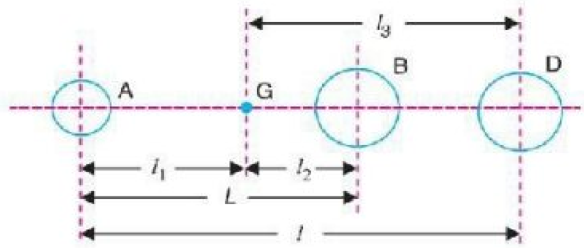


Fig. 15.21. Correction couple to be applied to make the two-mass system dynamically equivalent.

Consider two masses, one at *A* and the other at*D* be placed arbitrarily, as shown in Fig. 15.21. Let l_3 = Distance of mass placed at *D* from *G*, I_1 = New mass moment of inertia of the two masses; k_1 = New radius of gyration;

 = Angular acceleration of the body;

I = Mass moment of inertia of a dynamically equivalent system; k_G = Radius of gyration of a dynamically equivalent system. We know that the torque required to accelerate the body,

$$T = I. = m (k_G)^2 \quad ...(i)$$

Similarly, the torque required to accelerate the two-mass system placed arbitrarily,

$$T_1 = I_1. = m (k_1)^2 \quad ...(ii)$$

 Difference between the torques required to accelerate the two-mass system and the torque required to accelerate the rigid body,

$$T' = T_1 - T = m (k_1)^2 - m (k_G)^2 = m [(k_1)^2 - (k_G)^2] \quad ...(iii)$$

The difference of the torques *T'* is known as ***correction couple***. This couple must be applied, when the masses are placed arbitrarilyto make the system dynamical equivalent. This, of course, will satisfy the condition (*iii*)

8.A connecting rod of an I.C. engine has a mass of 2 kg and the distance between the centre of gudgeon pin and centre of crank pin is 250 mm. The C.G. falls at a point 100 mm from the gudgeon pin along the line of centres. The radius of gyration about an axis through the C.G. perpendicular to the plane of rotation is 110 mm. Find the equivalent dynamical system if only one of the masses is located at gudgeon pin.

If the connecting rod is replaced by two masses, one at the gudgeon pin and the other at the crank pin and the angular acceleration of the rod is 23 000 rad/s^2 clockwise, determine the correc- tion couple applied to the system to reduce it to a dynamically equivalent system.

Solution. Given :m = 2kg ;l =250 mm =0.25 m ;$l1$ =100 mm= 0.1m ; kG = 110mm= 0.11m ;

 = 23 000 rad/s2

Equivalent dynamical system

It is given that one of the masses is located at the gudgeon pin. Let the other mass be located at a distance $l2$ from the centre of gravity. We know that for an equivalent dynamical system.

$$l_1 . l_2 = (k_G)^2 \quad \text{or} \quad l_2 = \frac{(k_G)^2}{l_1} = \frac{(0.11)^2}{0.1} = 0.121 \text{ m}$$

Let m_1 = Mass placed at the gudgeon pin, and

m_2 = Mass placed at a distance l_2 from $C.G.$

We know that $$m_1 = \frac{l_2 . m}{l_1 + l_2} = \frac{0.121 \times 2}{0.1 + 0.121} = 1.1 \text{ kg} \quad \textbf{Ans.}$$

and $$m_2 = \frac{l_1 . m}{l_1 + l_2} = \frac{0.1 \times 2}{0.1 + 0.121} = 0.9 \text{ kg} \quad \textbf{Ans.}$$

Correction couple

Since the connecting rod is replaced by two masses located at the two centres (*i.e.* one at the gudgeon pin and the other at the crank pin), therefore,

$$l = 0.1 \text{ m, and } l3 = l - l1 = 0.25 - 0.1 = 0.15 \text{ m}$$

Let $k1$ = New radius of gyration.

We know that $(k1)^2 = l1.l3 = 0.1 \times 0.15 = 0.015 \text{ m}^2$

 Correction couple,

$$T' = m(k_1^2 - k_G^2)\,\alpha = 2\left[0.015 - (0.11)^2\right] 23\,000 = 133.4 \text{ N-m} \quad \textbf{Ans.}$$

1.12 INERTIA FORCES IN A RECIPROCATING ENGINE, CONSIDERING THE WEIGHT OFCONNECTING ROD

In a reciprocating engine, let OC be the crank and PC, the connecting rod whose centre of gravity lies at G. The inertia forces in a reciprocating engine may be obtained graphicallyas discussed below:

1. First of all, draw the acceleration diagram $OCQN$ by Klien's construction. We know that the acceleration of the piston P with respect to O,

$$aPO = aP = \square^2 \times NO,$$

acting in the direction from N to O. Therefore, the inertia force FI of the reciprocating parts will act in the opposite direction as shown in Fig. 15.22.

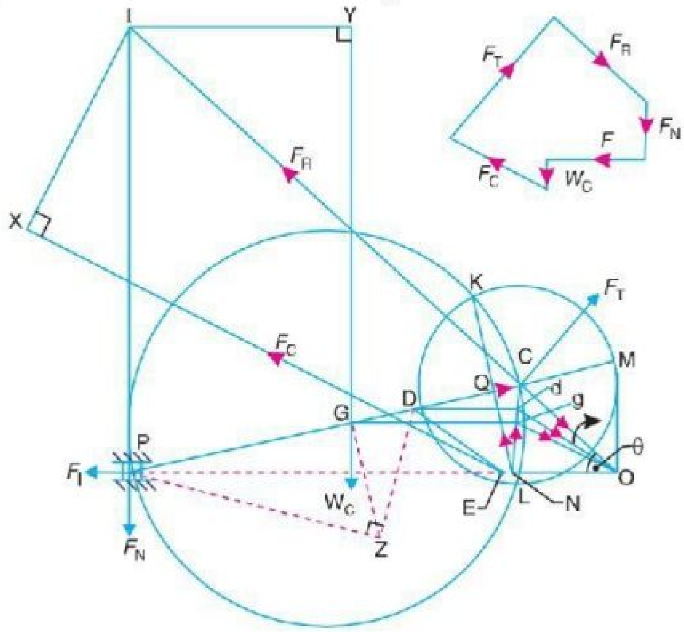


Fig. 15.22. Inertia forces is reciprocating engine, considering the weight of connecting rod.

2. Replace the connecting rod by dynamically equivalent system of two masses as discussed in Art. 15.12. Let one of the masses be arbitrarily placed at P. To obtain the position of the other mass, draw GZ perpendicular to CP such that $GZ = k$, the radius of gyration of the connecting rod. Join PZ and from Z draw perpendicular to DZ which intersects CP at D. Now, D is the position of the second mass.

Note: The position of the second mass may also be obtained from the equation,

$$GP \times GD = k2$$

3. Locate the points G and D on NC which is the acceleration image of the connecting rod. This is done by drawing parallel lines from G and D to the line of stroke PO. Let these parallel lines intersect NC at g and d respectively. Join gO and dO. Therefore, acceleration of G with respect to O, in the direction from g to O,

$$= a \quad = \square^2 \times \quad aGO \quad G \quad gO$$

and acceleration of D with respect to O, in the direction from d to O,

$$= a \quad = \square^2 \times \quad aDO \quad D \quad dO$$

4. From D, draw DE parallel to dO which intersects the line of stroke PO at E. Since the accelerating forces on the masses at P and D intersect at E, therefore their resultant must also pass through E. But their resultant is equal to the accelerang force on the rod, so that the line of action of the accelerating force on the rod, is given by a line drawn through E and parallel to gO, in the direc- tion from g to O. The inertia force of the connecting rod F_C therefore acts through E and in the opposite direction as shown in Fig. 15.22. The inertia force of the connecting rod is given by

$$F_C = m_C \times □^2 \times gO \qquad ...(i)$$

where m_C = Mass of the connecting rod.

A little consideration will show that the forces acting on the connecting rod are :

(a) Inertia force of the reciprocating parts (F_I) acting along the line of stroke PO,

(b) The side thrust between the crosshead and the guide bars (F_N) acting at P and right angles to line of stroke PO,

(c) Theweightoftheconnectingrod

$$(W_C = m_C.g),$$

(d) Inertia force of the connecting rod (F_C),

(e) The radial force (F_R) acting through O and parallel to the crank OC,

(f) The force (F_T) acting perpen- dicular to the crank OC.

Now, produce the lines of action of F_R and F_N to intersect at a point I, known as instantaneous centre. From I draw $I\,X$ and $I\,Y$, perpendicular to the lines of action of F_C and W_C. Taking moments about I, we have

$$F_T \times IC = F_I \times IP + F_C \times I\,X + W_C \times I\,Y \qquad ...(ii)$$

The value of F_T may be obtained from this equation and from the force polygon as shown in Fig. 15.22, the forces F_N and F_R may be calculated. We know that, torque exerted on the crankshaft to overcome the inertia of the moving parts = $F_T \times OC$

1.12.1 Analytical Method for Inertia Torque

The effect of the inertia of the connecting rod on the crankshaft torque may be obtained as discussed in the following steps:

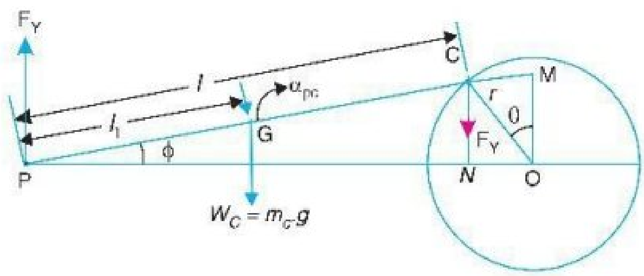


Fig. 15.23. Analytical method for inertia torque.

1. The mass of the connecting rod (m_C) is divided into two masses. One of the mass is placed at the crosshead pin *P* and the other at the crankpin *C* as shown in Fig. 15.23, so that the centre of gravity of these two masses coincides with the centre of gravity of the rod *G*.

2. Since the inertia force due to the mass at *C* acts radially outwards along the crank *OC*, therefore the mass at *C* has no effect on the crankshaft torque.

3. The inertia force of the mass at *P* may be obtained as

follows: Let m_C = Mass of the

connecting rod, l = Length of the connecting rod, l_1 = Length of the centre of gravity of the connecting rod from *P*.

∴ Mass of the connecting rod at *P*,

$$= \frac{l - l_1}{l} \times m_C$$

The mass of the reciprocating parts (m_R) is also acting at *P*. Therefore,

Total equivalent mass of the reciprocating parts acting at *P*

$$= m_R + \frac{l - l_1}{l} \times m_C$$

∴ Total inertia force of the equivalent mass acting at *P*,

$$F_I = \left(m_R + \frac{l - l_1}{l} \times m_C \right) a_R \qquad \text{...(i)}$$

where a_R = Acceleration of the reciprocating parts

$$= \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$\therefore \quad F_I = \left[m_R + \frac{l - l_1}{l} \times m_C \right] \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

and corresponding torque exerted on the crank shaft,

$$T_I = F_I \times OM = F_I r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right) \qquad \text{...(ii)}$$

4. In deriving the equation ***(ii)*** of the torque exerted on the crankshaft, it is assumed that one of the two masses is placed at *C* and the other at *P*. This assumption does not satisfy the condition for kinetically equivalent system of a rigid bar. Hence to compensate for it, a correcting torque is neces- sary whose value is given by

$$T' = m_C \left[(k_1)^2 - (k_G)^2 \right] \alpha_{PC} = m_C . l_1 \ (l - L) \ \alpha_{PC}$$

where L = Equivalent length of a simple pendulum when swung about an axis through P

$$= \frac{(k_G)^2 + (l_1)^2}{l_1}$$

α_{PC} = Angular acceleration of the connecting rod PC

$$= \frac{-\omega^2 \sin \theta}{n}$$ ...(From Art. 15.9)

The correcting torque T' may be applied to the system by two equal and opposite forces FY acting through P and C. Therefore,

$$F_Y \times PN = T' \quad \text{or} \quad F_Y = T'/PN$$

and corresponding torque on the crankshaft,

$$T_C = F_Y \times NO = \frac{T'}{PN} \times NO \qquad \text{...}(iii)$$

We know that, $NO = OC \cos\theta = r\cos\theta$

and $PN = PC\cos\phi = l\cos\phi$

$$\therefore \quad \frac{NO}{PN} = \frac{r\cos\theta}{l\cos\phi} = \frac{\cos\theta}{n\cos\phi} \qquad \text{...}\left(\because n = \frac{l}{r}\right)$$

$$= \frac{\cos\theta}{n\sqrt{1 - \frac{\sin^2\theta}{n^2}}} = \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}} \qquad \left(\because \cos\phi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}\right)$$

Since $\sin^2\theta$ is very small as compared to n^2, therefore neglecting $\sin^2\theta$, we have

$$\frac{NO}{PN} = \frac{\cos\theta}{n}$$

Substituting this value in equation (*iii*), we have

$$T_C = T' \times \frac{\cos\theta}{n} = m_C \times l_1 (l - L)\,\alpha_{PC} \times \frac{\cos\theta}{n}$$

$$= -m_C \times l_1 (l - L) \frac{\omega^2 \sin\theta}{n} \times \frac{\cos\theta}{n} \qquad \text{...}\left(\because \alpha_{PC} = \frac{-\omega^2 \sin\theta}{n}\right)$$

$$= -m_C \times l_1 (l - L) \frac{\omega^2 \sin 2\theta}{2n^2} \qquad \text{...}(\because 2\sin\theta\cos\theta = \sin 2\theta)$$

5. The equivalent mass of the rod acting at *C*,

$$m_2 = m_C \times \frac{l_1}{l}$$

$\therefore$ Torque exerted on the crank shaft due to mass m_2,

$$T_W = -m_2 \times g \times NO = -m_C \times g \times \frac{l_1}{l} \times NO = -m_C \times g \times \frac{l_1}{l} \times r\cos\theta$$

$$\text{...}(\because NO = r\cos\theta)$$

$$= -m_C \times g \times \frac{l_1}{n} \times \cos\theta \qquad \text{...}(\because l/r = n)$$

9. *The crank and connecting rod lengths of an engine are 125 mm and 500 mm respectively. The mass of the connecting rod is 60 kg and its centre of gravity is 275 mm from the crosshead pin centre, the radius of gyration about centre of gravity being 150 mm.*

If the engine speed is 600 r.p.m. for a crank position of 45° from the inner dead centre, determine, using Klien's or any other construction **1.** *the acceleration of the piston;* **2.** *the magni- tude, position and direction of inertia force due to the mass of the connecting rod.*

Solution. Given : $r = OC = 125$ mm ; $l = PC = 500$ mm; $m_C = 60$ kg ; $PG = 275$ mm ;

$m_C = 60$ kg ; $PG = 275$ mm ; $k_G = 150$ mm ; $N = 600$ r.p.m. or = 2 × 600/60 = 62.84 rad/s ; = 45°

1. ***Acceleration of the piston***

Let a_P = Acceleration of the piston.

First of all, draw the configuration diagram *OCP*, as shown in Fig. 15.24, to some suitable scale, such that

$OC = r = 125$ mm ; $PC = l = 500$ mm ; and = 45°.

Now, draw the Klien's acceleration diagram *OCQN,* as shown in Fig. 15.24, in the same manner as already discussed. By measurement,

$NO = 90$ mm = 0.09 m

 Acceleration of the piston, a_P = 2 × NO = (62.84)2 × 0.09 = 355.4 m/s **Ans.**

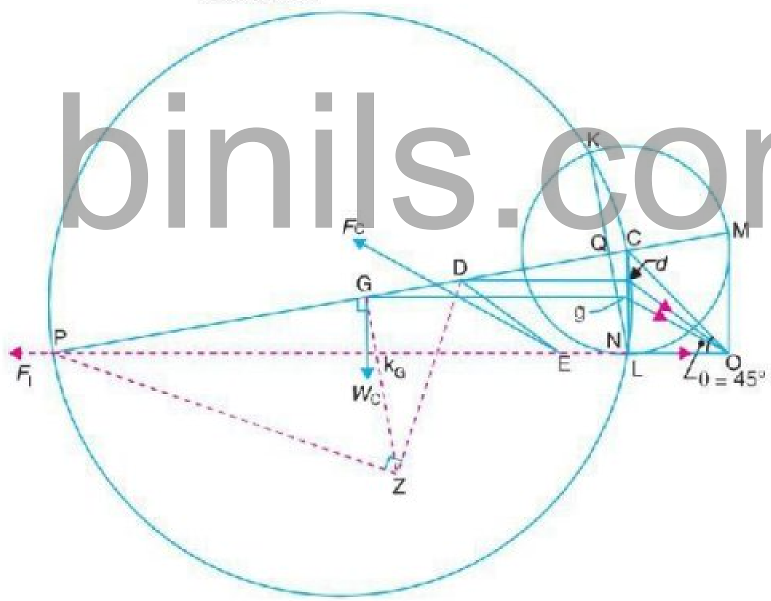


Fig. 15.24

2. ***The magnitude, position and direction of inertia force due to the mass of the connecting rod***

The magnitude, postition and direction of the inertia force may be obtained as follows:

(i) Replace the connecting rod by dynamical equivalent system of two masses, assuming that one of the masses is placed at *P* and the other mass at *D*. The position of the point *D* is obtained as discussed in Art. 15.12.

(ii) Locate the points *G* and *D* on *NC* which is the acceleration image of the connecting rod. Let these points are *g* and *d* on *NC*. Join *gO* and *dO*. By measurement,

$gO = 103$ mm = 0.103 m

 Acceleration of G, $a_G = \omega^2 \times gO$, acting in the direction from g to O.

(iii) From point D, draw DE parallel to dO. Now E is the point through which the inertia force of the connecting rod passes. The magnitude of the inertia force of the connecting rod is given by

$$F_C = m_C \times \omega^2 \times gO = 60 \times (62.84)^2 \times 0.103 = 24\ 400\ \text{N} = 24.4$$

kN **Ans.** **(*iv*)** From point E, draw a line parallel to gO, which shows the position of the inertia force of

the connecting rod and acts in the opposite direction of gO.

10. *The following data refer to a steam engine:*

Diameter of piston = 240 mm; stroke = 600 mm ; length of connecting rod = 1.5 m ; mass of reciprocating parts = 300 kg; mass of connecting rod = 250 kg; speed = 125 r.p.m ; centre of gravity of connecting rod from crank pin = 500 mm ; radius of gyration of the connecting rod about an axis through the centre of gravity = 650 mm.

Determine the magnitude and direction of the torque exerted on the crankshaft when the crank has turned through 30° from inner dead centre.

Solution. Given : D = 240 mm = 0.24 m ; L = 600 mm or $r = L/2$ = 300 mm = 0.3 m ; l = 1.5 m ; m_R = 300 kg ; m_C = 250 kg ; N = 125 r.p.m. or $\omega = 2\pi \times$

125/60 = 13.1 rad/s ; GC = 500 mm = 0.5 m ; k_G = 650 mm = 0.65 m ; θ = 30°

The inertia torque on the crankshaft may be determined by graphical method or analytical method as discussed below:

1. *Graphical method*

First of all, draw the configuration diagram OCP, as shown in Fig. 15.25, to some suitable scale, such that

$$OC = r = 300 \text{ mm} ;\ PC = l = 1.5 \text{ m} ; \text{ and angle } POC = \theta = 30°.$$

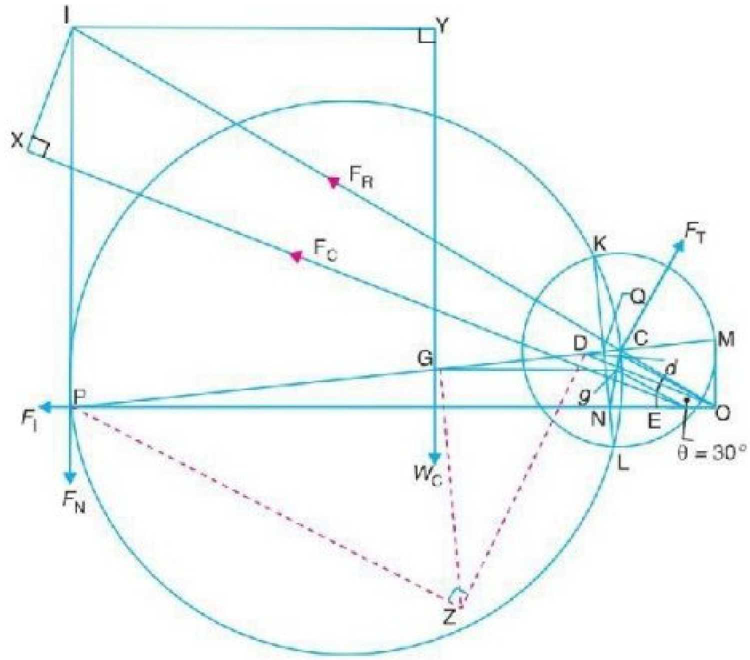


Fig. 15.25

Now draw the Klien's acceleration diagram *OCQN*, as shown in Fig. 15.25, and complete the figure in the similar manner as discussed in Art. 15.14.

By measurement; *NO* = 0.28 m ; *gO* = 0.28 m ; *IP* = 1.03 m ; *I X* = 0.38 m ; *I Y* = 0.98 m, and *IC* = 1.7 m.

We know that inertia force of reciprocating parts,

$$F_I = m_R \times \omega^2 \times NO = 300 \times (13.1)^2 \times 0.28 = 14\ 415 \text{ N}$$

and inertia force of connecting rod,

$$F_C = m_C \times \omega^2 \times gO = 250 \times (13.1)^2 \times 0.28 = 12\ 013 \text{ N}$$

Let F_T = Force acting perpendicular to the crank *OC*.

Taking moments about point *I*.

$$F_T \times IC = F_I \times IP + W_C \times IY + F_C \times IX$$

$$F_T \times 1.7 - 14\ 415 \times 1.03 + 250 \times 9.81 \times 0.98 + 12013 \times 0.38 - 21816$$

$$\therefore \quad F_T = 2.816/1.7 = 12\ 833 \text{ N} \quad \ldots(\because W_C = m_C.g)$$

We know that torque exerted on the crankshaft

$$= F_T \times r = 12\ 833 \times 0.3 = 3850 \text{ N-m } \textbf{Ans.}$$

11. *The connecting rod of an internal combustion engine is 225 mm long and has a mass 1.6 kg. The mass of the piston and gudgeon pin is 2.4 kg and the stroke is 150 mm. The cylinder bore is 112.5 mm. The centre of gravity of the connection rod is 150 mm from the small end. Its radius of gyration about the centre of gravity for oscillations in the plane of* *swing of the connect- ing rod is 87.5 mm. Determine the magnitude and direction of the resultant force on the crank pin when the crank is at 40° and the piston is moving away from inner dead centre under an effective gas presure of 1.8 MN/m^2. The engine speed is 1200 r.p.m.*

Solution. Given : *l* = *PC* = 225 mm = 0.225 m; *m*C = 1.6 kg; *m* R = 2.4 kg; *L* = 150 mm or *r* = *L*/2 = 75 mm = 0.075 m ; *D* = 112.5 mm = 0.1125 m ; *PG* = 150 mm ; *k*G = 87.5 mm = 0.0875 m ; □ = 40° ; *p* = 1.8 MN/m2 = 1.8 × 106 N/m2 ; *N* = 1200 r.p.m. or □ = 2□ × 1200/60 = 125.7 rad/s

First of all, draw the configuration diagram *OCP*, as shown in Fig. 15.27 to some suitable scale, such that *OC* = *r* = 75 mm ; *PC* = *l* = 225 mm ; and □ = 40°.

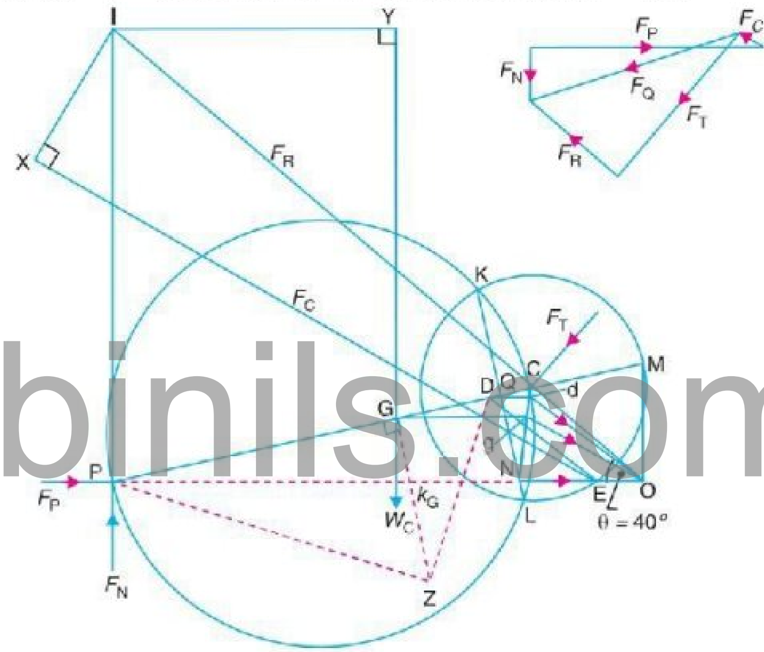


Fig. 15.27

Now, draw the Klien's acceleration diagram*OCQN*. Complete the diagram in the same manner as discussed earlier. By measurement,

NO = 0.0625 m ; *gO* = 0.0685 m ; *IC* = 0.29 m ; *IP* = 0.24 m ; *IY* = 0.148 m ; and *IX* = 0.08 m

We know that force due to gas pressure,

$$F_L - \frac{\pi}{4} \times D^2 \times p - \frac{\pi}{4} \times (0.1125)^2 \times 1.8 \times 10^6 - 17\ 895 \text{ N}$$

Inertia force due to mass of the reciprocating parts,

$$F_I = m_R \times \omega^2 \times NO = 2.4\ (125.7)^2 \times 0.0625 = 2370 \text{ N}$$

∴ Net force on the piston,

$$F_P - F_L - F_I - 17\ 895 - 2370 - 15\ 525 \text{ N}$$

Inertia force due to mass of the connecting rod,

$$F_C = m_C \times \omega^2 \times gO = 1.6 \times (125.7)^2 \times 0.0685 = 1732 \text{ N}$$

Let F_T = Force acting perpendicular to the crank OC.

Now, taking moments about point I,

$$F_P \times IP = W_C \times IY + F_C \times IX + F_T \times IC$$

$$15\,525 \times 0.24 = 1.6 \times 9.81 \times 0.148 + 1732 \times 0.08 + F_T \times 0.29$$

$$\therefore \quad F_T = 12\,362 \text{ N} \qquad \ldots (\because W_C = m_C . g)$$

Let us now find the values of FN and FR in magnitude and direction. Draw the force polygon as shown in Fig. 15.25.

By measurement, FN = 3550 N; and FR = 7550 N

The magnitude and direction of the resultant force on the crank pin is given by FQ , which is the resultant of FR and FT.

By measurement, FQ = 13 750 N **Ans.**

1.13 TURNING MOMENT DIAGRAM

The turning moment diagram (also known as ***crank-effort diagram***) is the graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa

1.13.1 Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. 16.1. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.

We have discussed in Chapter 15 (Art. 15.10.) that the turning moment on the crankshaft,

$$T = F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

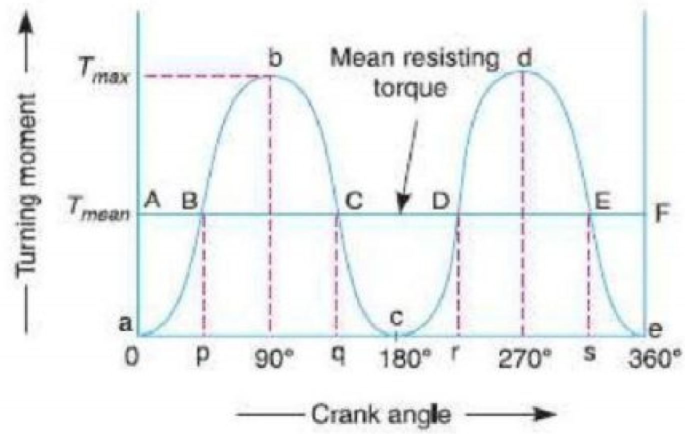


Fig. 16.1. Turning moment diagram for a single cylinder, double acting steam engine.

where F_P = Piston effort,

r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and

θ = Angle turned by the crank from inner dead centre.

is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.

This is shown by the curve *abc* in Fig. 16.1 and it represents the turning moment diagram for outstroke. The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.

Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. The height of the ordinate *a A* represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aAFe* is proportional to the work done against the mean resisting torque.

1.13.2 Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. 16.2. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, *i.e.* 720° (or 4 π radians).

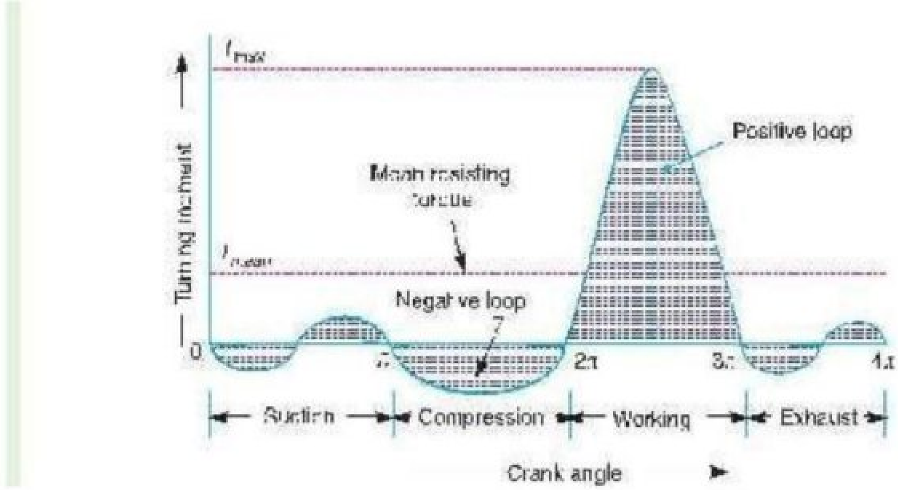
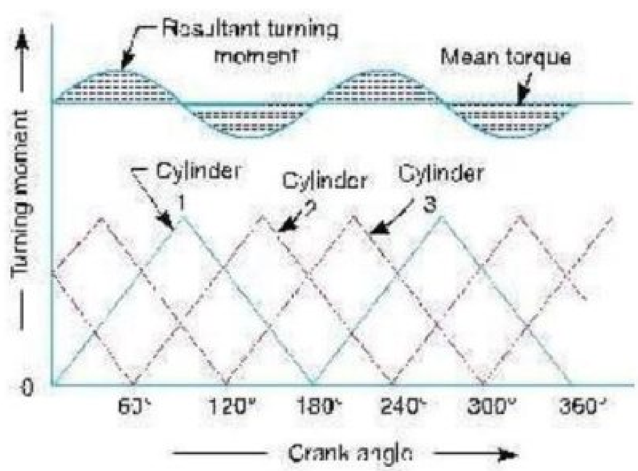


Fig. 16.2. Turning moment diagram for a four stroke cycle internal combustion engine.

Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. 16.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig. 16.2.

1.13.3. Turning Moment Diagram for a Multi-cylinder Engine

A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. 16.3. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders, are usually placed at 120° to each other.



1.14 FLUCTUATION OF ENERGY

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig. 16.1. We see that the mean resisting torque line *AF* cuts the turning moment diagram at points *B*, *C*, *D* and *E*. When the crank moves from *a* to *p*, the work done by the engine is equal to the area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the engine has done less work (equal to the area *a AB*) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represented by the area *pBCq*. Therefore, the engine has done more work than the requirement. This excess work (equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from *p* to *q*.

Similarly, when the crank moves from *q* to *r*, more work is taken from the engine than is developed. This loss of work is represented by the area *C c D*. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from *r* to *s*, excess energy is again developed given by the area *D d E* and the speed again increases. As the piston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuations of energy***. The areas *BbC*, *CcD*, *DdE*, etc. represent fluctuations of energy.

A little consideration will show that the engine has a maximum speed either at *q* or at *s*. This is due to the fact that the flywheel absorbs energy while the crank moves from *p* to *q* and from *r* to *s*. On the other hand, the engine has a minimum speed either at *p* or at *r*. The reason is that the flywheel gives out some of its energy when the crank moves from *a* to *p* and *q* to *r*. The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy***.

1.14.1 Determination of Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 16.4. The horizontal line *A G* represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine

Let the energy in the flywheel at $A = E$, then from Fig. 16.4, we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$ Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$ Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at *A* (*i.e.* cycle repeats after *G*)

Let us now suppose that the greatest of these energies is at *B* and least at *E*. Therefore,

Maximum energy in flywheel

$= E + a_1$ Minimum energy in the flywheel

$= E + a_1 - a_2 + a_3 - a_4$

∴ Maximum fluctuation of energy,

ΔE □ Maximum energy – Minimum energy

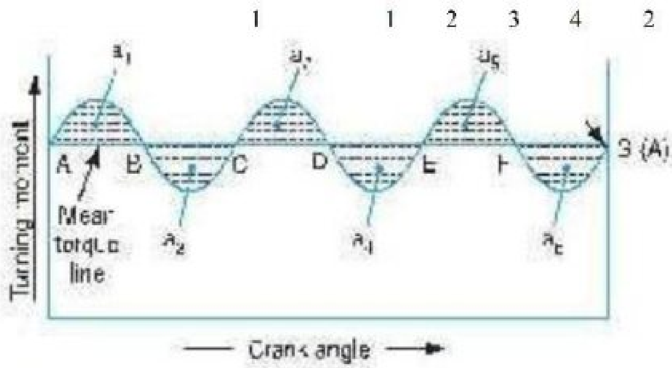


Fig. 16.4. Determination of maximum fluctuation of energy.

$= (E + a) - (E + a - a + a - a) = a - a3 + a4$

1.14.2 Coefficient of Fluctuation of Energy

It may be defined as the **ratio of the maximum fluctuation of energy to the work done per cycle.** Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The work done per cycle (in N-m or joules) may be obtained by using the following two relations :

1. Work done per cycle = $T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned (in radians), in one revolution.

= 2π , in case of steam engine and two stroke internal combustion engines

= 4π , in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation :

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and ω = Angular speed in rad/s = $2\pi N/60$

2. The work done per cycle may also be obtained by using the following relation :

$$\text{Work done per cycle} = \frac{P \times 60}{N}$$

where n = Number of working strokes per minute,

= N, in case of steam engines and two stroke internal combustion engines,

= $N/2$, in case of four stroke internal combustion engines.

The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

Coefficient of fluctuation of energy (C_E) for steam and internal combustion engines.

S.No.	*Type of engine*	*Coefficient of fluctuation of energy (C_E)*
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinders, single acting, four stroke gas engine	0.066
5.	Six cylinders, single acting, four stroke gas engine	0.031

1.15 FLYWHEEL

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

In machines where the operation is intermittent like *punching machines, shearing machines, rivetting machines, crushers, etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus, the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

1.16 COEFFICIENT OF FLUCTUATION OF SPEED

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed***. The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed***.

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

N – Mean speed in r.p.m. $= \dfrac{N_1 + N_2}{2}$

∴ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed.

1.17 ENERGY STORED IN A FLYWHEEL

A flywheel is shown in Fig. 16.5. We have discussed in Art. 16.5 that when a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m^2 = $m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad/s,

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad/s $= \dfrac{\omega_1 + \omega_2}{2}$,

C_S = Coefficient of fluctuation of speed, $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \qquad \text{(in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.}$$

$$= \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2 = \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} \times I(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \qquad ...(i)$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \text{... (Multiplying and dividing by } \omega)$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad ... (\because I = m.k^2) \quad ...(ii)$$

$$= 2.E.C_S \text{ (in N–m or joules)} \qquad ... \left(\because E = \frac{1}{2} \times I.\omega^2\right) \quad ... (iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore, substituting $k = R$

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

where v = Mean linear velocity (*i.e.* at the mean radius) in m/s = $\omega.R$

12. *The turning moment diagram for a multicylinder engine has been drawn to a scale 1 m m = 600 N-m vertically and 1 m m = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows :*

+ 52, – 124, + 92, – 140, + 85, – 72 and + 107 m m², when the engine is running at a speed of 600 r.p.m. If the total fluctuation of speed is not to exceed 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.

Solution. Given : N = 600 r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; R = 0.5 m

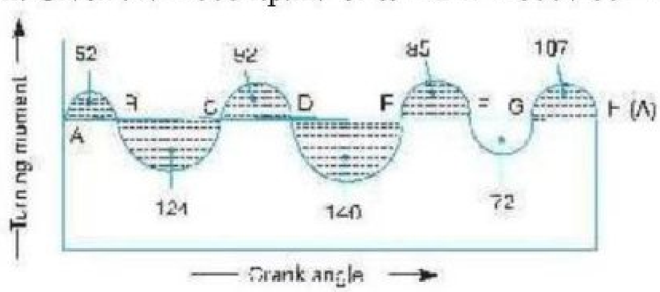


Fig. 16.7

Since the total fluctuation of speed is not to exceed 1.5% of the mean speed, therefore $\omega_1 - \omega_2 = 3\% \; \omega = 0.03 \; \omega$

and coefficient of fluctuation of speed,

$$C_s = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

The turning moment diagram is shown in Fig. 16.7.

Since the turning moment scale is 1 mm = 600 N-m and crank angle scale is 1 mm = 3° = 3° × π/180 = π / 60 rad, therefore

1 mm^2 on turning moment diagram

$$= 600 \times \pi/60 = 31.42 \text{ N-m}$$

Let the total energy at $A = E$, then referring to Fig. 16.7,

Energy at $B = E + 52$ ...**(Maximum energy)**

Energy at $C = E + 52 - 124 = E - 72$

Energy at $D = E - 72 + 92 = E + 20$

Energy at $E = E + 20 - 140 = E - 120$ ...**(Minimum energy)**

Energy at $F = E - 120 + 85 = E - 35$

Energy at $G = E - 35 - 72 = E - 107$

Energy at $H = E - 107 + 107 = E$ = Energy at A

We know that maximum fluctuation of energy,

ΔE = Maximum energy – Minimum energy

$= (E + 52) - (E - 120) = 172 = 172 \times 31.42 = 5404$ N-m

Let m = Mass of the flywheel in kg. We know that maximum fluctuation of energy (ΔE),

$5404 = m.R2.\omega^2.CS = m \times (0.5)^2 \times (62.84)^2 \times 0.03 = 29.6\,m$

□ $m = 5404 / 29.6 = 183$ kg **Ans.**

13. *A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1/2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1/2 revolution and remains constant for one revolution, the cycle being repeated thereafter.*

Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 m m.

Solution. Given : $N = 250$ r.p.m. or $\omega = 2\pi \times 250/60 = 26.2$ rad/s ; $m = 500$ kg ; $k = 600$ mm = 0.6 m

The turning moment diagram for the complete cycle is shown in Fig. 16.8.

We know that the torque required for one complete cycle

= Area of figure $OABCDEF$

= Area $OAEF$ + Area ABG + Area $BCHG$ + Area CDH

$$= OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH$$

$$= 6\pi \times 750 + \frac{1}{2} \times \pi(3000 - 750) + 2\pi(3000 - 750) + \frac{1}{2} \times \pi(3000 - 750)$$

$$= 11\,250\,\pi \text{ N-m} \quad ...(i)$$

If T_{mean} is the mean torque in N-m, then torque required for one complete cycle

$$= T_{mean} \times 6\pi \text{ N-m} \quad ...(ii)$$

From equations (*i*) and (*ii*),

$$T_{mean} = 11\,250\,\pi / 6\,\pi = 1875 \text{ N-m}$$

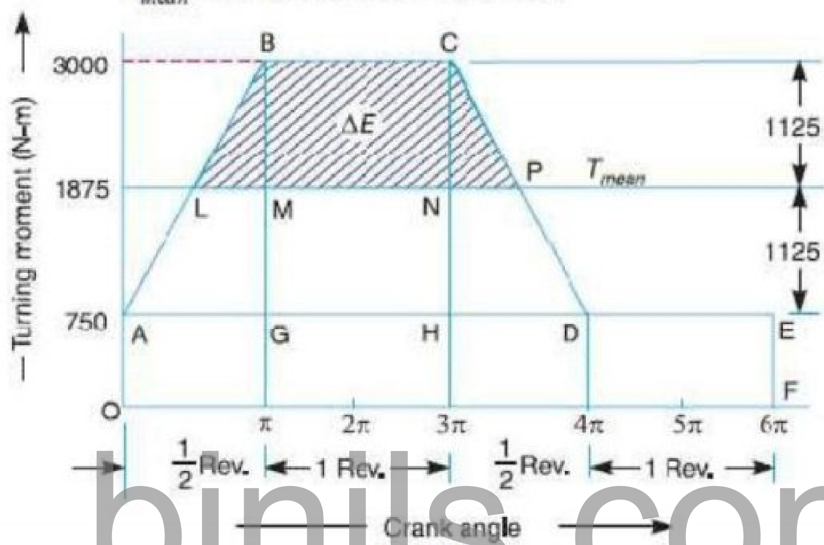


Fig. 16.8

We know that power required to drive the machine,

$$P = T_{mean} \times \omega = 1875 \times 26.2 = 49\,125 \text{ W} = 49.125 \text{ kW} \textbf{ Ans.}$$

Coefficient of fluctuation of speed

Let C_S = Coefficient of fluctuation of speed.

First of all, let us find the values of LM and NP. From similar triangles ABG and BLM,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad \text{or} \quad \frac{LM}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad LM = 0.5\,\pi$$

Now, from similar triangles CHD and CNP,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad \text{or} \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad NP = 0.5\,\pi$$

From Fig. 16.8, we find that

$$BM = CN = 3000 - 1875 = 1125 \text{ N-m}$$

Since the area above the mean torque line represents the maximum fluctuation of energy, therefore, maximum fluctuation of energy,

$$\Delta E = \text{Area } LBCP = \text{Area } LBM + \text{Area } MBCN + \text{Area } PNC$$

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times 0.5\pi \times 1125 + 2\pi \times 1125 + \frac{1}{2} \times 0.5\pi \times 1125$$
$$= 8837 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.k^2.\omega^2.C_S = 500 \times (0.6)^2 \times (26.2)^2 \times C_S = 123\,559\, C_S$$

$$C_S = \frac{8837}{123559} = 0.071 \textbf{ Ans.}$$

14. *A single cylinder, single acting, four stroke gas engine develops 20 kW at 300 r.p.m. The work done by the gases during the expansion stroke is three times the work don during the compression stroke, the work done during the suction and exhaust strokes ing per cent If the total fluctuation of speed is not to exceed ± 2 turning moment diagram durin of the mean and expansion is assumed to be triangular in shape, find the moment of inertia of th speed and the*

Solution. Given : $P = 20$ kW $= 20 \times 10^1$ W; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

Since the total fluctuation of speed ($\omega_1 - \omega_2$) is not to exceed ± 2 per cent of the mean speed □□), therefore

$$\omega_1 - \omega_2 = 4\%\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 4\% = 0.04$$

The turning moment-crank angle diagram for a four stroke engine is shown in Fig. 16.11. It is assumed to be triangular during compression and expansion strokes, neglecting the suction and exhaust strokes.

We know that for a four stroke engine, number of working strokes per cycle, $n = N/2 = 300 / 2 = 150$

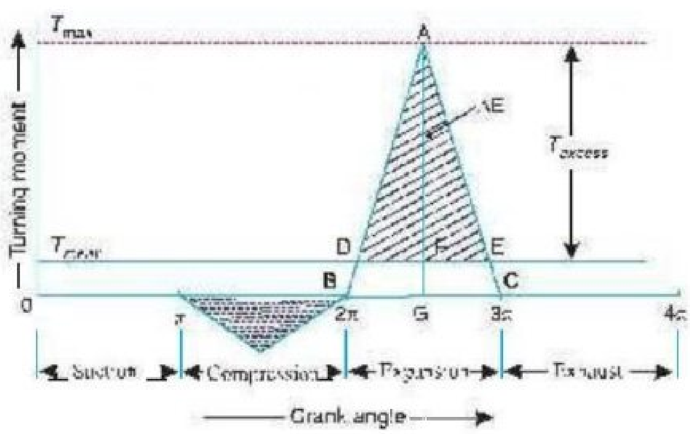


Fig. 16.11

Since the work done during suction and exhaust strokes is negligible, therefore net work done per cycle (during compression and expansion strokes)

$$= W_E - W_C = W_E - \frac{W_E}{3} = \frac{2}{3} W_E \quad \text{... } (\because W_E = 3W_C) \text{ ...}(ii)$$

Equating equations (*i*) and (*ii*), work done during expansion stroke,

$$W_E = 8000 \times 3/2 = 12\,000 \text{ N-m}$$

We know that work done during expansion stroke (W_E),

$$12\,000 = \text{Area of triangle } ABC = \frac{1}{2} \times BC \times AG = \frac{1}{2} \times \pi \times AG$$

$$\therefore \quad AG = T_{max} = 12\,000 \times 2/\pi = 7638 \text{ N-m}$$

and mean turning moment,

$$* T_{mean} = FG = \frac{\text{Work done/cycle}}{\text{Crank angle/cycle}} = \frac{8000}{4\pi} = 637 \text{ N-m}$$

∴ Excess turning moment,

$$T_{excess} = AF = AG - FG = 7638 - 637 = 7001 \text{ N-m}$$

Now, from similar triangles ADE and ABC,

$$\frac{DE}{BC} = \frac{AF}{AG} \quad \text{or} \quad DE = \frac{AF}{AG} \times BC = \frac{7001}{7638} \times \pi = 2.88 \text{ rad}$$

Since the area above the mean turning moment line represents the maximum fluctuation of energy, therefore maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta ADE = \frac{1}{2} \times DE \times AF = \frac{1}{2} \times 2.88 \times 7001 = 10\,081 \text{ N-m}$$

Let I = Moment of inertia of the flywheel in kg-m^2.

We know that maximum fluctuation of energy (ΔE),

$$10\,081 = I.\omega^2.C_S = I \times (31.42)^2 \times 0.04 = 39.5\, I$$

$$\therefore \quad I = 10081/\,39.5 = 255.2 \text{ kg-m}^2 \quad \textbf{Ans.}$$

15. *The turning moment curve for an engine is represented by the equation, T = (20 000 + 9500 sin 2θ – 5700 cos 2θ) N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find:*

1. *Power developed by the engine ;* **2.** *Moment of inertia of flywheel in kg-m^2, if the total fluctuation of speed is not exceed 1% of mean speed which is 180 r.p.m; and* **3.** *Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre.*

Solution. Given : $T = (20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m ; N = 180 r.p.m. or $\omega = 2\pi \times 180/60 = 18.85$ rad/s

Since the total fluctuation of speed $(\omega_1 - \omega_2)$ is 1% of mean speed (ω), therefore coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 1\% = 0.01$$

1. *Power developed by the engine*

We know that work done per revolution

$$= \int_0^{2\pi} T\,d\theta = \int_0^{2\pi} (20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta)\,d\theta$$

$$= \left[20\,000\,\theta - \frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_0^{2\pi}$$

$$= 20\,000 \times 2\pi = 40\,000\,\pi \text{ N-m}$$

and mean resisting torque of the engine,

$$T_{mean} = \frac{\text{Work done per revolution}}{2\pi} = \frac{40\,000}{2\pi} = 20000 \text{ N-m}$$

We know that power developed by the engine

$$= T_{mean} \cdot \omega = 20\,000 \times 18.85 = 377\,000 \text{ W} = 377 \text{ kW}$$ **Ans.**

2. *Moment of inertia of the flywheel*

Let I = Moment of inertia of the flywheel in kg-m^2.

The turning moment diagram for one stroke (*i.e.* half revolution of the crankshaft) is shown in Fig. 16.13. Since at points *B* and *D*, the torque exerted on the crankshaft is equal to the mean resisting torque on the flywheel, therefore,

$$T = T_{mean}$$

$$20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta = 20\,000$$

or $$9500 \sin 2\theta = 5700 \cos 2\theta$$

$$\tan 2\theta = \sin 2\theta / \cos 2\theta = 5700/9500 = 0.6$$

$$\therefore \quad 2\theta = 31° \text{ or } \theta = 15.5°$$

$$\therefore \quad \theta_B = 15.5° \text{ and } \theta_D = 90° + 15.5° = 105.5°$$

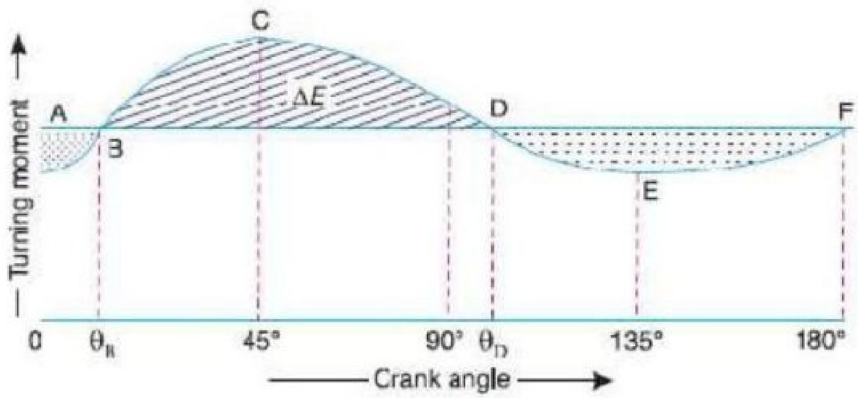


Fig. 16.13

Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean})\, d\theta$$

$$= \int_{15.5^\circ}^{105.5^\circ} (20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\,000)\, d\theta$$

$$= \left[-\frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_{15.5^\circ}^{105.5^\circ} = 11\,078 \text{ N m}$$

3. Angular acceleration of the flywheel Let α = Angular acceleration of the flywheel, and θ = Angle turned by the crank from inner dead centre = 45° . . . (Given)

The angular acceleration in the flywheel is produced by the excess torque over the mean torque. We know that excess torque at any instant,

$$T_{excess} = T - T_{mean}$$

$$= 20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20000$$

$$= 9500 \sin 2\theta - 5700 \cos 2\theta$$

∴ Excess torque at 45°

$$= 9500 \sin 90° - 5700 \cos 90° = 9500 \text{ N-m} \quad \ldots (i)$$

We also know that excess torque

$$= I.\alpha = 3121 \times \alpha \quad \ldots (ii)$$

From equations (*i*) and (*ii*),

$$\alpha = 9500/3121 = 3.044 \text{ rad/s}^2 \textbf{ Ans.}$$

1.18 DIMENSIONS OF THE FLYWHEEL RIM

Consider a rim of the flywheel as shown in Fig. 16.17.

Let D = Mean diameter of rim in metres,

R = Mean radius of rim in metres,

A = Cross-sectional area of rim in m^2,

ρ □ Density of rim material in kg/m^3,

N = Speed of the flywheel in r.p.m.,

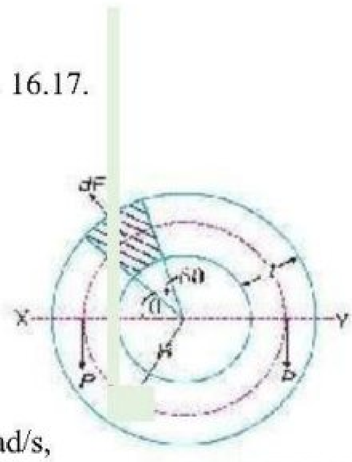


Fig. 16.17. Rim of a flywheel.

ω = Angular velocity of the flywheel in rad/s,

v = Linear velocity at the mean radius in m/s

= .R =

ω π $D.N$/60, and σ = Tensile stress or hoop stress in N/m^2 due to the centrifugal force.

Consider a small element of the rim as shown shaded in Fig. 16.17. Let it subtends an angle δθ at the centre of the flywheel.

Volume of the small element

$= A \times R.\delta\theta$ ∴ Mass of the small element

$$dm = \text{Density} \times \text{volume} = \rho.A.R.\delta\theta$$

and centrifugal force on the element, acting radially outwards,

$dF = dm.\omega^2.R = \rho.A.R^2.\omega^2.\delta\theta$

$$= 2\rho.A.R^2.\omega^2 \quad \ldots (i)$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress or circumferential stress), and it is resisted by $2P$, such that

$$2P = 2\sigma.A \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$2.\rho.A.R^2.\omega^2 = 2\sigma.A$$

or $$\sigma = \rho.R^2.\omega^2 = \rho.v^2 \quad \ldots (\because v = \omega.R)$$

∴ $$v = \sqrt{\frac{\sigma}{\rho}} \quad \ldots (iii)$$

We know that mass of the rim,

$$m = \text{Volume} \times \text{density} = \pi D . A . \rho$$

$$\therefore \quad A = \frac{m}{\pi . D . \rho} \qquad \ldots(iv)$$

From equations (*iii*) and (*iv*), we may find the value of the mean radius and cross-sectional area of the rim.

16 *The turning moment diagram for a multi-cylinder engine has been drawn to a scale of 1 mm to 500 N-m torque and 1 mm to 6° of crank displacement. The intercepted areas between output torque curve and mean resistance line taken in order from one end, in sq. mm are – 30, + 410, – 280, + 320, – 330, + 250, – 360, + 280, – 260 sq. mm, when the engine is running at 800 r.p.m.The engine has a stroke of 300 mm and the fluctuation of speed is not to exceed ± 2% of the mean speed. Determine a suitable diameter and cross-section of the flywheel rim for a limiting value of the safe centrifugal stress of 7 MPa. The material density may be assumed as 7200 kg/m³. The width of the rim is to be 5 times the thickness.*

Solution. Given : $N = 800$ r.p.m. or $\omega = 2\pi \times 800 / 60 = 83.8$ rad/s; *Stroke = 300 mm ; $\sigma = 7$ MPa $= 7 \times 10^6$ N/m² ; $\rho = 7200$ kg/m³

Since the fluctuation of speed is ± 2% of mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 4\% \ \omega = 0.04 \ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

Diameter of the flywheel rim

Let D = Diameter of the flywheel rim in metres, and

v = Peripheral velocity of the flywheel rim in m/s.

We know that centrifugal stress (σ),

$$7 \times 10^6 = \rho . v^2 = 7200 \, v^2 \quad \text{or} \quad v^2 = 7 \times 10^6 / 7200 = 972.2$$

$$\therefore \quad v = 31.2 \text{ m/s}$$

We know that $v = \pi D . N / 60$

$$\therefore \quad D = v \times 60 / \pi N = 31.2 \times 60 / \pi \times 800 = 0.745 \text{ m} \quad \textbf{Ans.}$$

Cross-section of the flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $5\,t$ ...(Given)

∴ Cross-sectional area of flywheel rim,

$$A = b.t = 5\,t \times t = 5\,t^2$$

First of all, let us find the mass (m) of the flywheel rim. The turning moment diagram is shown in Fig 16.18.

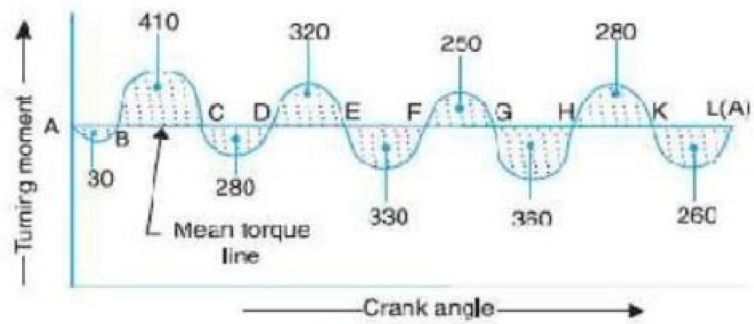


Fig. 16.18

Since the turning moment scale is 1 mm = 500 N-m and crank angle scale is 1 mm = 6° = π /30 rad, therefore

Let the energy at $A = E$, then referring to Fig 16.18,

Energy at $B = E - 30$ . . . (Minimum energy)

Energy at $C = E - 30 + 410 = E + 380$

Energy at $D = E + 380 - 280 = E + 100$

Energy at $E = E + 100 + 320 = E + 420$ . . . (Maximum energy)

Energy at $F = E + 420 - 330 = E + 90$

Energy at $G = E + 90 + 250 = E + 340$

Energy at $H = E + 340 - 360 = E - 20$

ΔE = Maximum energy – Minimum energy

$= (E + 420) - (E - 30) = 450\ \text{mm}^2$

$= 450 \times 52.37 = 23\ 566$ N-m

We also know that maximum fluctuation of energy (ΔE),

$$23\ 566 = m.v^2.C_S = m \times (31.2)^2 \times 0.04 = 39\ m$$

∴ $m = 23566 / 39 = 604$ kg

We know that mass of the flywheel rim (m),

604 = Volume × density $= \pi\, D.A.\rho$

$= \pi \times 0.745 \times 5t^2 \times 7200 = 84\ 268\ t^2$

∴ $t^2 = 604 / 84\ 268 = 0.007\ 17\ \text{m}^2$ or $t = 0.085$ m = 85 mm **Ans.**

and $b = 5t = 5 \times 85 = 425$ mm **Ans.**

17. *The turning moment diagram of a four stroke engine may be assumed for the sake of simplicity to be represented by four triangles in each stroke. The areas of these triangles are*

follows: Suction stroke = 5 × 10 $^{-5}$ m^2; Compression stroke = 21 × 10 $^{-5}$ m^2 85 × 10 $^{-5}$ m^2; Exhaust stroke = 8 × 10 $^{-5}$ m^2.

All the areas excepting expression stroke are negative. Each m^2 MN-m of work.

Assuming the resisting torque to be constant, determine the moment of flywheel to keep the speed between 98 r.p.m. and 102 r.p.m. Also find the size of a based on the minimum material criterion, given that density of flywheel material is the allowable tensile stress of the flywheel material is 7.5 MPa. The rim cross

as ; Expansion stroke = of area represents 14

-type

-section is rectangular, one side being four times the length of the other.

Solution. Given: $a_1 = 5 \times 10^{-5}$ m^2; $a_2 = 21 \times 10^{-5}$ m^2; $a_3 = 85 \times 10^{-5}$ m^2; $a_4 = 8 \times 10^{-5}$ m^2; $N_2 =$ 98 r.p.m.; $N_1 = 102$ r.p.m.; $\rho = 8150$ kg/m^3; $\sigma = 7.5$ MPa $= 7.5 \times 10^6$ N/m^2

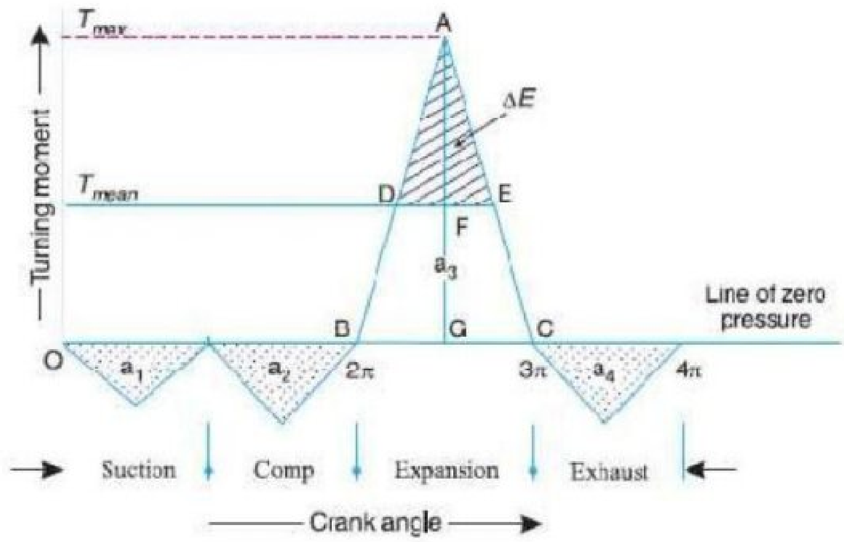


Fig. 16.20

∴ Net area $= a_3 - (a_1 + a_2 + a_4)$

$= 85 \times 10^5 - (5 \times 10^5 + 21 \times 10^{-5} + 8 \times 10^{-5}) = 51 \times 10^{-5}\ m^2$

Since $1 m^2 = 14$ MN-m $= 14 \times 10^6$ N-m of work, therefore

Net work done per cycle

$= 51 \times 10^{-5} \times 14 \times 10^6 = 7140$ N-m ...(*i*)

We also know that work done per cycle

$= T_{mean} \times 4\pi$ N-m ...(*ii*)

From equation (*i*) and (*ii*),

$T_{mean} = FG = 7140 / 4\pi = 568$ N-m

Work done during expansion stroke

$= a_3 \times$ Work scale $= 85 \times 10^{-5} \times 14 \times 10^6 = 11\ 900$ N-m ...(*iii*)

Also, work done during expansion stroke

$$= \frac{1}{2} \times BC \times AG = = \frac{1}{2} \times \pi \times AG = 1.571\ AG \quad ...(iv)$$

From equations (*iii*) and (*iv*),

$AG = 11\ 900/1.571 = 7575$ N-m

∴ Excess torque $= AF = AG - FG = 7575 - 568 = 7007$ N-m

Now from similar triangles ADE and ABC,

$$\frac{DE}{BC} = \frac{AF}{AG} \quad \text{or} \quad DE = \frac{AF}{AG} \times BC = \frac{7007}{7575} \times \pi = 2.9 \text{ rad}$$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta ADE = \frac{1}{2} \times DE \times AF$$

$$= \frac{1}{2} \times 2.9 \times 7007 = 10\,160 \text{ N-m}$$

Moment of Inertia of the flywheel

Let I = Moment of inertia of the flywheel in kg-m^2.

We know that mean speed during the cycle

$$N = \frac{N_1 + N_2}{2} = \frac{102 + 98}{2} = 100 \text{ r.p.m.}$$

∴ Corresponding angular mean speed,

$$\omega = 2\pi N / 60 = 2\pi \times 100/60 = 10.47 \text{ rad/s}$$

and coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{102 - 98}{100} = 0.04$$

We know that maximum fluctuation of energy (ΔE),

$$10\,160 = I.\omega^2.C_S = I\,(10.47)^2 \times 0.04 = 4.385\, I$$

∴ $I = 10160 / 4.385 = 2317$ kg m^2 **Ans.**

Size of flywheel

Let t = Thickness of the flywheel rim in metres,

b = Width of the flywheel rim in metres = $4\,t$ ...(Given)

D = Mean diameter of the flywheel in metres, and

v = Peripheral velocity of the flywheel in m/s.

We know that hoop stress (σ),

$$7.5 \times 10^6 = \rho \,.\, v^2 = 8150\, v^2$$

$$\therefore \quad v^2 = \frac{7.5 \times 10^6}{8150} = 920 \text{ or } v = 30.3 \text{ m/s}$$

and $v = \pi DN/60$ or $D = v \times 60/\pi N = 30.3 \times 60/\pi \times 100 = 5.786$ m

Also $m = \text{Volume} \times \text{density} = \pi D \times A \times \frac{w}{g} = \pi D \times b \times t \times \frac{w}{g}$

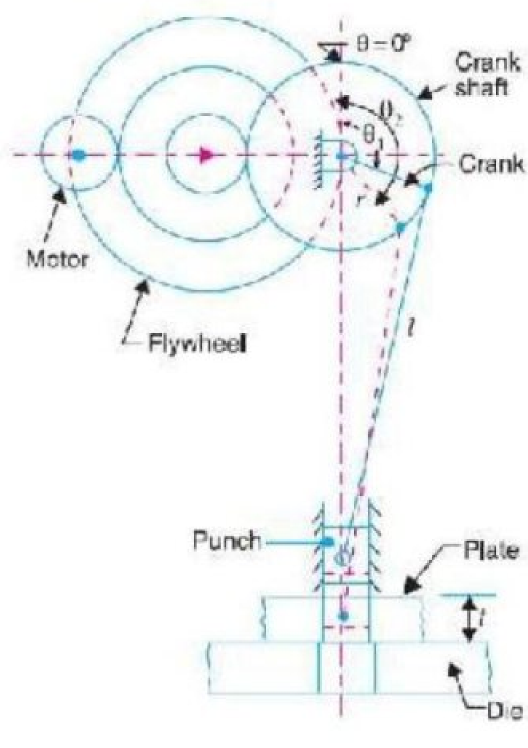
$$276.7 = \pi \times 5.786 \times 4t \times t \times \frac{8 \times 10^4}{9.81} = 59.3 \times 10^4\, t^2$$

$\therefore$ $t^2 = 276.7/59.3 \times 10^4 = 0.0216$ m or 21.6 mm **Ans.**

and $b = 4t = 4 \times 21.6 = 86.4$ mm **Ans.**

1.19 FLYWHEEL IN PUNCHING PRESS

We have discussed in Art. 16.8 that the function of a flywheel in an engine is to reduce the fluctuations of speed, when the load on the crankshaft is constant and the input torque varies during the cycle. The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle. Such an application is found in punching press or in a rivetting machine. A punching press is shown diagrammatically in Fig. 16.22. The crank is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider-crank mechanism. From Fig. 16.22, we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle. Unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while



$\theta = \theta_1$ to $\theta = \theta_2$ due to much more load than the energy supplied. Thus the flywheel has to absorb excess energy available at one stage and has to make up the deficient energy at the other stage to keep to fluctuations of speed within permissible limits. This is done by choosing the suitable moment of inertia of the flywheel.

Punching press and flywheel.

Let E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched, the thickness of the material and the physical properties of the material.

Let d_1 = Diameter of the hole punched,

t_1 = Thickness of the plate, and

τ_u = Ultimate shear stress for the plate material.

$\therefore$ Maximum shear force required for punching,

$$F_S = \text{Area sheared} \times \text{Ultimate shear stress} = \pi d_1 \cdot t_1 \tau_u$$

It is assumed that as the hole is punched, the shear force decreases uniformly from maximum value to zero.

$\therefore$ Work done or energy required for punching a hole,

$$E_1 = \frac{1}{2} \times F_s \times t$$

Assuming one punching operation per revolution, the energy supplied to the shaft per revolution should also be equal to E_1. The energy supplied by the motor to the crankshaft during actual punching operation,

$$E_2 = E_1\left(\frac{\theta_2 - \theta_1}{2\pi}\right)$$

$\therefore$ Balance energy required for punching

$$E_1 - E_2 = E_1 - E_1\left(\frac{\theta_2 - \theta_1}{2\pi}\right) = E_1\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)$$

This energy is to be supplied by the flywheel by the decrease in its kinetic energy when its speed falls from maximum to minimum. Thus maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = E_1\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)$$

The values of θ_1 and θ_2 may be determined only if the crank radius (r), length of connecting rod (l) and the relative position of the job with respect to the crankshaft axis are known. In the absence of relevant data, we assume that

$$\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}$$

where t = Thickness of the material to be punched,

s = Stroke of the punch = $2 \times$ Crank radius = $2r$.

By using the suitable relation for the maximum fluctuation of energy (ΔE) as discussed in the previous articles, we can find the mass and size of the flywheel.

18 *A punching press is driven by a constant torque electric motor. The press is provided with a flywheel that rotates at maximum speed of 225 r.p.m. The radius of gyration of the flywheel is 0.5 m. The press punches 720 holes per hour; each punching operation takes 2 second and requires 15 kN-m of energy. Find the power of the motor and the minimum mass of the flywheel if speed of the same is not to fall below 200 r. p. m.*

Solution. Given $N_1 = 225$ r.p.m ; $k = 0.5$ m ; Hole punched = 720 per hr; $E_1 = 15$ kN-m $= 15 \times 10^3$ N-m ; $N_2 = 200$ r.p.m.

Power of the motor

We know that the total energy required per second

= Energy required / hole × No. of holes / s

$= 15 \times 10^3 \times 720/3600 = 3000$ N-m/s

∴ Power of the motor = 3000 W = 3 kW **Ans.** (∵ 1 N-m/s = 1 W)

Minimum mass of the flywheel

Let m = Minimum mass of the flywheel.

Since each punching operation takes 2 seconds, therefore energy supplied by the motor in 2 seconds,

$E_2 = 3000 \times 2 = 6000$ N-m

∴ Energy to be supplied by the flywheel during punching or maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 15 \times 10^3 - 6000 = 9000 \text{ N-m}$$

Mean speed of the flywheel,

$$N = \frac{N_1 + N_2}{2} = \frac{225 + 200}{2} = 212.5 \text{ r.p.m}$$

We know that maximum fluctuation of energy (ΔE),

$$9000 = \frac{\pi^2}{900} \times m.k^2.N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m \times (0.5)^2 \times 212.5 \times (225 - 200) = 14.565\, m$$

∴ $m = 9000/14.565 = 618$ kg **Ans.**

19. *A machine punching 38 mm holes in 32 mm thick plate requires 7 N-m of energy per sq. mm of sheared area, and punches one hole in every 10 seconds. Calculate the power of the motor required. The mean speed of the flywheel is 25 metres per second. The punch has a stroke of 100 mm.*

Find the mass of the flywheel required, if the total fluctuation of speed is not to exceed 3% of the mean speed. Assume that the motor supplies energy to the machine at uniform rate.

Solution. Given : $d = 38$ mm ; $t = 32$ mm ; E_1 □ 7 N-m/mm^2 of sheared area ; v □ 25 m/s ; $s = 100$ mm ; $v_1 - v_2$ □ 3% v □ 0.03 v

Power of the motor required

We know that sheared area,

$$A = \pi d.t = \pi \times 38 \times 32 = 3820 \text{ mm}^2$$

Since the energy required to punch a hole is 7 N m/mm² of sheared area, therefore total energy required per hole,

$$E_1 = 7 \times 3820 = 26\ 740 \text{ N-m}$$

Also the time required to punch a hole is 10 second, therefore energy required for punching work per second

$$= 26\ 740/10 = 2674 \text{ N m/s}$$

∴ Power of the motor required

$$= 2674 \text{ W} = 2.674 \text{ kW} \textbf{ Ans.}$$

Mass of the flywheel required

Let m = Mass of the flywheel in kg.

Since the stroke of the punch is 100 mm and it punches one hole in every 10 seconds, therefore the time required to punch a hole in a 32 mm thick plate

$$= \frac{10}{2 \times 100} \times 32 = 1.6 \text{ s}$$

∴ Energy supplied by the motor in 1.6 seconds,

$$E_2 = 2674 \times 1.6 = 4278 \text{ N-m}$$

Energy to be supplied by the flywheel during punching or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 26\ 740 - 4278 = 22\ 462 \text{ N-m}$$

Coefficient of fluctuation of speed,

$$C_S = \frac{v_1 - v_2}{v} = 0.03$$

We know that maximum fluctuation of energy (ΔE),

$$22\ 462 = m.v^2.C_S = m \times (25)^2 \times 0.03 = 18.75\ m$$

$$\therefore \quad m = 22\ 462 / 18.75 = 1198 \text{ kg} \textbf{ Ans.}$$

20. *A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10 000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute?* **Solution.** Given : $P = 3$ kW ; $m = 150$ kg ; $k = 0.6$ m ; $N_1 = 300$ r.p.m. or $\omega_1 = 2\pi \times 300/60 = 31.42$ rad/s

Speed of the flywheel immediately after riveting

Let ω_2 = Angular speed of the flywheel immediately after riveting.

We know that energy supplied by the motor,

$$E_2 = 3\text{kW} = 3000\text{ W} = 3000\text{ N-m/s} \qquad (\because 1\text{ W} = 1\text{ N-m/s})$$

But energy absorbed during one riveting operation which takes 1 second,

$$E_1 = 10\,000\text{ N-m}$$

∴ Energy to be supplied by the flywheel for each riveting operation per second or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 10\,000 - 3000 = 7000\text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$7000 = \frac{1}{2} \times m.k^2 \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times 150 \times (0.6)^2 \times \left[(31.42)^2 - (\omega_2)^2\right]$$

$$= 27\left[987.2 - (\omega_2)^2\right]$$

∴ $(\omega_2)^2 = 987.2 - 7000/27 = 728$ or $\omega_2 = 26.98$ rad/s

Corresponding speed in r.p.m.,

$$N_2 = 26.98 \times 60/2\pi = 257.6\text{ r.p.m.} \textbf{ Ans.}$$

Number of rivets that can be closed per minute

Since the energy absorbed by each riveting operation which takes 1 second is 10 000 N-m, therefore, number of rivets that can be closed per minute,

$$= \frac{E_2}{E_1} \times 60 = \frac{3000}{10\,000} \times 60 = 18 \text{ rivets } \textbf{Ans.}$$

21. *A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at the rate of 30 holes per minute. It requires 6 N-m of energy per mm² of sheared area. If the punching takes 1/10 of a second and the r.p.m. of the flywheel varies from 160 to 140, determine the mass of the flywheel having radius of gyration of 1 metre.*

Solution. Given: d = 40 mm; t = 15 mm; No. of holes = 30 per min.; Energy required = 6 N-m/mm²; Time = 1/10 s = 0.1 s; N_1 = 160 r.p.m.; N_2 = 140 r.p.m.; k = 1m We know that sheared area per hole

$$= \pi\, d.t = \pi \times 40 \times 15 = 1885\text{ mm}^2$$

∴ Energy required to punch a hole,

$$E_1 = 6 \times 1885 = 11\,310\text{ N-m}$$

and energy required for punching work per second

= Energy required per hole × No. of holes per second

= 11 310 × 30/60 = 5655 N-m/s

Since the punching takes 1/10 of a second, therefore, energy supplied by the motor in 1/10 second,

$$E_2 = 5655 \times 1/10 = 565.5\text{ N-m}$$

∴ Energy to be supplied by the flywheel during punching a hole or maximum fluctuation of energy of the flywheel,

$$\Delta E = E_1 - E_2 = 11\,310 - 565.5 = 10\,744.5\text{ N-m}$$

Mean speed of the flywheel,

$$N = \frac{N_1 + N_2}{2} = \frac{160 + 140}{2} = 150 \text{ r.p.m.}$$

We know that maximum fluctuation of energy (ΔE),

$$10\,744.5 = \frac{\pi^2}{900} \times m.k^2\, N(N_1 - N_2)$$

$$= 0.011 \times m \times 1^2 \times 150\,(160 - 140) = 33\,m$$

$\therefore$ $m = 10744.5 / 33 = 327$ kg **Ans.**

1.20 CAM DYNAMICS:

Mechanism provides a non-linear I/O relationship. Different mechanism like single or multi-degree of freedom, intermittent motion mechanisms and linkages etc. have different I/O Relationship. When we can not obtain a certain functions from the well known mechanisms, we use a cam mechanism. It is a one degree of freedom mechanism of two moving links. One is cam and the other is follower.

- Rigid and elastic body cam system.
- Analysis of eccentric cam □ Problems on Cam – follower system.

1.21 REVIEW QUESTIONS:

1. When the crank is at the inner dead centre, in a horizontal reciprocating steam engine, then the velocity of the piston will be ?
2. A rigid body, under the action of external forces, can be replaced by two masses placed at a fixed distance apart. The two masses form an equivalent dynamical system, if?
3. The essential condition of placing the two masses, so that the system becomes dynamically equivalent is **?**
4. In an engine, the work done by inertia forces in a cycle is ?
5. In a turning moment diagram, the variations of energy above and below the mean resisting torque line is called?

1.22 TUTORIAL PROBLEMS

1. The stroke of a steam engine is 600 mm and the length of connecting rod is 1.5 m. The crank rotates
at 180 r.p.m. Determine: 1. velocity and acceleration of the piston when crank has travelled through an angle of 40° from inner dead centre, and 2. the position of the crank for zero acceleration of the piston. **[Ans. 4.2 m/s, 85.4 m/s²; 79.3° from I.D.C]** **2.** The following data refer to a steam engine :

Diameter of piston = 240 mm; stroke = 600 mm; length of connecting rod = 1.5 m; mass of reciprocat-ing parts = 300 kg; speed = 125 r.p.m.

Determine the magnitude and direction of the inertia force on the crankshaft when the crank has turned through 30° from inner dead centre. **[Ans. 14.92 kN]**

3. A vertical petrol engine 150 mm diameter and 200 mm stroke has a connecting rod 350 mm long. The mass of the piston is 1.6 kg and the engine speed is 1800 r.p.m. On the expansion stroke with crank angle 30° from top dead centre, the gas pressure is 750 kN/m^2. Determine the net thrust on the piston. **[Ans. 7535 N]**

4. A certain machine tool does work intermittently. The machine is fitted with a flywheel of mass 200 kg and radius of gyration of 0.4 m. It runs at a speed of 400 r.p.m. between the operations. The machine is driven continuously by a motor and each operation takes 8 seconds. When the machine is doing its work, the speed drops from 400 to 250 r.p.m. Find 1. minimum power of the motor, when there are 5 operations performed per minute, and 2. energy expanded in performing each operation.

[Ans. 4.278 kW; 51.33 kN-m]

5. A constant torque 4 kW motor drives a riveting machine. A flywheel of mass 130 kg and radius of gyration 0.5 m is fitted to the riveting machine. Each riveting operation takes 1 second and requires 9000 N-m of energy. If the speed of the flywheel is 420 r.p.m. before riveting, find:
1. the fall in speed of the flywheel after riveting; and 2. the number of rivets fitted per hour.
[Ans. 385.15 r.p.m.; 1600]