DEPARTMENT OF MATHEMATICS

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UNIT – IV : INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

UNIT IV– INTERPOLATION ,NUMERICAL DIFFERENTIATION AND INTEGRATION

Class Notes

Interpolation and Extrapolation:

Interpolation is the process of finding the values of f(x) for intermediate values of x in the given interval. Extrapolation is the process of finding the values of f(x) for extreme values of x.

Lagrange's Interpolation formula

$$f(x)=\frac{(x-x_1)(x-x_2).....(x-x_n)}{(x_o-x_1)(x_o-x_2).......(x_0-x_n)}y_o+\frac{(x-x_o)(x-x_2).....(x-x_n)}{(x_1-x_o)(x_1-x_2).......(x_0-x_n)}y_1+$$
$$\frac{(x-x_0)(x-x_1).....(x-x_n)}{(x_n-x_o)(x_n-x_1).......(x_0-x_n)}y_2+......+\frac{(x-x_0)(x-x_1).....(x-x_{n-1})}{(x_n-x_o)(x_n-x_1).......(x_n-x_{n-1})}y_n$$

Inverse Lagrange's Interpolation

$$x=\frac{(y-y_1)(y-y_2)....(y-y_n)}{(y_o-y_1)(y_o-y_2)....(y_o-y_n)}x_o+\frac{(y-y_o)(y-y_2)....(y-y_n)}{(y_1-y_o)(y_1-y_2)....(y_1-y_n)}x_1+.....$$
$$+\frac{(y-y_0)(y-y_1)....(y-y_{n-1})}{(y_n-y_o)(y_n-y_1)....(y_n-y_{n-1})}x_n$$

Uses of Newton's forward formula and Newton's Backward formula

Newton's forward formula is used to interpolate value of y nearer to the beginning value of the table. Newton's Backward formula is used to interpolate value of y nearer to the end of set of tabular values. This may also be used to extrapolate closure to right of y_n.

Newton's divided difference formula

$$f(x)=f(x_o)+(x-x_o)f(x_o,x_1)+(x-x_o)(x-x_1)f(x_o,x_1,x_2)+......$$
$$+(x-x_o)(x-x_1).......(x-x_{n-1})f(x_o,x_1,x_2,....x_n)$$

Properties of Divided difference.

(i) The divided differences are symmetrical in all their arguments. i.e. the value of any difference is independent of the order of the arguments.

(ii) The divided differences of the sum or difference of two functions is equal to thesum or difference of the corresponding separate divided differences.

Numerical Differentiation:

Numerical differentiation is the process of computing the values of .... For some particular values of x from the given data (xi,yi) where y is not known explicitly.

Numerical Integration:

The process of computing the values of a definite integrals $\int_a^b y\,dx$ from a set of values (x_i, y_i), i = 0,1,2,…,n, where x_o = a, x_n = b of the function y = f(x) is called Numerical integration

Problems:

1. From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
No.of Students	31	42	51	35	31

x	y	Δ	Δ^2	Δ^3	Δ^4
Below 40	31				
		42			
Below 50	73		9		
		51		-25	
Below 60	124		-16		37
		35		12	
Below 70	159		-4		
		31			
Below 80	190				

$u = \frac{x-x_0}{h} = \frac{45-40}{10} = 0.5$

By Newton's forward interpolation formula

No.of students with the marks less than 45 = 48
No.of students with the marks less than 40 = 31
Therefore No.of students who obtained marks between 40 and 45 = 48 – 31 = 17

2. Given the tables

x	**5**	**7**	**11**	**13**	**17**
y	**150**	**392**	**1452**	**2366**	**5202**

Evaluate f(9) using Newton's divided difference formula.

Solution:

The divided difference table is given as follows:

x	y	Δ	Δ^2	Δ^3	Δ^4
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42		
		709			
17	5202				

f(9) = 150+484+192-16 = 810

3. Find the missing values from the following table

X	0	5	10	15	20	25
Y	6	10	-	17	-	31

Solution:
The finite difference table is

x	y	Δ	Δ^2	Δ^3	Δ^4
0	6				
		4			
5	10		a-14		
		a-4		41-3a	
10	a		27-2a		-102+6a+b
		17-a		-61+3a+b	
15	17		-34+b+a		143-4b-4a
		b-17		82-3b-a	
20	b		48-2b		
		31-b			
25	31				

Since only four values are given, we have fourth difference is zero
Therefore Δ4y = 0

6a+b=102, 4a+b=143
Solving this we get a=13.25, b= 22.5

4 **Find the expression f(x) using Lagrange's formula for the following data**

x	0	1	4	5
f(x)	4	3	24	39

Solution:
The Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_{01})(x_1-x_2)(x_1-x_3)} y +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

x0 =0 ,x1 =1 ,x2 =4 ,x3 =5, y0 =4, y1 =3, y2 =24, y3=39
f(x) = 2x2-3x+4

5 **Find an approximate polynomial for f(x) using Lagrange's interpolation for the following data**

x	0	1	2	5
f(x)	2	3	12	147

Solution:
The Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_{01})(x_1-x_2)(x_1-x_3)} y +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) + \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$= \frac{1}{20}[-x^3 + 166x^2 - 184x - 40]$$

6 **Find $y'(1)$ if**

x	-1	0	2	3
y	-8	3	1	12

Solution:
The Lagrange's interpolation formula is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= 2x^3 - 6x^2 + 3x + 3$$

$$y'(x) = 6x^2 - 12x + 3$$

$$y'(1) = -3$$

7 **Using Lagrange's formula, fit a polynomial to the data**

x	-1	0	2	3
y	-8	3	1	12

Hence find y at x = 1.5 and x = 1

Solution:

The Lagrange's interpolation formula is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$x = 1.5$

$$= \frac{(1.5-0)(1.5-2)(1.5-3)}{(-1-0)(-1-2)(-1-3)}(-8) + \frac{(1.5+1)(1.5-2)(1.5-3)}{(0+1)(0-2)(0-3)}(3) + \frac{(1.5+1)(1.5-0)(1.5-3)}{(2+1)(2-0)(2-3)}(1)$$

$$+ \frac{(1.5-0)(1.5+1)(1.5-2)}{(3+1)(3-0)(3-2)}(12)$$

$= 0.75$

$x = 1$

$$= \frac{(1-0)(1-2)(1-3)}{(-1-0)(-1-2)(-1-3)}(-8) + \frac{(1+1)(1-2)(1-3)}{(0+1)(0-2)(0-3)}(3) + \frac{(1+1)(1-0)(1-3)}{(2+1)(2-0)(2-3)}(1)$$

$$+ \frac{(1-0)(1+1)(1-2)}{(3+1)(3-0)(3-2)}(12)$$

$= 2$

8 **Using Lagrange's inverse interpolation formula, find the value of x when y=20 from the given data**

x	1	2	3	4
y	1	8	27	64

Solution: $$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_o-y_1)(y_o-y_2)(y_o-y_3)} x_o + \frac{(y-y_o)(y-y_2)(y-y_3)}{(y_1-y_o)(y_1-y_2)(y_1-y_3)} x_1 +$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_o)(y_2-y_1)(y_2-y_3)} x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_o)(y_3-y_1)(y_3-y_2)} x_3$$

$y = 20 \quad y_0 = 1 \; y_1 = 8 \;\; y_2 = 27 \;\; y_3 = 64$

$x_0 = 1 \; x_1 = 2 \; x_2 = 3 \;\; x_3 = 4$

$x = 2.8468$

10 **Given the tables**

X	0	2	3	4	7	9
Y	**4**	**26**	**58**	**112**	**466**	**922**

Evaluate f(10) and f'(6) using Newton's divided difference formula.

Solution: The divided difference table is given as follows:

x	y	Δ	Δ^2	Δ^3	Δ^4
0	4				
		11			
2	26		7		
		32		1	
3	58		11		0
		54		1	
4	112		16		0
		118		1	
7	446		22		
		228			
9	922				

Newton's divided difference formula.

$$f(x) = f(x_o) + (x - x_o) f(x_o, x_1) + (x - x_o)(x - x_1) f(x_o, x_1, x_2) + \ldots\ldots$$
$$+(x - x_o)(x - x_1)\ldots\ldots(x - x_{n-1}) f(x_o, x_1, x_2, \ldots. x_n)$$

$$f(x) = x^3 + 2x^2 + 3x + 4$$

$$f'(x) = 3x^2 + 4x + 3$$

f(10) = 1234

f'(6) = 135

11 **Find f(x) as a Polynomial in x for the following data by Newton's divided difference formula and hence find f(8).**

x :	4	5	7	10	11	13
$f(x)$:	**48**	**100**	**294**	**900**	**1210**	**2028**

Solution:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$

4	48				
		52			
			15		
5	100				
				1	
		97			0
			21		
7	294				
				1	
		202			
					0
10	900		27		
				1	
		310			
11	1210		33		
		409			
13	2028				

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3)$$
$$+ (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$f(x) = 48 + 9x - 4)52 + (x-4)(x-5)15 + (x-4)(x-5)(x-7)(1)$$

$$hence\ f(8) = 448$$

12 **Using Newton's divided difference formula compute f(2) from the data**

x	**-4**	**-1**	**0**	**2**	**5**
y	**1245**	**33**	**5**	**9**	**1335**

Solution:

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3)$$
$$+ (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		-404			
			94		
-1	33				
				-14	
		-28			3
			10		
0	5				
				13	
		2			
2	9		88		
		442			
5	1335				

$here\ x_0 = -4,\ x_1 = -1,\ x_2 = 0,\ x_3 = 2,\ x_4 = 5$

$f(x_0) = 1245$

$f(x_0, x_1) = -404$

$f(x_0, x_1, x_2) = 94$

$f(x_0, x_1, x_2, x_3) = -14$

$f(x_0, x_1, x_2, x_3, x_4) = 3$

$f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$

$f(2) = 9$

13 **Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data and hence find the value of y for x = 5.**

X	4	6	8	10
Y	1	3	8	16

Solution:

x	y	Δ	Δ^2	Δ^3
4	1			
		2		
6	3		3	

		5		0
8	8		3	
		8		
10	16			

$$u = \frac{x - x_0}{h} = \frac{12 - 10}{5} = 0.4$$

$$y(x) = y_o + u\,\Delta y_o + \frac{u(u-1)}{2!}\Delta^2 y_o + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_o + ...$$

$$0.375x^2 - 2.75x + 6$$

$$f(5) = 9.375 - 13.75 + 6$$

$$f(5) = 29.125$$

14 **Using Newton's forward interpolation formula, find the cubic polynomial which takes the following values:**

$$\begin{array}{lcccc} x: & 0 & 1 & 2 & 3 \\ f(x): & 1 & 2 & 1 & 10 \end{array}$$

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

here $x_0 = 0$; $y_0 = 1$; $h = 1$; $u = \frac{x - x_0}{h} = x$

hence $y(x) = 2x^3 - 7x^2 + 6x + 1 = f(x)$

$f(4) = 41$

15 **From the following table find the value at $\tan 45^o 15'$**

x^o	: 45	46	47	48	49	50
tan x^o	: 1	**1.03553**	**1.07237**	**1.11061**	**1.15037**	**1.19175**

Solution:

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y_0 + \ldots$$

θ^o	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
45	1.0000					
		0.03553				
			0.00131			
46	1.03553			0.0009		
		0.03684			0.00003	
			0.00140			-0.00005
47	1.07237			0.00012		
		0.03824			-0.00002	
			0.00152			
48	1.11061			0.00010		
		0.03976				
			0.00162			
49	1.15037					
		0.04138				
50	1.19175					

here $u = \frac{x - x_0}{h} = \frac{45^o 15' - 45^o}{1} = 15' = \frac{1^0}{4}$

hence $y(45^o 15') = 1.008764609$

16 **From the given data, find the number of students whose weight is between 60 and 70.**

Weight in lbs :	**0 – 40**	**40-60**	**60-80**	**80-100**	**100-120**
No of students :	**250**	**120**	**100**	**70**	**50**

Solution:

x	y	Cumulative frequency	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250	250				
			120			
				-20		
Below 60	120	370			-10	
			100			
				-30		20
Below 80	100	470			10	
			70			
				-20		
Below 100	70	540				
			50			
Below 120	50	590				

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

here $u = \frac{x - x_0}{h} = \frac{70-40}{20} = 1.5$

hence $y(70) = 423.5937$

now we take $x = 60$

$u = \frac{60-40}{20} = 1$

$y(60) = 370$

Therefore, the no.of students whose weight below 60=370

The no.of students whose weight below 70=423.5937

No. Of students whose weight is between 60 & 70=423.5937-370=53.5937=54 approximately.

17 **Find the value of sec 310 from the following data**

θ	**31**	**32**	**33**	**34**
tanθ	**0.6008**	**0.6249**	**0.6494**	**0.6745**

Solution:
The difference table is

θ	tanθ	Δy	$\Delta^2 y$	$\Delta^3 y$
31	0.6008			
		0.0241		
32	0.6249		0.0004	
		0.0245		0.0002
33	0.6494		0.0006	
		0.0251		
34	0.6745			

$\frac{d}{d\theta}(\tan\theta) = \sec^2\theta$

By Newton's formula

$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + ...)$

$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{1^0}[0.0241 - \frac{1}{2}(0.0004) + \frac{1}{3}(0.0002) + ...) = 1.3693$

$1^0 = \frac{\pi}{180}$

$\frac{d}{d\theta}(\tan 31^0) = \sec^2 31^0 = 1.3693$

$\sec 31^0 = 1.1702$

18 **Evaluate** $I = \int_1^2 \frac{dx}{1+x^2}$ **using Simpson's** $\frac{1}{3}$ **rule by taking four equal parts.**

Solution:

x	1	1.25	1.5	1.75	2
f(x)	0.5	0.3902	0.3077	0.2462	0.2

Simpson's one – third rule is

$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$

$= \frac{0.25}{3}\left[(0.5+0.2)+2(0.3077)+4(0.3902+0.2462)\right]$

= 0.2941

19 **Evaluate** $\int_0^1 \frac{x^2}{1+x^3}dx$ using Simpson's $\frac{1}{3}$ rule with $h = 0.25$

Solution:

x	0	0.25	0.5	0.75	1
f(x)	0	0.06154	0.22222	0.39560	0.50000

Simpson's one – third rule is

$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2) + 4(y_1 + y_3)]$

= 0.25/3 [0+0.5+2(0.2222)+4(0.06154+0.39560)] = 0.231083

20 **Evaluate** $\int_0^6 \frac{e^x}{1+x}dx$ using Simpson's $\frac{1}{3}$ rule with h = 1

Solution:

x	0	1	2	3	4	5
f(x)	1	1.359	2.463	5.021	10.91	24.73

Simpson's one – third rule is

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

= **1/3** [1+57.632+2(2.463+10.91)+4(1.359+5.021+24.735)]
= 69.946

21 **Evaluate $I = \int_{-3}^{3} x^4 dx$ correct to three decimals dividing the range of integration into 8 equal parts using Trapezoidal , Simpson's $\frac{3}{8}$, Simpson's $\frac{1}{3}$rule .And compare the result with actual integration**

Solution:

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
y	**81**	**16**	**1**	**0**	**1**	**16**	**81**

Trapezoidal rule:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + ... + y_{n-1})]$$

= 115

Simpson's one – third rule:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

= 98

Simpson's 3/8 rule is

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_3 + y_6 + ...) + 3(y_1 + y_2 + y_4 + y_5 ...)]$$

= 99

Actual integration:

$$I = \int_{-3}^{3} x^4 dx = \left[\frac{x5}{5}\right]_{-3}^{3} = \left[\frac{243}{5} - \frac{(-243)}{5}\right]$$

= 97.2

22 **Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by using Trapezoidal and simpson's rule with h = k = 0.25.**

Solution:

y / x	1	1.25	1.5	1.75	2
1	0.5	0.4444	0.4	0.3636	0.3333
1.25	0.4444	0.4	0.3636	0.3333	0.3077

1.5	0.4	0.3636	0.3333	0.3077	0.2857
1.75	0.3636	0.3333	0.3077	0.2857	0.2667
2	0.3333	0.3077	0.2857	0.2667	0.25

By Trapezoidal rule,

I= $\frac{hk}{4}$ {sum of values of f at the four corners+2(sum of the values of f at the remaining nodes on the boundary)+4(sum of the values of f at the interior nodes)}

I = 0.34065

23 **Evaluate $I = \int_0^2\int_1^2 \sin(9x+y)\, dxdy$ by using Simpson's $\frac{1}{3}$rule by taking h = 0.25 k = 0.5**

Solution:

y/x	0	0.25	0.5	0.75	1
0	0	0.7781	-0.9775	0.45	0.4121
0.5	0.4794	0.3817	-0.9589	0.8231	-0.07515
1	0.8415	-0.1082	-0.7055	0.9946	-0.5540

By simpson's rule

$$I = \left\{ \frac{0.25\times 0.5}{3\times 3} \left\{ \begin{array}{l} (0+0.4121-0.5540+0.8415)+2(-0.9775+0.8415) \\ +4(0.7781+0.45-0.07515+0.9946-0.1082+0.4794) \\ +\left\{ \begin{array}{l} 8(-0.9589)+ \\ 16(0.3817+0.8231) \end{array} \right\} \end{array} \right\} \right\}$$

I = 0.2548

24 **Evaluate numerically $\int_1^2\int_1^2 \frac{xy}{x+y}dxdy$ by using Trapezoidal rule taking h = k = 0.25**

Solution:

y\x	1	1.25	1.5	1.75	2
1	0.5	0.5556	0.6	0.6364	0.6667
1.25	0.5556	0.625	0.6818	0.7292	0.7692
1.5	0.6	0.6818	0.75	0.8077	0.8571
1.75	0.6364	0.7292	0.8077	0.875	0.9333
2	0.6667	0.7692	0.8571	0.9333	1

By using Trapezoidal rule
I=hk/4{sum of values of f at the four corners+2(sum of the values of f at the remaining nodes on the boundary)+4(sum of the values of f at the interior nodes)}

$= \frac{(0.25)(0.25)}{4}$ [2.8334+2(8.7032)+4(6.6874)]

=0.0156[2.8334+17.4064+26.7496]

=0.73303

25 **Evaluate** $\int_0^1\int_0^1 \frac{1}{x+y+1}dxdy$ **by using Trapezoidal rule taking h = 0.5 and k = 0.25**

Solution:

$$I = \frac{hk}{4}[(f_{00}+f_{04}+f_{20}+f_{24})+2(f_{01}+f_{02}+f_{03}+f_{10}+f_{21}+f_{22}+f_{23}+f_{14})+4(f_{11}+f_{12}+f_{13})]$$

$$= 0.03125[2.3333+8.622+6.0616]$$

$$= 0.5318$$