Catalog

3.1 LTI SYSTEM

If a system has both the linearity and time invariant properties, then this system is called linear time invariant (LTI) system

This is a linear first order differential equation with constant coefficients (assuming a and b are constants)

$$\frac{d}{dt}y(t) - ay(t) = bx(t)$$

The general nth order linear DE with constant equations is

$$a_0 y(t) + a_1 \frac{d}{dt}y(t) + \ldots + a_{n-1}\frac{d^{n-1}}{dt^{n-1}}y(t) + a_n \frac{d^n}{dt^n}y(t) =$$

$$b_0 x(t) + b_1 \frac{d}{dt}x(t) + \ldots + b_{m-1}\frac{d^{m-1}}{dt^{m-1}}x(t) + b_m \frac{d^m}{dt^m}x(t)$$

which we can write as:

$$\sum_{k=0}^{n} a_k \frac{d^k}{dt^k}y(t) = \sum_{k=0}^{m} b_k \frac{d^k}{dt^k}x(t).$$

3.2. BLOCK DIAGRAM REPRESENTATION

System Realization

There are four types of system realization in continuous time linear time invariant systems. They are

- Direct form I realization
- Direct form II realization
- Cascade form realization
- Parallel form realization

Direct form I realization

It is the direct implementation of differential equation or transfer function describing the system. It uses separate integrators for input and output variables. It provides direct relation between time domain and s-domain equations. In general, this form requires 2N delay elements (for both input and output signals) for a filter of order N. This form is practical for small filters.

Advantages:

- Simplicity
- Most straight forward realization

Disadvantages:

- More number of integrators are used
- Inefficient and impractical (numerically unstable) for complex design

Direct form II realization

It is the direct implementation of differential equation or transfer function describing the system. Instead of using separate integrators for integrating input and output variables separately, an intermediate variable is integrated. It provides direct relation between time domain and s-domain equations.

Advantages:

- It uses minimum number of integrators

- Straight forward realization

Disadvantages:

- It increases the possibility of arithmetic overflow for filters of high Q or resonance

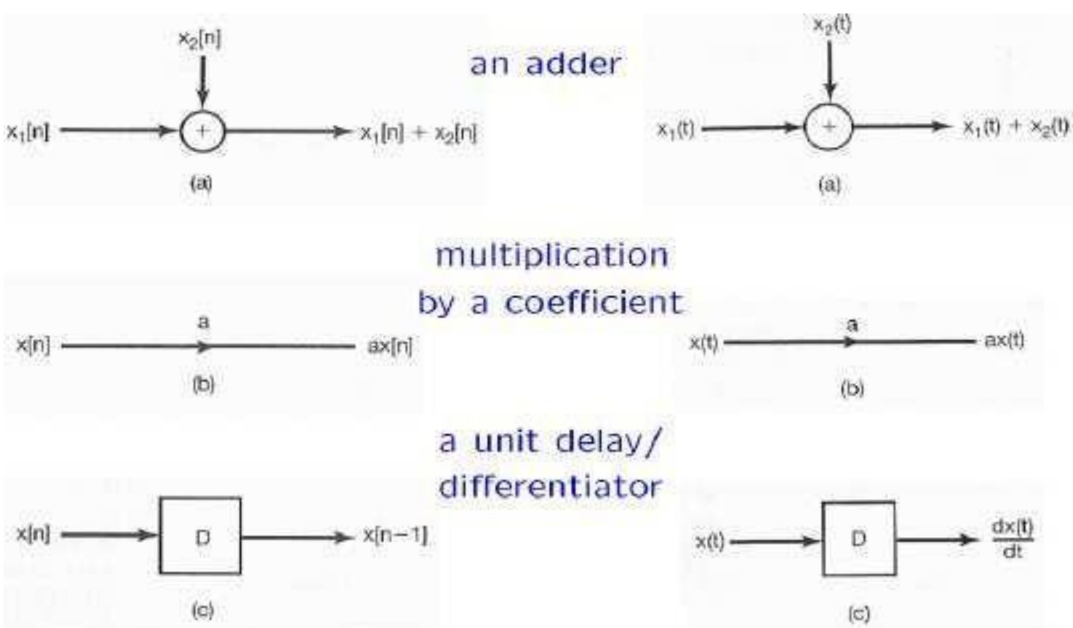
Cascade form

In cascade form realization the given transfer function is expressed as a product of several transfer function and each of these transfer function is realized in direct form II and then all those realized structures are cascaded i.e., is connected in series.

Parallel form realization

The given transfer function is expressed into its partial fractions and each factor is realized in direct form II and all those realized structures are connected in parallel.

Block diagram representations of first-order systems described by differential and difference equations



3.2 CONVOLUTION INTEGRAL

The response of a continuous-time LTI system can be computed by convolution of the impulse response of the system with the input signal, using a convolution integral, rather than a sum.

The response to the input signal $x(t)$ can be written as a convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

or it can be expressed symbolically

$$y(t) = x(t) * h(t)$$

Calculation of convolution integral

The output $y(t)$ is a weighted integral of the input, where the weight on $x(\tau)$ is $h(t-\tau)$ To evaluate this integral for a specific value of t ,

· First obtain the signal $h(t-\tau)$ (regarded as a function of τ with t fixed) from $h(\tau)$ by a reflection about the origin and a shift to the right by t if $t>0$ or a shift to the left by $|t|$ is $t<0$.

· Then multiply together the signals $x(\tau)$ and $h(t-\tau)$.

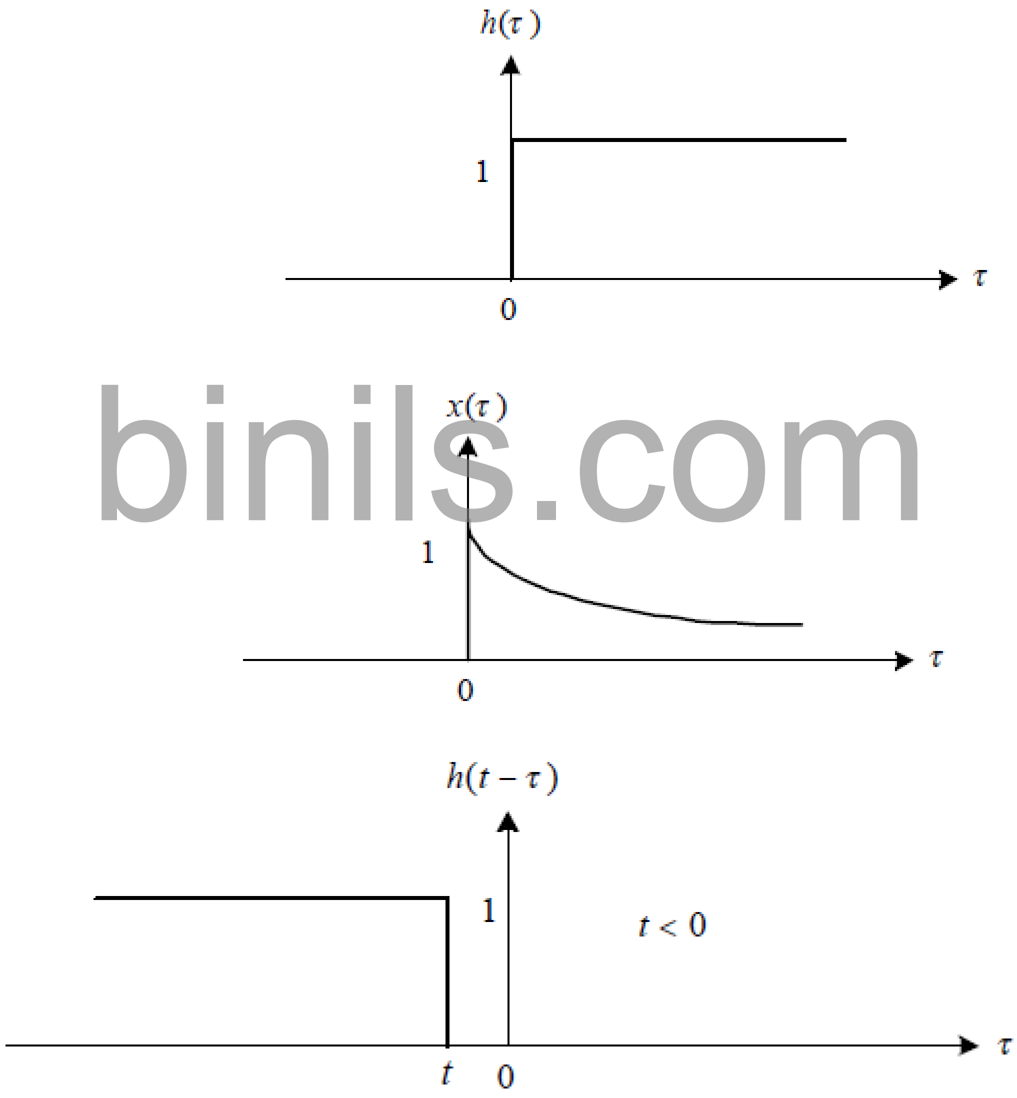
· $y(t)$ is obtained by integrating the resulting product from $\tau=-\infty$ to $\tau=+\infty$

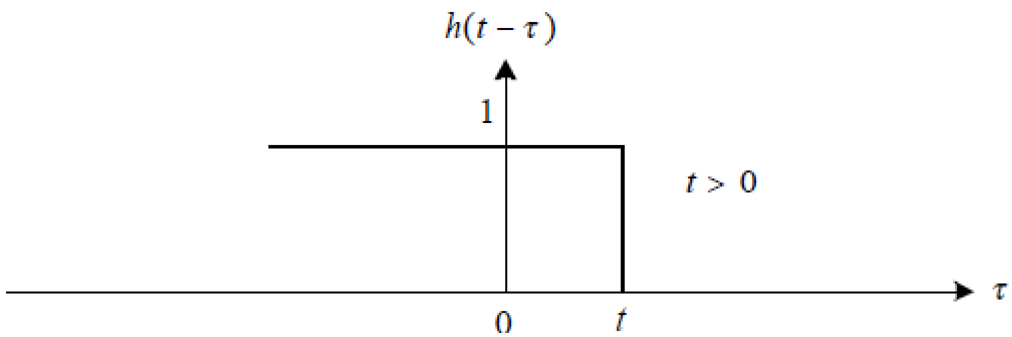
Example: Let $x(t)$ be the input to an LTI system with unit impulse response $h(t)$, where

$$x(t) = e^{-at}u(t),\ a > 0 \text{ and } h(t) = u(t).$$

Solution:

Step1: The functions $h(\tau), x(\tau)$ and $h(t-\tau)$ are depicted





Step 2: From the figure we can see that for $t < 0$, the product of the product $x(\tau)$ and $h(t-\tau)$ is zero, and consequently, $y(t)$ is zero. For $t > 0$

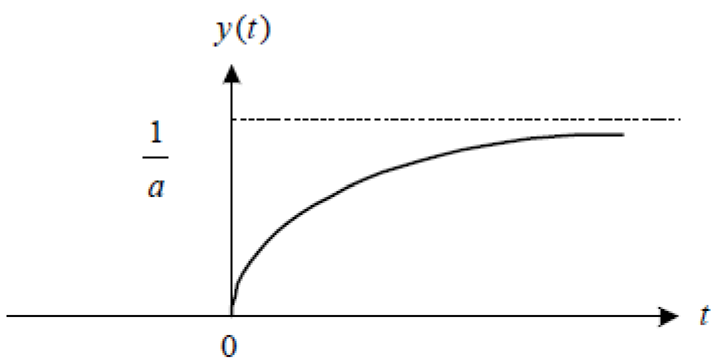
$$x(\tau)h(t-\tau) = \begin{cases} e^{-at}, & t > 0 \\ 0, & otherwise \end{cases}$$

Step 3: Compute $y(t)$ by integrating the product for $t > 0$

$$y(t) = \int_0^t e^{-a\tau} d\tau = -\frac{1}{a} e^{-a\tau}\Big|_0^t = \frac{1}{a}(1 - e^{-at}).$$

The output of $y(t)$ for all t is

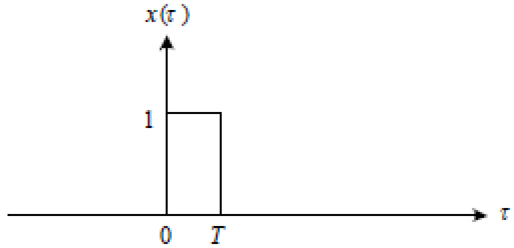
$y(t) = \frac{1}{a}(1 - e^{-at})u(t)$, and is shown in figure below.

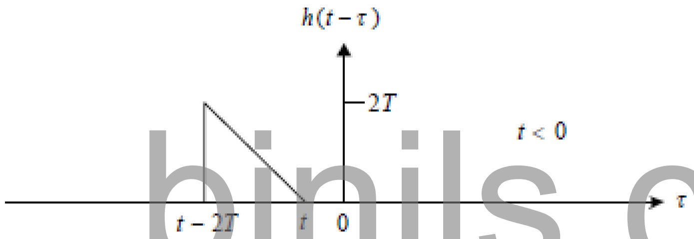


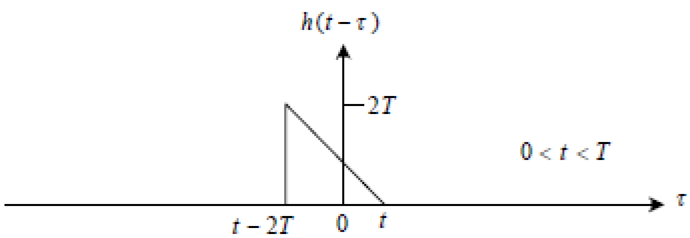
Example: Compute the convolution of the two signals below:

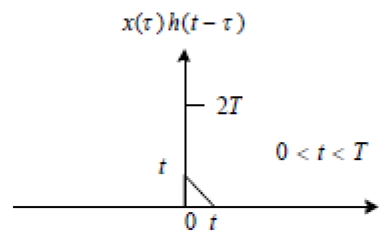
$$x(t)=\begin{cases}1, & 0<t<T\\ 0, & otherwise\end{cases} \quad \text{and} \quad h(t)=\begin{cases}t, & 0<t<2T\\ 0, & otherwise\end{cases}$$

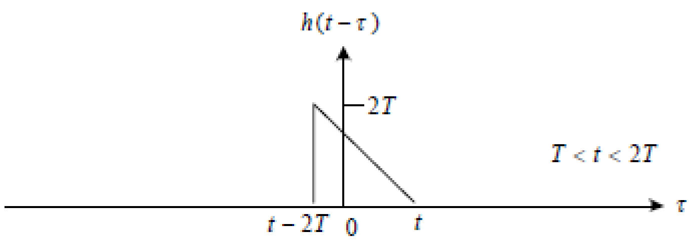
Solution:

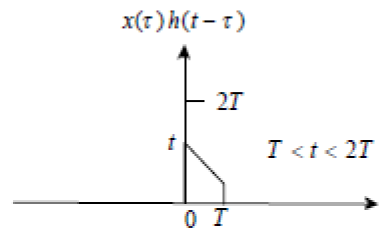


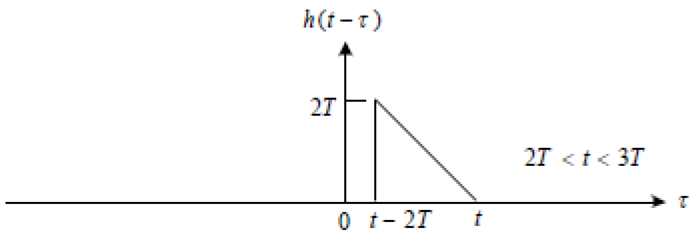


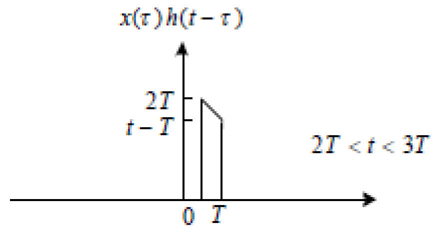


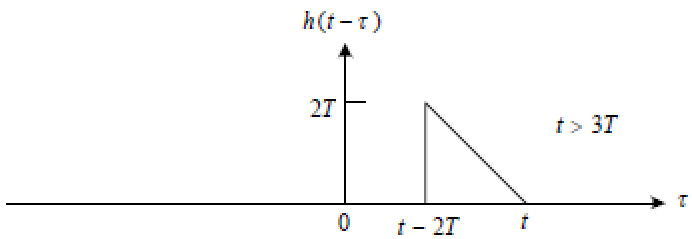




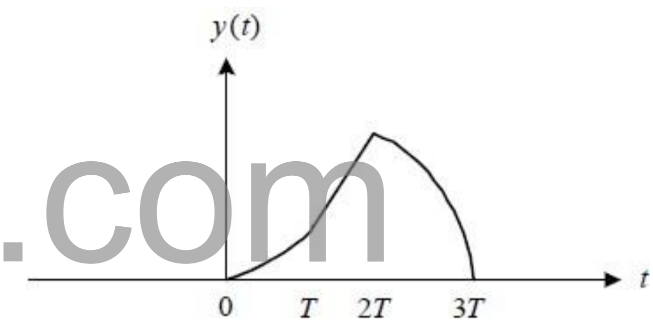








$$y(t)=\begin{cases} 0, & t<0 \\ \frac{1}{2}t^2, & 0<t<T \\ Tt-\frac{1}{2}T^2, & T<t<2T \\ -\frac{1}{2}t^2+Tt+\frac{3}{2}T^2, & 2T<t<3T \\ 0, & t>3T \end{cases}$$



3.3 ANALYSIS OF CT SYSTEMS

IMPULSE RESPONSE

LTI SYSTEM WITH AND WITHOUT MEMORY

Convolution is given by $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Convolution is commutative $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$

A CT system is memoryless if present output depends on present input. Above condition is true only for $h(\tau) = k\delta(\tau)$ and such a memory system has the form

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)k\delta(\tau)d\tau$$

$$= kx(t) * \delta(t) \text{-------}(1)$$

We know that, $x(t) * \delta(t) = x(t)$

$$(1) \rightarrow y(t) = kx(t)$$

K is the constant

The system is memory less or static if

$$h(t) = \delta(t) ----(2)$$

If (2) is not satisfied system is dynamic system

Hence the output is equal to the input, this system becomes an identity system.

INVERTIBILITY OF LTI SYSTEM

Consider a CT LTI system with impulse response h(t). This system is invertible, if it has an LTI inverse system.

We know, for an identity system

$$x(t) * \delta(t) = x(t)$$

The impulse response of an inverse system should satisfy the condition

$$h(t) * h_1(t) = \delta(t)$$

CAUSALITY FOR LTI SYSTEM

For a system to be causal, the output of the system must depend on present and past inputs only.

$$\therefore h(t) = 0 \; for \; t < 0$$

The convolution integral becomes

$$y(t) = \int_{\infty}^{t} x(\tau)h(t-\tau)d\tau$$

$$y(t) = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau$$

STABILITY OF LTI SYSTEM

For a LTI system to be stable

$$\int_{-\infty}^{\infty} \left|h(\tau)\right| d\tau < \infty$$

FOURIER METHOD FOR ANALYSIS

FREQUENCY RESPONSE OF LTI SYSTEM

Consider a LTI system

$$y(t) = x(t) * h(t)$$

Take Fourier transform on both sides

$$Y(j\omega) = X(j\omega).H(j\omega) - - - - - (1)$$

Convolution in time domain gives multiplication in frequency domain.

Where, $H(j\omega)$ *is the frequency response of the system*

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Magnitude of equation (1) is

$$|Y(j\omega)| = |X(j\omega)|.|H(j\omega)|$$

$$|H(j\omega)| \text{ is the gain of the system}$$

phase of equation (1) is

$$\angle Y(j\omega) = \angle X(j\omega).\angle H(j\omega)$$

$$\angle H(j\omega) \text{ is the phase shift of the system}$$

ANALYSIS AND CHARACTERIZATION OF LTI SYSTEM USING LAPLACE TRANSFORM

Output of the system is given by

$$y(t) = x(t) * h(t)$$

Taking Laplace Transform on both sides.

$$Y(S) = X(S).H(S)$$

Where, $H(S)$ *is the system function or transfer function*

If $S = j\omega$, then $H(S)$ is frequency response of LTI system.

CAUSALITY:

For causal LTI system

$$h(t) = 0 \; for \; t < 0$$

1. ROC for a causal system is in the right half plane.
2. ROC for causal system with rational system is right half plane to right of right most pole.

STABILITY:

An LTI system is stable if and only if ROC of its system function $H(S)$ includes $j\omega$ axis $.ie, Re(S) = 0$

A causal system with rational system function is stable if and only if all poles(X) of $H(S)$ lie in left half of S-plane.