DEPARTMENT OF MATHEMATICS

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UNIT – III : SOLUTIONS OF EQUATIONS & EIGEN VALUE PROBLEMS

UNIT – III SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

Newton's iterative formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Ex: To find the Iterative formula for finding $\sqrt{N}$, where N is a real number

$Let\ x = \sqrt{N} \Rightarrow x^2 - N = 0$

$Let\ f(x) = x^2 - N;\ f'(x) = 2x$

We Know that $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \left[\frac{x_n^2 - N}{2x_n}\right] = \frac{x_n}{2} + \frac{N}{2x_n} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$

The iterative formula for finding $\sqrt{N}$ is $\frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$

Diagonally Dominant:

A matrix is diagonally dominant if the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical values of the other element in that row.

Consider the system of equations,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

i.e. $|a_1| \geq |b_1| + |c_1|;\quad |b_2| \geq |a_2| + |c_2|\ ;\ |c_3| \geq |a_3| + |b_3|$

Procedure of Gauss Elimination method.

In this method the augmented matrix is transformed into an equivalent upper triangular matrix, and then by back substitution we get the solution.

Procedure of Gauss-Jordan method to solve the simultaneous equations

Answer: In this method, the augmented matrix is reduced to a diagonal matrix (or even a unit matrix) by elementary row operations. Here we get the solutions directly, without using back substitution method.

Procedure to find all the eigen values of a 3x3 matrix A by using power method.

Answer:

Step 1: Find the dominant Eigen value of A, Say λ_1, by using power method.

Step 2: Consider the matrix B = A – $\lambda_1 I$

Step 3: Again by applying power method, find the dominant Eigen value of B. Then the smallest eigen value of A is equal to λ_1+ the dominant eigen value of B

Step 4: The remaining eigen value is found by using the property.

1. Find Newton's iterative formula to find the reciprocal of a given number N and hence find the Value of $\frac{1}{19}$

Solution:

Let $x = \frac{1}{N}$ $\quad \therefore N = \frac{1}{x}$

$f(x) = \frac{1}{x} - N$; $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$; $x_{i+1} = x_i(2 - Nx_i)$

To find $\frac{1}{19}$, take $\alpha_0 = \frac{1}{20} = 0.05$

$X_1=0.0525$; $x_2=0.05263$; $x_3=0.05263$; $x_4=0.05263$.

$\therefore \frac{1}{19} = 0.05263$

2. **Find the least positive root of $x^4 - x - 10 = 0$ correct to 2 decimal places using Newton Raphson method.**

Solution:

Given f(x)= $x^4 - x - 10$, $f'(x) = 4x^3 - 1$

f(1) = -10= -ve; f(2) = 4= +ve

Therefore A root lies between 1and 2. Take $x_0=2$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$; $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$=1.871; $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$=1.856 ; $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ =1.856.

Therefore the root is 1.856

3. **Find a root of $x\log_{10} x - 1.2 = 0$ Newton's method correct to three decimal places.**

Solution:

$f(x) = x\log_{10} x - 1.2 \quad f(2) = -ve, \quad f(3) = +ve$

The root lies between 2 and 3

$take\ x_0 = \frac{2+3}{2} = 2.5 \qquad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$i = 0 \quad x_1 = 2.7465$

$i = 1 \quad x_2 = 2.74065$

$i = 2 \quad x_3 = 2.74065$ Hence the root is 2.74065

4. **Find the approximate root of $xe^x = 3$ by Newton- Raphson method correct to 3 decimal places.**

Solution:

Given f(x)= $xe^x = 3$ $f'(x) = xe^x + e^x$

f(1) = -0.2817 = -ve;

f(2) = 11.7781 = +ve

Therefore root lies between 1and 2.

Here $|f(1)| < |f(2)|$

The root is nearer to 1 Take x_0=1

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)};$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.0518; \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.0499; \quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.0499$$

The value of x_2 and x_3 are equal. Therefore the root is 1.0499

5. **Solve the following system of equations by Gauss Elimination method**

$2x + 3y - z = 5$, $4x + 4y - 3z = 3$, $2x - 3y + 2z = 2$

Solution:

$$(A,B) = \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ 2 & -3 & 4 & 2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & -6 & 3 & -3 \end{array}\right] R_2 \to R_2 - 2R_1$$

$$= \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & 6 & 18 \end{array}\right] R_3 \to R_3 - 3R_2$$

Hence

$$2x + 3y - z = 5$$
$$-2y - z = -7$$
$$6z = 18$$

$\therefore$ $z = 3$ $y = 2$ $x = 1$

6. **Apply Gauss Jordan method to find the solution of the following system:**

$x + 3y + 3z = 16,\ x + 4y + 3z = 18,\ x + 3y + 4z = 19;$

Since the coefficient of x in the third equation is unity, we interchange first and third equation, we get x +3 y +3 z = 16, x +4y +3z =18, x +3y +4z = 19.

Solution:

$$(A,B) = \left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array} = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] R_1 - 3R_2 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] R_1 - 3R_3$$

Therefore x= 1,y= 2,z=3.

7. Solve $10x+y+z=12,\ 2x+10y+z=13,\ x+y+5z=7$ by Gauss Jordan method

Solution:

x+ y +5z = 7 2x +10y + z =13 10x + y + z = 12

$$(A,B)=\left[\begin{array}{ccc|c}1&1&5&7\\2&10&1&13\\10&1&1&12\end{array}\right]$$

$$=\left[\begin{array}{ccc|c}1&1&5&7\\0&8&-9&-1\\0&-9&-49&-58\end{array}\right]\begin{matrix}R_2+(-2)R_1\\R_3+(-10)R_1\end{matrix}=\left[\begin{array}{ccc|c}1&1&5&7\\0&1&\frac{-9}{8}&\frac{-1}{8}\\0&-9&-49&-58\end{array}\right]R_2\left(\frac{1}{8}\right)$$

$$=\left[\begin{array}{ccc|c}1&1&5&7\\0&1&\frac{-9}{8}&\frac{-1}{8}\\0&-9&-\frac{473}{8}&-\frac{473}{8}\end{array}\right]=\left[\begin{array}{ccc|c}1&1&5&7\\0&1&\frac{-9}{8}&\frac{-1}{8}\\0&0&1&1\end{array}\right]R_3\left(\frac{-8}{473}\right)$$

$$=\left[\begin{array}{ccc|c}1&0&\frac{49}{8}&\frac{57}{8}\\0&1&\frac{-9}{8}&\frac{-1}{8}\\0&0&1&1\end{array}\right]R_1+(-1)R_2=\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&1\\0&0&1&1\end{array}\right]\begin{matrix}R_2+\left(\frac{9}{8}\right)R_3\\R_1+\left(\frac{-49}{8}\right)R_3\end{matrix}$$

Therefore x= 1,y= 1,z=1.

8. Solve the following system by Gauss Seidel method

$$20x+y-2z=17,\ 3x+20y-z=-18,\ 2x-3y+20z=25;$$

Solution:

The system is diagonally dominant,
Now, the system is diagonally dominant.

Therefore $x=\frac{1}{20}(17-y+2z),\ y=\frac{1}{20}(-18-3x+z),\ z=\frac{1}{20}(25-2x+3y)$

Setting y= 0,z=0 we get

$x^{(1)}=0.8500,\ y^{(1)}=-1.0275,\ z^{(1)}=1.0109$

Second iteration: $x^{(2)}=1.0025,\ y^{(2)}=-0.9998,\ z^{(2)}=0.9998$

Third iteration: $x^{(3)}=1.000,\ y^{(3)}=-1.000,\ z^{(3)}=1.000$

Hence x= 1,y= -1, z=1

9. Solve the following system by Gauss Seidel method

$$4x+2y+z=14,\ x+5y-z=10,\ x+y+8z=20;$$

Solution:

The system is diagonally dominant,
Now, the system is diagonally dominant.

Therefore $x=\frac{1}{4}(14-2y-z),\ y=\frac{1}{5}(10-x+z),\ z=\frac{1}{8}(20-x-y)$

Setting y= 0,z=0 we get

$x^{(1)} = 3.5, y^{(1)} = -1.3, z^{(1)} = 1.9$

Second iteration: $x^{(2)} = 2.375, y^{(2)} = 1.905, z^{(2)} = 1.965$

Third iteration: $x^{(3)} = 2.056, y^{(3)} = 1.982, z^{(3)} = 1.995$

Proceeding like this we get x= 2 , y= 2 , z=2.

10. Using Gauss Jordan method find the inverse of the matrix $A = \begin{pmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{pmatrix}$

Solution:

$$\text{Given } [A \mid I] = A = \left[\begin{array}{ccc|ccc} 2 & 2 & 6 & 1 & 0 & 0 \\ 2 & 6 & -6 & 0 & 1 & 0 \\ 4 & -8 & -8 & 0 & 0 & 1 \end{array}\right]$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1/2 & 0 & 0 \\ 2 & 3 & -3 & 0 & 1/2 & 0 \\ 1 & -2 & -2 & 0 & 0 & 1/4 \end{array}\right] \quad R_1 \to R_1/2, R_2 \to R_2/2, R_3 \to R_3/4$$

$$\Rightarrow A = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1/2 & 0 & 0 \\ 0 & 2 & -6 & -1/2 & 1/2 & 0 \\ 0 & -3 & -5 & -1/2 & 0 & 1/4 \end{array}\right] \quad R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$$

$$\Rightarrow A = \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/4 & -1/4 & 0 \\ 0 & 1 & -3 & -1/4 & 1/4 & 0 \\ 0 & 0 & 1 & 5/56 & -3/56 & -1/56 \end{array}\right] \quad R_3 \to R_3\left(\frac{3}{14}\right)$$

$$\Rightarrow A = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/14 & 1/14 & 6/56 \\ 0 & 1 & 0 & 1/56 & 5/56 & -3/56 \\ 0 & 0 & 0 & 5/56 & -3/56 & -1/56 \end{array}\right] \quad R_1 \to R_1 - 6R_3, R_2 \to R_2 + 3R_3$$

$$\text{Hence } A^{-1} = \frac{1}{56}\begin{bmatrix} 12 & 4 & 6 \\ 1 & 5 & -3 \\ 5 & -3 & -1 \end{bmatrix}$$

11. Using Gauss Jordan method find the inverse of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$

Solution:

$$\text{Given } [A \mid I] = A = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \quad R_2 \to R_2 - R_1, R_3 \to R_3 + 2R_1$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 1 & 3 & 1 & 0 & 0\\ 0 & 1 & -3 & -1/2 & 1/2 & 0\\ 0 & -2 & 2 & 2 & 0 & 1\end{array}\right] R_2 \to R_2/2$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 0 & 6 & 3/2 & -1/2 & 0\\ 0 & 1 & -3 & -1/2 & 1/2 & 0\\ 0 & 0 & 4 & 1 & 1 & 1\end{array}\right] R_1 \to R_1 - R_2, R_3 \to R_3 + 2R_1$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 3 & 1 & 3/2\\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4\\ 0 & 0 & 0 & -1/4 & -1/4 & -1/4\end{array}\right] R_1 \to R_1 - 6R_3, R_2 \to R_2 + 3R_3$$

Hence $A^{-1} = \frac{1}{4}\begin{bmatrix}12 & 4 & 6\\ -5 & -1 & -3\\ -1 & -1 & -1\end{bmatrix}$

12. Using Gauss Jordan method, find the inverse of the matrix $A=\begin{pmatrix}2 & 2 & 3\\ 2 & 1 & 1\\ 1 & 3 & 5\end{pmatrix}$

Solution:

Given $[A \mid I] = A = \left[\begin{array}{ccc|ccc}2 & 2 & 3 & 1 & 0 & 0\\ 2 & 1 & 1 & 0 & 1 & 0\\ 1 & 3 & 5 & 0 & 0 & 1\end{array}\right]$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 1 & 3/2 & 1/2 & 0 & 0\\ 2 & 1 & 1 & 0 & 1 & 0\\ 1 & 3 & 5 & 0 & 0 & 1\end{array}\right] R_1 \to R_1/2$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 1 & 3/2 & 1/2 & 0 & 0\\ 0 & -1 & -2 & -1 & 1 & 0\\ 0 & 2 & 7/2 & -1/2 & 0 & 1\end{array}\right] R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 1 & 3/2 & 1/2 & 0 & 0\\ 0 & 1 & 2 & 1 & 1 & 0\\ 0 & 2 & 7/2 & -1/2 & 0 & 1\end{array}\right] R_2 \to -R_2$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 0 & 1/2 & 1/2 & 1 & 0\\ 0 & 1 & 2 & 1 & -1 & 0\\ 0 & 0 & -1/2 & -5/2 & 2 & 1\end{array}\right] R_1 \to R_1 - R_2, R_3 \to R_3 - 2R_1$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 0 & 1/2 & 1/2 & 1 & 0\\ 0 & 1 & 2 & 1 & -1 & 0\\ 0 & 0 & -1/2 & -5 & -4 & -2\end{array}\right] R_3 \to -2R_3$$

$$\Rightarrow A=\left[\begin{array}{ccc|ccc}1 & 0 & 1 & 2 & -1 & -1\\ 0 & 1 & 0 & -9 & 7 & 4\\ 0 & 0 & 1 & 5 & -4 & -2\end{array}\right] R_1 \to R_1 + \frac{R_3}{2}, R_2 \to R_2 - 2R_3$$

Hence $A^{-1} = \begin{bmatrix} 2 & -1 & -1 \\ -9 & 7 & 4 \\ 5 & -4 & -2 \end{bmatrix}$

13. Find the largest eigen value and eigen vector of $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{pmatrix}$, by using Power method.

Solution:

Let A= $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.Let the initial eigenvector be $X_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$AX_1 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.857 \\ 0.285 \\ 1 \end{pmatrix} = 7X_2,$$

$$AX_2 = AX_2' = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}\begin{pmatrix} 0.857 \\ 0.285 \\ 1 \end{pmatrix} = \begin{pmatrix} 4.427 \\ 0.86 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.632 \\ 0.122 \\ 1 \end{pmatrix} = 7X_3'$$

$$AX_3 = AX_3' = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}\begin{pmatrix} 0.632 \\ 0.122 \\ 1 \end{pmatrix} = \begin{pmatrix} 3.876 \\ 1.512 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.557 \\ 0.216 \\ 1 \end{pmatrix} = 7X_4'$$

$$AX_4 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}\begin{pmatrix} 0.557 \\ 0.216 \\ 1 \end{pmatrix} = \begin{pmatrix} 4.069 \\ 1.136 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.581 \\ 0.162 \\ 1 \end{pmatrix} = 7X_5'$$

$$AX_5 = \begin{pmatrix} 3.905 \\ 1.352 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.558 \\ 0.193 \\ 1 \end{pmatrix} = 7X_6' \qquad AX_6 = \begin{pmatrix} 3.944 \\ 1.228 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.562 \\ 0.175 \\ 1 \end{pmatrix} = 7X_7'$$

$$AX_7 = \begin{pmatrix} 3.913 \\ 1.260 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.561 \\ 0.180 \\ 1 \end{pmatrix} = 7X_8' \qquad AX_8 = \begin{pmatrix} 3.929 \\ 1.280 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.560 \\ 0.182 \\ 1 \end{pmatrix} = 7X_9'$$

$$AX_9 = \begin{pmatrix} 3.921 \\ 1.280 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.560 \\ 0.182 \\ 1 \end{pmatrix} = 7X_{10}' \qquad AX_{10} = \begin{pmatrix} 3.924 \\ 1.272 \\ 7 \end{pmatrix} = 7\begin{pmatrix} 0.560 \\ 0.182 \\ 1 \end{pmatrix}$$

Therefore the eigen vector is $\begin{pmatrix} 0.560 \\ 0.182 \\ 1 \end{pmatrix}$ and the eigen value is 7.

14. Find the largest eigen value and eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, by using power method. Also

find the least latent root and hence find the third eigen value also.

Solution:

Let A= $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.Let the initial eigenvector be $\mathbf{X_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\mathbf{AX_1} = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1.\mathbf{X_2}$$

$$\mathbf{AX_2} = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = 7\begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = 7.\mathbf{X_3}$$

$$\mathbf{AX_3} = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.4286 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.5714 \\ 1.8572 \\ 0 \end{bmatrix} = 3.5714\begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix} = 3.5714.\mathbf{X_3} \quad \ldots\ldots\ldots\ldots$$

Proceeding like this we get, eigen value = 4 and eigen vector = (1 , 0.5 , 0)

To find the least eigen value B = A - 4I since $\lambda_1 = 4$

$$\therefore \quad \mathbf{B} = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 2 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\textbf{Let } \mathbf{y_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{By_1} = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} = -3\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = -3\mathbf{y_2}$$

$$\mathbf{By_2} = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 1.6666 \\ 0 \end{pmatrix} = -5\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = -5\mathbf{y_3}$$

$$\mathbf{y_2} = \begin{pmatrix} -3 & 6 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 1.6666 \\ 0 \end{pmatrix} = -5\begin{pmatrix} 1 \\ -0.3333 \\ 0 \end{pmatrix} = -5\mathbf{y_3}$$

Dominant eigen value is – 5

Adding 4, smallest eigenvalue of A = – 5 + 4 = –1

Sum of eigen value = Trace of A = 1 + 2 + 3 = 6

$$4+(-1)+\lambda_3 = 6$$

$$\lambda_3 = 3$$

The eigen values are 4, 3, –1

15. Find the numerically largest eigen value of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ **and the corresponding eigen vector, by using power method.**

Solution:

Let the initial eigenvector be $X^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$AX^{(0)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 25 \\ 1 \\ 2 \end{pmatrix} = 25\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = 25X^{(1)}.$$

$$AX^{(1)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = \begin{pmatrix} 25.2 \\ 1.12 \\ 1.68 \end{pmatrix} = 25.2\begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix} = 25.2X^{(2)}$$.(ie) $X^{(2)} = \begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix}$ .Repeating this , we get $25.1778\begin{pmatrix} 1 \\ 0.0450 \\ 0.06888 \end{pmatrix}, 25.1826\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}, 25.1821\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$. Therefore The largest eigenvalue is 25.1821 and the corresponding eigen vector is $\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$.

16. Solve the following system of equations by Gauss Elimination method

$$2x + y + 4z = 12,\ 8x - 3y + 2z = 20,\ 4x + 11y - z = 33$$

Solution:

$$(A,B) = \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 8 & -3 & 2 & 20 \\ 4 & 11 & -1 & 33 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & -28 \\ 0 & 9 & -9 & 9 \end{array}\right] R_2 \to R_2 - 4R_1,\ R_3 \to R_3 - 2R_1$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & -1 & 1 \end{array}\right] R_2 \to \frac{R_2}{-7},\ R_3 \to \frac{R_3}{9}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 3 & 3 \end{array}\right] R_3 \to R_2 - R_3$$

By Back Substitution, we get,

$3z = 3 \Rightarrow z = 1$

$y + 2z = 4 \Rightarrow y = 4 - 2 = 2$

$2x + y + 4z = 12 \Rightarrow 2x = 12 - (2) - 4(1) = 2 \Rightarrow x = 3$

Hence the solution is, x=1, y= -2, z=3

17. **Solve the following system of equations by Gauss Elimination method**

$$\mathbf{5x_1 + x_2 + x_3 + x_4 = 4,\ \ x_1 + 7x_2 + x_3 + x_4 = 12,}$$
$$\mathbf{x_1 + x_2 + 6x_3 + x_4 = -5,\ x_1 + x_2 + x_3 + 4x_4 = -6}$$

Solution:

$$5x_1 + x_2 + x_3 + x_4 = 4,\ \ x_1 + 7x_2 + x_3 + x_4 = 12,$$
$$x_1 + x_2 + 6x_3 + x_4 = -5,\ x_1 + x_2 + x_3 + 4x_4 = -6$$

$$(A,B) = \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{array}\right] \sim \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 34 & 4 & 4 & 56 \\ 0 & 4 & 29 & 4 & -29 \\ 0 & 4 & 4 & 19 & -34 \end{array}\right] R_2 \to 5R_2 - R_1,\ R_3 \to 5R_3 - R_1,\ R_4 \to 5R_4 - R_1$$

$$\sim \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 34 & 4 & 4 & 56 \\ 0 & 0 & 970 & 120 & -1210 \\ 0 & 0 & 120 & 630 & -1380 \end{array}\right] R_3 \to 34R_3 - 4R_2,\ R_4 \to 34R_4 - 4R_2$$

$$\sim \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 17 & 2 & 2 & 28 \\ 0 & 0 & 97 & 12 & -121 \\ 0 & 0 & 0 & 5967 & -11934 \end{array}\right] R_4 \to 970R_4 - 120R_3$$

Hence $5967x_4 = -11934$

$$x_4 = -2$$

$$97x_3 + 12x_4 = -121$$

$$x_3 = -1$$

$$17x_2 + 2x_3 + 2x_4 = 28$$

$$x_2 = 2$$

$$5x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 = 1$$

Hence the solution is $x_1 = 1; x_2 = 2; x_3 = -1; x_4 - 2$