Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth Semester

Electronics and Communication Engineering

MA 8451 — PROBABILITY AND RANDOM PROCESSES

(Common to : B.E. Computer and communication Engineering/ B.E. Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours　　　　Maximum : 100 marks

(Normal distribution table may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What do you mean by moment generating function?

2. If X is a Poisson variate with a parameter $\lambda > 0$, then prove that $E(X^2) = \lambda E(X+1)$.

3. Random variables X and Y have joint probability density function given by

$$f(x,y) = \begin{cases} c & ; 0 \le x \le 5,\ 0 \le y \le 3 \\ 0 & ; otherwise \end{cases}$$, find the constant 'c'.

4. State central limit theorem.

5. Identify the random process for the following:

   (a) Level of water in dam at time `t'

   (b) Number of babies born in a hospital on the 1st day of a week.

6. Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0.3 & 0.4 & 0.5 \end{bmatrix}$. Is it a valid transition probability matrix of a Markov chain? Justify

7. Write down the relationship between cross-power spectrum and cross-correlation function.

8. Assume that the input X(t) to a linear time-invariant system is white noise. What is the power spectral density of the output process Y(t) if the system response H(w) is given by the following?

$$H(w) = \begin{cases} 1 & w_1 < |w| < w_0 \\ 0 & otherwie \end{cases}$$

9. "$R(\tau)$ is an even function of τ" – prove or disprove.

10. What do you mean by linear systems with random inputs?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A bolt is manufactured by 3 machines A,B and C. A turns out twice as many items as B, and machines B and C produce equal number of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and 1 is chosen from this pile. What is the probability that it is defective? (8)

(ii) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events: (1) exactly two messages arrive within one hour (2) no message arrives within one hour (3) at least three messages arrive within one hour. (8)

Or

(b) (i) A random variable X has the following probability distributions

$X{=}x$	0	1	2	3	4	5	6	7
P(x):	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find

(1) the value of k,

(2) $P(1.5 < X < 4.5 / X > 2)$

(3) the smallest value of λ for which $P(X \le \lambda) > 1/2$. (8)

(ii) 'The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least 1 of them would have scored above 75? (8)

12. (a) The joint CDF of two discrete random variables X and Y is given as follows:

$F_{XY}(x, y) = 1/8$ when $x = 1, y = 1$; $F_{XY}(x, y) = 5/8$ when $x = 1, y = 2$;

$F_{XY}(x, y) = 2/8$ when $x = 2, y = 1$; $F_{XY}(x, y) = 8/8$ when $x = 2, y = 2$;

Determine the following:

(1) Joint PMF of X and Y

(2) Marginal PMF of X

(3) Marginal PMF of Y

(4) $P(X + Y < 2)$

(5) $P(X > Y)$. (16)

Or

(b) (i) The Joint PDF of (X, Y) is given by $f(x, y) = \begin{cases} \dfrac{x(1+3y^2)}{4}, 0 < x < 2,\ 0 < y < 1 \\ 0 \quad otherwise \end{cases}$ Are X and Y independent? (6)

(ii) In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible: Variance of $X = 1$, Regression equations: $3X + 2Y - 26 = 0$, $6X + Y = 31$ Find (1) the mean values of X and Y (2) the correlation coefficient between X and Y (3) the standard deviation of Y? (4) the standard deviation of X? (10)

13. (a) (i) Suppose that there are 5 balls numbered 1,2,3,4,5 distributed among two urns randomly. At time n, a number is selected at random from the set {1,2,3,4,5}, then the ball with that number is found in one of the two urns and is moved to the other urn. Let X_n be the number of balls in urn I after n transitions. Is $\{X_n, n = 0,1,2,3\}$ is a Markov chain? If so, find its transition probability matrix and also specify the classes. (8)

(ii) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sense stationary process, if A and ω_0 are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

Or

(b) (i) The times between component failures in a certain system are exponentially distributed with a mean of 4 hours. (1) What is the probability that at least one component failure occurs within a 30 minute period? (2) What is the probability that no component failure occurs within a 30 minute period? (8)

(ii) Define random telegraph signal process and prove that it is wide-sense stationary process. (8)

14. (a) (i) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\gamma|\tau|}$. Determine the power spectral density function of random telegraph signal. (8)

(ii) The random process X(t) has the autocorrelation function $R_{XX}(\tau) = e^{-2|\tau|}$. The random process Y(t) is defined as follows: $Y(t) = \int_0^t X^2(u)du$. Find $E(Y(t))$. (8)

Or

(b) (i) A stationary random process X(t) has an autocorrelation function given by $R_{XX}(\tau) = 2e^{-|\tau|} + 4e^{-4|\tau|}$, Find the power spectral density of the process. (8)

(ii) Find the cross-correlation function corresponding to the cross-power density spectrum $\wp_{XY}(\omega) = \dfrac{8}{(\alpha + j\omega)^3}$. (8)

15. (a) A random process X(t) is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t \geq 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, determine the following:

(i) The cross correlation function $R_{XY}(\tau)$ between the input process $X(t)$ and the output process $Y(t)$. (8)

(ii) The cross correlation function $R_{YX}(\tau)$ between the output process $Y(t)$ and the input process $X(t)$. (8)

Or

(b) A wide sense stationary process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-7t}, t \geq 0$. If the autocorrelation function of the process $R_{XX}(\tau) = e^{-4|\tau|}$, and the output process is $Y(t)$, find the following:

(i) The power spectral density of $Y(t)$

(ii) The cross-spectral power density $S_{XY}(w)$

(iii) The cross correlation function $R_{XY}(\tau)$. (16)

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