DEPARTMENT OF MATHEMATICS

NAME OF THE SUBJECT : STATISTICS & NUMERICAL METHODS

SUBJECT CODE : MA8452

REGULATION : 2017

UNIT – II : DESIGN OF EXPERIMENTS

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NOTES

Analysis of Variance :

Analysis of Variance is a statistical method used to test the difference between 2 or more means. In short it is ANOVA

Uses of ANOVA:

- To test the homogeneity of several mean
- It is now frequently used intesting the linearity of the fitted regression line or in the significance of the correlation ratio

Assumptions of ANOVA

- The sample observations are independent
- The environmental effects are additive in nature
- Sample observation are coming from normal distribution/population

Experimental error:

Factors beyond the control of the experiment are known as experimental error

Aim of the design of experiment:

Aim is to control the extraneous variables so that the result could be attributed only to the experimental variables

Basic principles of design of experiments:

- Randomization
- Replication
- Local Control

Three essential steps to plan Design of experiment:

To plan an experiment the following three are essential

- A Statement of the objective. Statement should clearly mention the hypothesis to be tested
- A description of the experiment. Description should include the type of experimental material, size of the experiment and the number of replications.
- The outline of the method of analysis. The outline of the method consists of analysis of variance

Completely randomized design:

In Completely randomized design the treatments are given to the experimental units by a procedure of random allocation. It is used when the units are homogeneous.

ANOVA table for One Way classification (CRD)

Source of Variation	Sum of Squares	Degrees of freedom	Mean Square	F- Ratio
Between Samples	SSC	K-1	$MSC = \frac{SSC}{K-1}$	$F_C = \frac{MSC}{MSE}$
Within Samples	SSE	N-K	$MSE = \frac{SSE}{N-K}$	

Two-way classification or Randomized Block Design (RBD)

When data are classified according to two factors one classification is taken column wise and the other row wise. Such a classification is called two-way classification

ANOVA table for Randomized Block Design

Source of Variation	Sum of Squares	Degrees of freedom	Mean Square	F- Ratio
Column Treatment	SSC	c-1	$MSC = \frac{SSC}{c-1}$	$F_C = \frac{MSC}{MSE}$
Row Treatments	SSR	r-1	$MSC = \frac{SSR}{r-1}$	$F_R = \frac{MSR}{MSE}$
Error (or) Residual	SSE	(r-1) (c-1)	$MSE = \frac{SSE}{(r-1)(c-1)}$	

ANOVA table for Latin Square Design

Source of Variation	Sum of Squares	Degrees of freedom	Mean Square	F- Ratio
Column Treatments	SSC	n-1	$MSC = \frac{SSC}{n-1}$	$F_C = \frac{MSC}{MSE}$
Row Treatments	SSR	n-1	$MSR = \frac{SSR}{n-1}$	$F_R = \frac{MSR}{MSE}$
Between Treatments	SST	n-1	$MSK = \frac{SST}{n-1}$	$F_K = \frac{MSK}{MSE}$
Error (or) Residual	SSE	(n-1) (n-2)	$MSE = \frac{SSE}{(n-1)(n-2)}$	

ANOVA table for 2^2 factorial design

Source of Variation	Sum of Squares	Degree of freedom	Mean Square	F- Ratio
A	SS_A	1	$MS_A = \frac{SS_A}{d.f}$	$F_A = \frac{MS_A}{SS_E}$
B	SS_B	1	$MS_B = \frac{SS_B}{d.f}$	$F_B = \frac{MS_B}{SS_E}$
AB	SS_{AB}	1	$MS_{AB} = \frac{SS_{AB}}{d.f}$	$F_{AB} = \frac{MS_{AB}}{SS_E}$
Error (or) Residual	SS_E	4(r –1)	$MS_E = \frac{SS_E}{d.f}$	

PROBLEMS:

1. **The following table shows the lives in hours of 4batches of electric bulbs. [2015]**

1	1610	1610	1650	1680	1700	1720	1800
2	1580	1620	1620	1700	1750		
3	1460	1550	1600	1620	1640	1740	1820
4	1510	1520	1530	1570	1600	1680	

Perform an analysis of variance of these data and show that a significance tet dose not reject their homogeneity

Solution:

We subtract 1640 from the given values and workout with the new values of x_{ij}

Batches			**lives**	**of**	**bulbs**				T_i	n_i	$\frac{T_i^2}{n_i}$
1	-30	-30	10	40	60	80	**160**	**-**	**290**	7	12014
2	-60	0	0	60	110	-	**-**	**-**	**110**	5	2420
3	-180	-90	-40	-20	0	20	**100**	**180**	**-30**	8	113
4	-130	-120	-110	-70	-40	40	**-**	**-**	**-430**	6	30817
Total									**-60**	26	45364

N = 26   T = 98

$$C.F = \frac{T^2}{N} = 369.39$$

$$TSS = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 1950.62$$

$$SSC = \frac{(\sum T_i)^2}{n_i} - \frac{T^2}{N} = 452.25$$

$$SSE = TSS - SSC = 1498.36$$

Source of Variation	**Sum of Squares**	**Degree of freedom**	**Mean Square**	**F- Ratio**
Between Column	$SSC = 452.25$	h-1=3	$MSC = 150.75$	$\frac{15075}{6811} = 2.21$
Error	$SSE = 1498.36$	N-h=22	$MSE = 68.11$	
Total	$TSS = 1950.62$	N-1 = 25		

From the table $F_{0.05}(v_1 = 3, v_2 = 22) = 3.05$

Calculated F < Tabulated F

Conclution: Hence we accept H_0 the lives of 4 batches of bulbs do not differ significantly.

2. As head of the department of a consumers research organization you have the responsibility of testing and comparing life times of 4 brands of electric bulbs.suppose you test the life time of 3 electric bulbs each of 4 brands,the data is given below,each entry representing the life time of an electric bulb,measured in hundreds of hours.

A	B	C	D
20	25	24	23
19	23	20	20
21	21	22	20

Solution:

H_0: Here the population means are equal .

H_1 : The population mean are not equal.

	X_1	X_2	X_3	X_4	X_1^2	X_2^2	X_3^2	X_4^2
	20	25	24	23	400	625	576	529
	19	23	20	20	361	529	400	400
	21	21	22	20	441	441	484	400
TOTAL	60	69	66	63	1202	1595	1460	1329

N = Total No of Observations = 12　　　　T= Grand Total = 258

$$\text{Correction Factor} = \frac{(\text{Grand total})^2}{\text{Total No of Observations}} = 5547$$

$$TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N} = 39$$

$$SSC = \frac{\left(\sum X_1\right)^2}{N_1} + \frac{\left(\sum X_2\right)^2}{N_1} + \frac{\left(\sum X_3\right)^2}{N_1} - \frac{T^2}{N} = 15$$

(N_1 = No of element in each column)

SSE = TSS – SSC = 39 - 15 = 24

ANOVA TABLE

Source of Variation	**Sum of Squares**	**Degree of freedom**	**Mean Square**	**F- Ratio**
Between Samples	SSC=39	C-1=4-1=3	$MSC = \frac{SSC}{C-1} = 5$	$F_C = \frac{MSC}{MSE}$ =1.67
Within Samples	SSE=15	N-C=12-4=8	$MSE = \frac{SSE}{N-C} = 3$	

Cal F_C = 1.67 & Tab F_C (3,8)=4.07

Conclusion : Cal F_C < Tab F_C $\Rightarrow$ Hence we accept **H_0**

3. The accompanying data results from an experiment comparing the degree of soiling for fabric co-polymerized with the three different mixtures of methacrcylic acid. Analysis is the given classification

Mixture 1	**0.56**	**1.12**	**0.90**	**1.07**	**0.94**
Mixture 2	**0.72**	**0.69**	**0.87**	**0.78**	**0.91**

Mixture 3	**0.62**	**1.08**	**1.07**	**0.99**	**0.93**

Solution:

H_0: The true average degree of soiling is identical for 3 mixtures.

H_1 : The true average degree of soiling is not identical for 3 mixtures.

We shift the origin

	X_1	X_2	X_3	TOTAL	X_1^2	X_2^2	X_3^2
	0.56	0.72	0.62	**1.9**	0.3136	0.5184	0.3844
	1.12	0.69	1.08	**2.89**	1.2544	0.4761	1.1664
Total	0.90	0.87	1.07	**2.84**	0.8100	0.7569	1.1449
	1.07	0.78	0.99	**2.84**	1.1449	0.6084	0.9801
	0.94	0.91	0.93	**2.78**	0.8836	0.8281	0.8649
	4.59	**3.97**	**4.69**		**4.4065**	**3.1879**	**4.5407**

N = Total No of Observations = 15
T= Grand Total = 13.25

$$\text{Correction Factor} = \frac{(\text{Grand total})^2}{\text{Total No of Observations}} = 11.7042$$

$$TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N} = 0.4309$$

$$SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - \frac{T^2}{N} = 0.0608$$ (N_1 = No of element in each column)

SSE = TSS – SSC = 0.4309 – 0.0608 = 0.3701

ANOVA TABLE

Source of Variation	**Sum of Squares**	**Degree of freedom**	**Mean Square**	**F- Ratio**
Between Samples	SSC=0.0608	C-1=3-1=2	$MSC = \frac{SSC}{C-1}$ =0.030 4	$F_C = \frac{MSE}{MSC}$ =10.144
Within Samples	SSE=0.3701	N-C=15-3=12	$MSE = \frac{SSE}{N-C}$ =0.30 84	

Cal F_C = 10.144 & Tab F_C (12,2)=19.41

Conclusion : Cal F_C < Tab F_C $\Rightarrow$ Hence we accept **H_0**

4. Analyse the following RBD and find the conclusion

Treatments		T1	T2	T3	T4
Blocks	B1	12	14	20	22
	B2	17	27	19	15
	B3	15	14	17	12
	B4	18	16	22	12
	B5	19	15	20	14

Solution:

H_0: There is no significant difference between blocks and treatment

H_1 : There is no significant difference between blocks and treatment

We subtract 15 from the given value

	T1	T2	T3	T4	Total=T_i	$[T_i^2]/k$	ΣX_{ij}^2
B1	-3	-1	5	7	**8**	16	84
B2	2	12	4	0	**18**	81	164
B3	0	-1	2	-3	**-2**	1	14
B4	4	0	5	-1	**8**	16	42
B5	4	0	5	-1	**8**	16	42
Total=T_j	**6**	**11**	**23**	**0**	**40**	130	372
$[T_j^2]/h$	7.2	24.2	105.8	0	137.2		
$\sum y_{ij}^2$	38	147	119	68	372		

N = 20

T=Grand Total = 40 ;

$$\text{Correction Factor} = \frac{(Grand\ \ total)^2}{Total\ \ No\ of\ \ Observations} = \frac{(40)^2}{20}$$

$$TSS = \sum\sum X_{ij}^{2} - \frac{T^2}{N} = 292$$

$$SSC = \frac{\sum T_J^{2}}{h} - C.F = 57.2$$

$$SSR = \sum\sum Y_{ij}^{2} - \frac{T^2}{N} = 50$$

SSE = TSS – SSC – SSR = 184.8

ANOVA Table

Source of Variation	Sum of Squares	Degree of freedom	Mean Square	F- Ratio	F_{Tab} Ratio
Between Rows (Blocks)	SSR = 50	h - 1= 3	MSR=12.5	F_R = 1.232 F_C= 1.238	$F_{5\%}(12,4)$ = 5.91 $F_{5\%}(3, 12)$ = 3.49
Between Columns (Treatments)	SSC= 57.2	k – 1=4	MSC = 19.07		
Residual	SSE = 184.8	(h – 1)(k – 1) = 12	MSE = 15.4		
Total	292				

Conclusion : Cal F_C < Tab F_C and Cal F_R < Tab F_R $\Rightarrow$ hence the difference between the blocks and that treatments are not significant

5. **Consider the results given in the following table for an experiment involving 6 treatments in 4 randomized blocks. The treatments are indicated by numbers with in the paranthesis.**

1	**(1) 24.7**	**(3) 27.7**	**(2) 20.6**	**(4) 16.2**	**(5) 16.2**	**(6) 24.9**
2	**(3) 22.7**	**(2) 28.8**	**(1) 27.3**	**(4) 15**	**(6) 22.5**	**(5) 17**
3	**(6) 26.3**	**(4) 19.6**	**(1) 38.5**	**(3) 36.8**	**(2) 39.5**	**(5) 15.4**
4	**(5) 17.7**	**(2) 31**	**(1) 28.5**	**(4) 14.1**	**(3) 34.9**	**(6) 22.9**

Test whether the treatments differ significantly [($F_{0.05}$ (3,15)=5.42, $F_{0.05}$ (5,15)=4.5]

Solution:

We subtract the origin to 25 and workout with new values of $\mathbf{X_{ij}}$

	1	**2**	**3**	**4**	**5**	**6**	**Total =T_i**	**$[T_i]/k$**	**ΣX_{ij}^2**
1	-0.3	-404	2.7	-8.8	-8.8	-0.1	**-19.7**	64.68	181.6 3
2	22.3	3.8	-2.3	-10	-8	-2.5	**-16.7**	46.48	195.2 7

3	13.5	14.5	11.8	-5.4	-9.6	1.3	**26.1**	113.54	654.75
4	3.5	6	9.9	-10.9	-7.3	-2.4	**-1.2**	0.24	324.12
Total =T_j	**19**	**19.9**	**22.1**	**-35.1**	**-33.7**	**-3.7**	**-11.5**	224.94	1355.77
$[T_j^2]/h$	90.25	99	122.10	3.8	283.92	3.42	906.69		

T=Grand Total = -11.5 ;

Correction Factor = $\dfrac{(Grand\ \ total)^2}{Total\ \ No\ of\ \ Observations} = \dfrac{(-11.5)^2}{24}$

$$SSR = \frac{\sum T_i^2}{k} - C.F = 224.94 - \frac{(-11)^2}{24} = 219.43$$

$$SSC = \frac{\sum T_j^2}{h} - C.F = 906.69 - \frac{(-11)^2}{24} = 901.18$$

SSE = TSS – SSC – SSR = 229.65

ANOVA Table

Source of Variation	**Sum of Squares**	**Degree of freedom**	**Mean Square**	**F- Ratio**	**F_{Tab} Ratio**
Between Rows (Blocks)	SSR=219.43	h - 1= 3	MSR=73.14	4.78 11.75	$F_{5\%}(3, 15)$ = 5.42 $F_{5\%}(5,15)$ = 4.5
Between Columns (Treatments)	SSC=901.18	k – 1=5	MSC =180.24		
Residual	SSE=229.65	(h – 1)(k – 1) =15	MSE =15.31		
Total	1350.26				

Conclusion : Cal F_C < Tab F_C and Cal F_R > Tab $F_R \Rightarrow$ There is no significant difference between the blocks and there is significant difference between the Treatments.

6. Three Varieties A,B,C of a crop are tested in a randomized block design with four replications. The plot yield in pounds are as follows

A	6	C	5	A	8	B	9
C	8	A	4	B	6	C	9
B	7	B	6	C	10	A	6

Analyze the experimental yield and state your conclusion

Solution:

The table can be given as

	I	II	III	IV
A	6	4	8	6
B	7	6	6	9
C	8	5	10	9

We shift the origin $X_{ij} = x_{ij} - 6$; h = 3; k = 4; N = 12

	I	II	III	IV	Total=T_{i*}	$[T_{i*}^2]/k$	ΣX_{*ij}^2
A	0	-2	2	0	**0**	0	8
B	1	0	0	3	**4**	4	10
C	2	-1	4	3	**8**	16	30
Total=T_{*j}	**3**	**-3**	**6**	**6**	**12**	20	48
$[T_{*j}^2]/h$	3	3	12	12	30		

T=Grand Total = 12

$$\text{Correction Factor} = \frac{(Grand\ \ total)^2}{Total\ \ No\ of\ \ Observations} = \frac{(12)^2}{12} = 12$$

$$TSS = \sum_i \sum_j X_{ij}^2 - C.F = 48 - 12 = 36$$

$$SSR = \frac{\sum T_{i*}^{\ 2}}{k} - C.F = 20 - 12 = 8$$

$$SSC = \frac{\sum T_{*j}^{\ 2}}{h} - C.F = 30 - 12 = 12$$

SSE = TSS – SSC – SSR = 36-8-18 =10

ANOVA Table

Source of Variation	**Sum of Squares**	**Degree of freedom**	**Mean Square**	**F- Ratio**	**F_{Tab} Ratio**
Between Rows (Workers)	SSR=8	h - 1= 2	MSR=4		

Between Columns (Machine)	SSC=18	k – 1=3	MSC = 6	F_R = 2.4 F_C = 3.6	$F_{5\%}(2, 6)$ =5.14 $F_{5\%}(3, 6)$ =4.76
Residual	SSE = 10	(h – 1)(k – 1) = 6	MSE = 5/3		
Total	36				

Conclusion : Cal F_C < Tab F_C and Cal F_R < Tab F_R ⇒ There is no significant difference between the crops and no significant difference between the plots

7. **An experiment was designed to study the performances of 4 different detergents for cleaning fuel injectors. The following "cleanliness" readings were obtained with specially designed experiment for 12 tanks of gas distributed over 3 different models of engines:**

ENGINES		**I**	**II**	**III**
DETERGENTS	**A**	**45**	**43**	**51**
	B	**47**	**46**	**52**
	C	**48**	**50**	**55**
	D	**42**	**37**	**49**

Perform the ANOVA and test at 0.01 level of significance whether there are difference in the detergents or in the engines.

<u>Solution</u> :

We shift the origin $X_{ij} = x_{ij} - 50$; h = 4; k = 3; N = 12

	I	**II**	**III**	**Total=T_{i*}**	**$[T_{i*}^2]/k$**	**ΣX_{*ij}^2**
A	-5	-7	1	**-11**	40.33	75
B	-3	-4	2	**-5**	8.33	29
C	-2	0	5	**3**	3	29
D	-8	-13	-1	**-22**	161.33	234
Total=T_{*j}	**-18**	**-24**	**7**	**-35**	212.99	367
$[T_{*j}^2]/h$	81	144	12.25	237.25		

T=Grand Total = -35 , Correction Factor = $\frac{(Grand\ \ total)^2}{Total\ \ No\ of\ \ Observations} = \frac{(-35)^2}{12}$

$$TSS = \sum_i \sum_j X_{ij}^2 - C.F = 367 - \frac{(-35)^2}{12} = 264.92$$

$$SSR = \frac{\sum T_{i*}^{\ 2}}{k} - C.F = 212.99 - \frac{(-35)^2}{12} = 110.91$$

$$SSC = \frac{\sum T_{*j}^{\ 2}}{h} - C.F = 237.25 - \frac{(-35)^2}{12} = 135.17$$

SSE = TSS – SSC – SSR = 264.92-110.91-135.17 = 18.

ANOVA Table

Source of Variation	Sum of Squares	Degree of freedom	Mean Square	F-Ratio	F Tab Ratio
Between Rows (DETERGENTS)	SSR=110.91	h - 1= 3	MSR=36.97	F_R = 11.774	$F_{5\%}(3, 6)$ = 4.76
Between Columns (ENGINES)	SSC=135.17	k – 1=2	MSC = 7.585	F_C = 21.52	$F_{5\%}(2, 6)$ = 5.14
Residual	SSE= 18.84	(h – 1)(k – 1) = 6	MSE = 3.14		
Total	264.92				

Conclusion :
Cal F_C > Tab F_C and Cal F_R > Tab F_R ⇒ There is significant difference between the **DETERGENTS** and significant difference between the ENGINES

8. A set of data involving four " four tropical feed stuffs A,B,C,D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data (Apr/May 2017)

A	**55**	**49**	**42**	**21**	**52**
B	**61**	**112**	**30**	**89**	**63**
C	**42**	**97**	**81**	**95**	**92**
D	**169**	**137**	**169**	**85**	**154**

Solution:

H_0 : There is no significant difference between column means as well as row means

H_1 : There is no significant difference between column means as well as row means

	X_1	X_2	X_3	X_4	X_5	Total	X^2_1	X^2_2	X^2_3	X^2_4	X^2_5
A	5	-1	-8	-29	2	**-31**	25	1	64	841	4
B	11	62	-20	39	13	**105**	121	3844	400	1521	169
C	-8	47	31	45	42	**157**	64	2209	961	2025	1764
D	119	87	119	35	104	**464**	14161	7569	14161	1225	10816
Total	**127**	**195**	**122**	**90**	**161**	**695**	**14371**	**13623**	**15586**	**5612**	**12753**

N = 20 T = 695

$$C.F = \frac{T^2}{N} = 24151.25$$

$$TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 \sum X_4^2 + \sum X_5^2 - \frac{T^2}{N} = 37793.75$$

$$SSC = \frac{\left(\sum X_1\right)^2}{N_1} + \frac{\left(\sum X_2\right)^2}{N_1} + \frac{\left(\sum X_3\right)^2}{N_1} + \frac{\left(\sum X_4\right)^2}{N_1} + \frac{\left(\sum X_5\right)^2}{N_1} - \frac{T^2}{N} = 1613.50$$

(N_1 = No of element in each column)

$$SSR = \frac{\left(\sum Y_1\right)^2}{N_2} + \frac{\left(\sum Y_2\right)^2}{N_2} + \frac{\left(\sum Y_3\right)^2}{N_2} + \frac{\left(\sum Y_4\right)^2}{N_2} + \frac{\left(\sum Y_5\right)^2}{N_2} - \frac{T^2}{N} = 26234.95$$

(N_2 = No of element in each row)

SSE = TSS – SSC-SSR = 37793.75 – 1613.5 – 26234.95 = =9945.3

ANOVA TABLE

S.V	DF	SS	MSS	F cal	F tab
Column treatment	c-1= 5-1 =4	SSC=1613.5	$MSC = \frac{SSC}{C-1} = 403.375$	$F_C = \frac{MSC}{MSE} = 2.055$	3.26
Between Row	r-1=4-1=3	SSR=26234.95	$MSR = \frac{SSR}{R-1} = 8744.98$	$F_C = \frac{MCR}{MSE} = 10.552$	3.49
Error	N-c-r+1=12	SSE=9945.3	$MSE = \frac{SSE}{12} = 828.775$		

Conclusion: Cal F_c <Tab F_c , Accept H_0

Cal F_R > Tab F_R , Reject H_0

9. Three varieties of coal were analysed by 4 chemists and the ash content is given below. Perform an ANOVA Table

		Chemists			
		A	**B**	**C**	**D**
COAL	**I**	**8**	**5**	**5**	**7**
	II	**7**	**6**	**4**	**4**
	III	**3**	**6**	**5**	**4**

Solution:

		Chemists				
		A	**B**	**C**	**D**	**TOT**
COAL	**I**	8	5	5	7	**25**
	II	7	6	4	4	**21**
	III	3	6	5	4	**18**
	TOT	**18**	**17**	**14**	**15**	**64**

N = 12

T = 64

$$C.F = \frac{T^2}{N} = 341.33$$

$$TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 \sum X_4^2 - \frac{T^2}{N} = 24.67$$

$$SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - \frac{T^2}{N} = 3.34$$

(N_1 = No of element in each column)

$$SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} - \frac{T^2}{N} = 6.17$$

(N_2 = No of element in each row)

SSE = TSS – SSC - SSR = 24.67 – 3.34 – 6.17 = 15.16

ANOVA TABLE

S.V	DF	SS	MSS	F cal	F tab
Column treatment	C-1= 4-1 =3	SSC=3.34	$MSC = \frac{SSC}{C-1} = 1.11$	$F_C = \frac{MSE}{MSC} = 2.28$	3.49
Between Row	R-1=3-1=2	SSR=6.17	$MSR = \frac{SSR}{R-1} = 3.09$	$F_C = \frac{MCR}{MSE} = 1.22$	3.26
Error	N-c-R+1=6	SSE=15.16	$MSE = \frac{SSE}{12} = 2.53$		

Conclusion: Cal F_c > Tab F_c , Reject H_0

Cal F_R > Tab F_R , Reject H_0

10. **The following is the latin square of a design when 4 varieties of seed are being tested. Set up the analysis of variance table and state your conclusion. You can carry out the suitable change of origin and scale**

A 110	**B 100**	**C 130**	**D 120**
C 120	**D 130**	**A 110**	**B 110**
D 120	**C 100**	**B 110**	**A 120**
B 100	**A 140**	**D 100**	**C 120**

Solution:

Subtracting 100 and dividing by 10

	1	**2**	**3**	**4**	**Total=T_{i*}**	**$[T_{i*}^2]/n$**	**ΣX_{*ij}^2**
1	A1	B0	C3	D2	6	9	14
2	C2	D3	A1	B1	7	12.25	15
3	D2	C0	B1	A2	5	6.25	9
4	B0	A4	D0	C2	6	9	20
Total=T_{*j}	5	7	5	7	24	36.5	58
$[T_{*j}^2]/n$	6.25	12.25	6.25	12.25	37		
Σy_{ij2}	9	25	11	13	58		

	Letters				Total=T_K	$[T_K^2]/n$
A	1	1	2	4	**8**	16
B	0	1	1	0	**2**	1
C	3	2	0	2	**7**	12.25
D	2	3	2	0	**7**	12.25
Total					**24**	41.5

$Q=\sum\sum Y_{ij}^2-\frac{T^2}{N}=22 \qquad Q_1=\frac{1}{n}\sum T_{i2}-\frac{T^2}{N}=0.5$

$Q_2=\frac{1}{n}\sum T_{j2}-\frac{T^2}{N}=1$

$Q_3=\frac{1}{n}\sum T_{K2}-\frac{T^2}{N}=5.5$

$Q_4=Q-Q_1-Q_2-Q_3=15$

Source of Variation	Sum of Squares	Degree of freedom	Mean Square	F- Ratio	F_{Tab} Ratio (5% level)
Between Rows	0.5	3	0.167	F_R =14.97	$F_R(6,36)=8.9$
Between Columns	1	3	0.333		4
Between Letters	5.5	3	1.833	F_C =7.508	$F_C(6,3)=8.94$
Residual	15	6	2.5	F_L =1.364	$F_L(6\ ,3)=8.94$
Total	22	15			

Conclusion:

$Cal\ F_R > Tab\ F_R$, $Cal\ F_C < Tab\ F_C$, $Cal\ F_L < Tab\ F_L$ **There is a significant difference between rows and no significant difference between column and also between letters.**

11. **A Company wants to produce cars for its own use. It has to select the make of the car out of the four makes A,B,C,D available in the market. For this he tries 4 cars of each make by assigning the cars to 4 drivers to run on 4 different routes. The efficiency of the cars is measured in terms of time in hours.**
Analyse the experinment data and draw conclusion ($F_{0.05}(3,5)=5.41$).

18(C)	12(D)	16(A)	20(B)
26(D)	34(A)	25(B)	31(C)
15(B)	22(C)	10(D)	28(A)
30(A)	20(B)	15(C)	9(D)

Solution:

We subtract 20 from the given value and workout with new value of X_{ij}

	1	2	3	4	Total=T_i	$[T_i^2]/n$	ΣX_{ij}^2
1	C -2	D -8	A -4	B 0	**-14**	49	84
2	D 6	A 14	B 5	C 11	**36**	324	378
3	B -5	C 2	D -10	A 8	**-5**	6.25	193
4	A 10	B 0	C -5	D -11	**-6**	9	246
Total=T_j	**9**	**8**	**-14**	**8**	**11**	388.25	901
$[T_j^2]/n$	20.25	16	49	16	101.25		
ΣX_i^2	165	264	166	306	901		

	Letters				Total=T_K	$[T_K^2]/n$
A	-4	14	8	10	**28**	196
B	0	5	-5	0	**0**	0
C	-2	11	2	-5	**6**	9
D	-8	6	-10	-11	**-23**	132.25
Total					**11**	337.25

T=Grand Total = 11 ; Correction Factor = $\frac{(Grand\ \ total)^2}{Total\ \ No\ of\ \ Observations} = \frac{(11)^2}{16}$

$$TSS = \sum_i \sum_j X_{ij}^2 - C.F = 901 - \frac{(11)^2}{16} = 893.438$$

$$SSR = \frac{\sum T_{i*}^{\ 2}}{n} - C.F = 388.25 - \frac{(11)^2}{16} = 380.688$$

$$SSC = \frac{\sum T_j^{\ 2}}{n} - C.F = 101.25 - \frac{(11)^2}{16} = 93.688$$

$$SSL = \frac{\sum T_K^{\ 2}}{n} - C.F = 337.25 - \frac{(11)^2}{16} = 329.688$$

SSE = TSS – SSC – SSR-SSL = 89.374

Source of Variation	Sum of Squares	Degree of freedom	Mean Square	F- Ratio	F_{Tab} Ratio (5% level)
Between Rows	SSR=380.688	n - 1= 3	MSR=126.896	F_R = 8.519	F_R(3, 6)=4.76
Between Columns	SSC=93.688	n - 1= 3	MSC =31.229	F_C =2.096	F_C(3, 6)=4.76
Between Letters	SSL = 329.688	n - 1= 3	MSL=109.896	F_L =7.378	F_L(3, 6)=4.76
Residual	SSE= 89.374	(n – 1)(n – 2) = 6	MSE = 14.896		
Total	893.438				

Conclusion :

Cal F_C < Tab F_C , Cal F_L > Tab F_L and Cal F_R > Tab F_R $\Rightarrow$ There is significant difference between the **rows** , no significant difference between the column and significant difference between the letters

12. **A variable trial was conducted on wheat with 4 varieties in a Latin square Design. The plan of the experiment and the per plot yield are given below:**

C 25	**B 23**	**A 20**	**D 20**
A 19	**D 19**	**C 21**	**B 18**
B 19	**A 14**	**D 17**	**C 20**
D 17	**C 20**	**B 21**	**A 15**

Solution:

Subtract 20 from all the items

	X_1	X_2	X_3	X_4	**Total**	X^2_1	X^2_2	X^2_3	X^2_4
Y_1	5	3	0	0	**8**	25	9	0	0
Y_2	-1	-1	1	-2	**-3**	1	1	1	4
Y_3	-1	-6	-3	0	**10**	1	36	9	0
Y_4	-3	0	1	-5	**-7**	9	0	1	25
Total	**0**	**-4**	**-1**	**-7**	**-12**	**36**	**46**	**11**	**29**

H_0 : There is no significant difference between rows, columns & treatments.

H_1 : There is significant difference between rows, columns & treatments.

N = 16 T = -12

C.F = $\frac{T^2}{N} = 9$ $TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 \sum X_4^2 + \sum X_5^2 - \frac{T^2}{N} = 113$

$$SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} + \frac{(\sum X_4)^2}{N_1} + \frac{(\sum X_5)^2}{N_1} - \frac{T^2}{N} = 7.5$$

(N_1 = No of element in each column)

$$SSR = \frac{(\sum Y_1)^2}{N_2} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_2} + \frac{(\sum Y_4)^2}{N_2} + \frac{(\sum Y_5)^2}{N_2} - \frac{T^2}{N} = 46.5$$

(N_2 = No of element in each row)

SSK:

					T
A	0	-1	-6	-5	-12
B	3	-2	-1	1	1
C	5	1	0	0	6
D	0	-1	-3	-3	-7

SSK = $\frac{(-12)^2}{4} + \frac{(1)^2}{4} + \frac{(6)^2}{4} + \frac{(-7)^2}{4} - \frac{T^2}{N} = 48.5$

SSE = TSS – SSC - SSR = 113-7.5-46.5-48.5 = 10.5

ANOVA TABLE

S.V	DF	SS	MSS	F cal	F tab
Column treatment	k-1=3	SSC=7.5	$MSC = \frac{SSC}{K-1} = 2.5$	$F_C = \frac{MSC}{MSE} = 1.43$	4.76
Between Row	k-1=3	SSR=46.5	$MSR = \frac{SSR}{K-1} = 15.5$	$F_R = \frac{MCR}{MSE} = 8.86$	4.76
Between Treatment	k-1=3	SSK=48.5	$MSK = \frac{SSK}{K-1} = 16.17$	$F_T = \frac{MSK}{MSE} = 9.24$	4.76
Error	(k-1)(k-2) = 6	SSE=10.5	$MSE = \frac{SSE}{(K-1)(K-2)} = 1.75$		

Conclusion: Cal F_c <Tab F_c

Cal F_R > Tab F_R

Cal F_T > Tab F_T

There is significant difference between treatment and rows but there is no significant difference between columns.

13. Analyse 2^2 factorial experiment for the following table

Block	Treatment			
I	**(1)**	**kp**	**k**	**p**
	64	**6**	**25**	**30**

II	k	(1)	kp	P
	14	75	33	50
IIII	kp	p	k	(1)
	17	41	12	76
IV	p	k	(1)	kp
	25	33	75	10

Solution:

Treatment	I	II	III	IV
(l)	64	75	76	75
(k)	25	14	12	33
(p)	30	50	41	25
(kp)	6	33	17	10

We shift the origin $X_{ij} = x_{ij} - 37$;

Treatment	I	II	III	IV	Total=T_{i*}	$[T_{i*}^2]/n$	ΣX_{*ij}^2
(l)	27	38	39	38	142	5041	5138
(k)	-12	-23	-25	-4	-64	1024	1314
(p)	7	13	4	-12	12	36	378
(kp)	-31	-4	-20	-27	-82	1681	2106
Total=T_{*j}	-9	24	-2	-5	8	7782	8936
$[T_{*j}^2]/n$	20.25	144	1	6.25	171.5		

T=Grand Total = 8:   N=16

Correction Factor = $\frac{(Grand\ \ total)^2}{Total\ \ No\ of\ \ Observations} = \frac{(8)^2}{16} = 4$

$$TSS = \sum_i \sum_j X_{ij}^2 - C.F = 8936 - 4 = 8932$$

$$SSR = \frac{\sum T_{i*}^{\ 2}}{n} - C.F = 7782 - 4 = 7778$$

$$SSC = \frac{\sum T_{*j}^{\ 2}}{n} - C.F = 171.5 - 4 = 167.5$$

SSE = TSS – SSC – SSR = 8932 – 7778 – 167.5 = 986.5

[k] = [kp] – [p] + [k] – [1] =-300 ; [p] = [kp] + [p] - [k] – [1] =-148

[kp] = [kp] – [p] - [k] + [1] =126

$S_k = [k]^2 /4r = 5625$; $S_p = [p]^2 /4r = 1369$; $S_{kp} = [kp]^2 /4r = 992.2$

ANOVA Table

Source of Variatio	Sum of Squares	Degree of freedom	Mean Square	F- Ratio	F_{Tab} Ratio

n					
K	5625	1	5625	$F_k = 51.32$ $F_p = 12.49$ $F_{kp} = 9.05$	$F_{5\%}(1, 9) =$ 6.99 $F_{5\%}(1, 9) =$ 6.99 $F_{5\%}(1, 9) =$ 6.99
P	1369	1	1369		
Kp	992.25	1	992.25		
Error	986.5	9	109.6		

Conclusion : Cal F_k > Tab F_k , Cal F_p > Tab F_p and Cal F_{kp} > Tab F_{kp} ⇒ There is significant difference between the treatments.

14. Given the following observation for the 2 factors A & B at two levels compute (i) the main effect (ii) make an analysis of variance.

Treatment Combination	Replication I	Replication II	Replication III
(1)	**10**	**14**	**9**
A	**21**	**19**	**23**
B	**17**	**15**	**16**
AB	**20**	**24**	**25**

Solution:

H_0 : No difference in the Mean effect.

H_1 :Tthe is a difference in the Mean effect.

We code the data by subtracting 20

Treatment	Replication			Total	X^2_1	X^2_2	X^2_3
(l)	-10	-6	-11	**-27**	100	36	121
(a)	1	-1	3	**3**	1	1	9
(b)	-3	-5	-4	**-12**	9	25	16
(ab)	0	4	5	**9**	0	16	25
Total				**-27**	**110**	**78**	**171**

T= Grand Total = -27 N =12

$$\text{Correction Factor} = \frac{(Grand\quad total)^2}{Total\quad No\quad of\quad Observations} = \frac{(-27)^2}{12} = 60.75$$

A Contract = a + ab – b – (1) = 3+9+-(12)-(-27) = 51

B Contract = b + ab – a – (1) = -12+9-3-(-27) = 21

A Contract = (1) + ab – a – b = - 27+9-3-(-12) = -9

(i) Main effects of A = A Contract / 2n = 51/6 = 8.5

Main effects of B = B Contract / 2n = 21/6 = 3.5

Main effects of AB = AB Contract / 2n = -9 / 6 = -1.5

$$TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N} = 110 + 78 + 171 - 60.75 = 298.25$$

$$SSA = \frac{(A\,\text{contract})^2}{4n} = \frac{(51)^2}{12} = 216.75$$

$$SSB = \frac{(B\,\text{contract})^2}{4n} = \frac{(21)^2}{12} = 36.75$$

$$SSAB = \frac{(AB\,\text{contract})^2}{4n} = \frac{(-9)^2}{12} = 6.75$$

SSE = TSS – SSA – SSB - SSAB = 298.25 – 216.75 – 36.75 – 6.75 = 38

ANOVA Table

Source of Variation	Sum of Squares	Degree of freedom	Mean Square	F- Ratio	F_{Tab} Ratio
A	SSA=216.75	1	MSA=216.75	$F_A = 45.63$	$F_{5\%}(1, 8) = 5.32$
B	SSB=36.75	1	MSB=36.75	$F_B = 7.74$	$F_{5\%}(1, 8) = 5.32$
AB	SSAB=6.75	1	MSAB=6.75	$F_{AB} = 1.42$	$F_{5\%}(1, 8) = 5.32$
Error	SSE=38	4(n-1)=8	MSE=4.75		
Total	TSS=298.25	4n-1=11			

Conclusion : Cal F_A > Tab F_A , Reject H_0
Cal F_B > Tab F_B Reject H_0
Cal F_{AB} < Tab F_{AB} Accept H_0

15. **The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory. Test whether the difference among the four sample means can be attributed to chance at α = 0.01 ..**

Technician	I	II	III	IV
Day 1	6	14	10	9
Day 2	14	9	12	12
Day 3	10	12	7	8
Day 4	8	10	15	10
Day 5	11	14	11	11

Solution:

H_0: There is no significant difference between the technicians

H_1 : Significant difference between the technicians

We shift the origin

	X_1	X_2	X_3	X_4	TOTAL	X_1^2	X_2^2	X_3^2	X_4^2
Total	-4	4	0	-1	**-1**	16	16	0	1
	4	-1	2	2	**7**	16	1	4	4

	0	2	-3	-2	**-3**	0	4	9	4
	-2	0	5	0	**3**	4	0	25	0
	1	4	1	1	**7**	1	16	1	1
	-1	**9**	**5**	**0**	**13**	**37**	**37**	**39**	**10**

N= Total No of Observations = 20 T=Grand Total = 13

Correction Factor = $\frac{(\text{Grand total})^2}{\text{Total No of Observations}} = 8.45$

$$TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - C.F = 37 + 37 + 39 + 10 - 8.45 = 114.55$$

$$SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_1} + \frac{(\sum X_3)^2}{N_1} - C.F = \frac{(-1)^2}{5} + \frac{(9)^2}{5} + \frac{(5)^2}{5} + 0 - 8.45 = 12.95$$

SSE = TSS – SSC = 114.55-12.95= 101.6

ANOVA Table

Source of Variation	**Sum of Squares**	**Degree of freedom**	**Mean Square**	**F- Ratio**
Between Samples	SSC=12.95	C-1= 4-1=3	MSC = $\frac{SSC}{C-1}$ =4.317	$F_C = \frac{MSC}{MSE}$ =1.471
Within Samples	SSE=101.6	N-C=20-4=16	MSE = $\frac{SSE}{N-C}$ =6.35	

Cal F_C = 1.471 & Tab F_C (16,3)=5.29

Conclusion : Cal F_C < Tab F_C ⇒ There is no significance difference between the technicians

16. **The following data represent the number of units of production per day turned out by different workers using 4 different types of machines. [May/June-2013]**

	Machine type	**A**	**B**	**C**	**D**
Workers	**1**	**44**	**38**	**47**	**36**
	2	**46**	**40**	**52**	**43**
	3	**34**	**36**	**44**	**32**
	4	**43**	**38**	**46**	**33**
	5	**38**	**42**	**49**	**39**

(1) **Test whether the five men differ with respect to mean productivity and**
(2) Test whether the mean productivity is the same for the four different machine types.
Solution:

H_0: There is no significant difference between the Machine types and no significant difference between the Workers

H_1 : Significant difference between the Machine types and no significant difference between the Workers

We shift the origin $X_{ij} = x_{ij} - 46$; h = 5; k = 4; N = 20

	A	B	C	D	Total=T_{i*}	$[T_{i*}^2]/k$	ΣX_{*ij}^2
1	-2	-8	1	-10	**-19**	90.25	169
2	0	-6	6	-3	**-3**	2.25	81
3	-12	-10	-2	-14	**-38**	361	444
4	-3	-8	0	-13	**-24**	144	242
5	-8	-4	3	-7	**-16**	64	138
Total =T_{*j}	**-25**	**-36**	**8**	**-47**	**-100**	661.5	1074
$[T_{*j}^2]/h$	125	259.2	12.8	441.8	838.8		

T=Grand Total = -100

$$\text{Correction Factor} = \frac{(Grand\ \ total)^2}{Total\ \ No\ of\ \ Observations} = \frac{(-100)^2}{20} = 500$$

$$TSS = \sum_i \sum_j X_{ij}^2 - C.F = 1074 - 500 = 574$$

$$SSR = \frac{\sum T_{i*}^{\ 2}}{k} - C.F = 661.5 - 500 = 161.5$$

$$SSC = \frac{\sum T_{*j}^{\ 2}}{h} - C.F = 838.8 - 500 = 338.8$$

SSE = TSS – SSC – SSR = 574 – 161.5 – 338.8 = 73.7

ANOVA Table

Source of Variation	**Sum of Squares**	**Degree of freedom**	**Mean Square**	**F- Ratio**	**F_{Tab} Ratio**
Between Rows (Workers)	SSR=161.5	h - 1= 4	MSR = 40.375	F_R = 6.574	$F_{5\%}(4, 12)$ = 3.26
Between Columns (Machine)	SSC=338.8	k – 1=3	MSC = 112.933	F_C = 18.388	$F_{5\%}(3, 12)$ = 3.59
Residual	SSE = 73.7	(h – 1)(k – 1) = 12	MSE = 6.1417		
Total	1074				

Conclusion : Cal F_C < Tab F_C and Cal F_R < Tab $F_R \Rightarrow$ There is no significant difference between the Machine types and no significant difference between the Workers