MA8353 Transforms and Partial Differential Equations

IMPORTANT QUESTIONS AND QUESTION BANK

UNIT-I PARTIAL DIFFERENTIAL EQUATIONS

2Marks

1. Form the partial differential equation from the equation $2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$
2. Find the complete integral of the PDE: $z = px + qy + \sqrt{pq}$.
3. Find the partial differential equation by eliminating the arbitrary function 'f' from the relation $z = f(x^2 - y^2)$.
4. What are singular integrals? How does it differ from particular integral?
5. Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$.
6. If u= x^2+t^2 is a solution of $c^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$, the find the value of c?
7. Find the complete solution of p=2px.
8. Solve $(D^2-6DD'+9D'^2)z = 0$.
9. What are singular integrals? How does it differ from particular integral?
10. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 0$
11. Solve $(D - D')^3 z = 0$
12. Solve $(D-1)(D-D'+1)z = 0$
13. Solve $(D^4 - D'^4)z = 0$
14. Solve $(D^2 - 7DD' + 6D'^2)z = 0$
15. Solve $(D^3 - D^2D' - 8DD'^2 + 12D'^3)z = 0$

13Marks

1. Solve $\frac{\partial^2 z}{\partial x^3} - 2\frac{\partial^2 z}{\partial x^2 \partial y}$.= $2e^{2x}+3x^2y$.
2. Find the general solution of $(D^2 + 2DD' + D^2)\, z = x^2 y + e^{x-y}$.
3. From the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$.
4. Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y), where\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ $(8+8)$.

5. 1)Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x dy} - 6\frac{\partial^2 z}{\partial y^2} = ycosx.$

   2)Solve $(x^2 + yz)p + (y^2 - zx)q = z^2 - xy \; where \; p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ $(8 + 8).$
6. Find the solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4x \frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = \text{ e2x} + \text{y}$$

7. Solve the Lagrange's linear equation $(x^2 + yz)p + (y^2 - zx)q = z^2 - xy$
8. Solve the partial differential equation $(D^2 + 2DD' + D^2 - 2D')z =$ $sin(x + 2y)$.
9. 1)From the partial differential equation by eliminating the arbitrary function from $u = f(x + ct) + g(x - ct)$.

   2)solve $(D^2 - 2D^2D')z = sin(x + 2y) + 3x^2y$.
10. 1)Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

    2)Solve $p - x^2 = q + y^2$

UNIT-II FOURIER SERIES

2Marks

1. Sketch the graph of one even and one odd extension of $f(x) = x^3$ in [0, 1].
2. State the sufficient condition for the function $f(x)$ to be expressed as a Fourier series.
3. Define Root mean square value of a function.
4. What is the behavior of Fourier series of a function $f(x)$ at the point of discontinuity?
5. Sketch the even and odd extension of the periodic function $f(x)$ = x^2 for 0<x<2.
6. State the Dirichlet's conditions.
7. Sketch the even extension of the function $f(x)$ = sin x, 0<x<π.
8. State giving reason whether the function $f(x)$ =x sin $\left(\frac{1}{x}\right)$ can be expanded in Fourier series in the interval of (0,2 π).
9. Find the Fourier Constant b_n for xsinx in $(-\pi, \pi)$.

10. Find the Root mean square value of $f(x) = x$ in $(0, l)$
11. Write down the Parseval's formula on Fourier coefficients
12. What is meant by Harmonic Analysis?
13. Find the R.M.S value of $f(x) = 1 - x$ in 0<x<1
14. If the function $f(x) = x$ in the interval 0<x<2π then find the constant term of the Fourier series expansion of the function f.
15. If f(x) is an odd function defined in $(-l, l)$. What are the values of $a_0 and\ a_n$?

13Marks

1. Find the Fourier series expansion of f(x) = $\sqrt{1 - cosx}$, 0 ≤ x ≤ 2π and hence evaluate the value of the series 1 1.3 + 1 3.5 + 1 5.7 − ⋯.
2. Find the Fourier series of period 2π for the function $f(x)$ = $xcos\ x\ in$ 0 < x < 2π.
3. 1)Obtain the Fourier series of the periodic function $f(x) = e^{ax}$ in the interval 0≤x≤2π.

   2)Develop the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.
4. Find the complex form of the Fourier series for $f(x)$ = e^{-x}, in -1≤x≤1.
5. Develop the half range Fourier series for the function $f(x) = x^3$ in (0, L).
6. The displacement y(x) of a part of a mechanism is tabulated with corresponding angular movement x^0 of the crank. Express y(x) as a Fourier series neglecting the harmonics above the third.

x^o:	0	30	60	90	120	150	180	210	240	270	300	330
y(x):	1.8	1.1	0.3	0.16	0.15	1.3	2.16	1.25	1.3	1.52	1.72	2

7. Find the Fourier series of f(x) = x^2 in (0,2*l*). hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty = \frac{\pi^2}{6}$

8. Find the complex series of f(x) = cos ax in ($-\pi,\pi$), where 'a' is neither zero nor an integer.
9. Obtain the constant term and the first three harmonics in the Fourier Cosine series of y= f(x) in (0,6) from the following table.

x	0	1	2	3	4	5
y	4	8	15	7	6	2

10. Find the Fourier series expansion of f(x)=sin ax in $(l,-l)$

UNIT-III APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

2Marks

1. Write all three possible solutions of one-dimensional heat equations.
2. Classify the partial differential equation $u_{xy} = u_x u_y + xy$.
3. Write all possible solutions of one-dimensional heat equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$
4. Classify the partial differential equation $2x\frac{\partial^2 u}{\partial x^2} + 4x\frac{\partial^2 u}{\partial y} + 8x\frac{\partial^2 u}{\partial y^2} = 0$
5. Mention the various possible general solutions for one dimensional heat equation.
6. Classify the PDE $3\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial x^2} = 0$.
7. Classify the two-dimensional steady state heat conduction equation.
8. Give the mathematical formulation of the problem of one-dimensional heat conduction in a rod of length *l* with insulated ends and with initial temperature $f(x)$.
9. Classify the PDE $u_{xx} + u_{xy} + u_{yy} = 0$
10. What is the various solution of one-dimensional wave equation?
11. In the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ what does C^2 stand for?
12. Write down the three possible solution of Laplace equation in two dimensions
13. State the assumption in deriving the one-dimensional heat equation
14. Write down the governing equation of two-dimensional steady state heat equation
15. If the ends of the string of length ***l*** are fixed at both sides. The midpoint of the string is displaced transversely through a heigh h and the string is released from rest, state the initial and boundary conditions.

13Marks

1. Solve using by the method of separation of variables $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$
2. A string is stretched and fastened to two points x = 0 and $x = l$ apart. Motion is started by displacing the string into the form $y = k\ (lx - x\ 2)$ from which it is released at time t = 0. Find the displacement of any point on the string at a distance of x from one end at time t.
3. 1)Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial x} + u,$ where u(x,0) =$6e^{-3x}$
   2)Find the temperature u (x, t) in a laterally insulated heat conducting bar of length L with its ends kept at o^0 and with the initial temperature in the bar is u (x, 0) = 100sin$\left(\frac{\pi x}{80}\right)$ and L =80cm.
4. Derive the general solutions for one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ using separation of variables method.
5. Find the displacement of a string stretched between two fixed points at a distance L apart. The string is initially at rest in equilibrium position and points of the string are given initial displacement u(x,0) = k(Lx-x^2). Assume initial velocity zero.
6. Solve the equation $\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions u (x, 0) = 3 sinπx, u (0, t) = 0 and u (1, t) = 0 where 0<x<1, t>0
7. A tightly stretched flexible string has its ends fixed at x=o and x=L. at time t= 0, the string is given a shape defined by y= μ(L-x), where μ is a constant, and then released. Find the displacement of any point x of the string at any time t>0.
8. 1)Solve $u_t = a^2 u_{xx}$ by the method of separation of variables and obtain all possible equations.
   2)A rectangular plate with insulated surface is 8 cm wide and so long compared to its width that it may be considered as an infinite plate.
   If the temperature along the short edge y=0 is u(x,0) =100sin$\left(\frac{\pi x}{8}\right)$ 0<x<8 while two long edges x =0 & x=8 as well as the other short edge are kept at o^0, then find the steady state temperature at any point of the plate.
9. Solve the problem of a tightly stretched string with fixed end points x =0 & x=1 which is initially in the position y= f(x) and which is initially set vibrating by giving to each of its points a velocity $\frac{dy}{dt} = g(x)$ at t=0.
10. Classify the partial differential equation $(1 - x^2)f_{xx} - 2xyf_{xy} + (1 - y^2)f_{yy} = 0$.

UNIT-IV FOURIER TRANSFORMS

2Marks

1. State convolution theorem for Fourier transform.
2. State the condition for the existence of Fourier cosine and sine transforms of derivatives.
3. Find Fourier Sine transform of 1 x.
4. Does Fourier sine transform of $f(x)$=k, 0≤x≤∞,, exist? Justify your answer.
5. Show that $3_c[f(x)\cos ax] = \frac{1}{2}\{F_c(s+a)+F_c(s-a)\}$ where 3_c $[f(x)] = F_c(s)$ is the Fourier cosine transform of $f(x)$.
6. State Fourier integral Theorem
7. Write Fourier transform pair.
8. Find the Fourier Transform of $e^{-a|x|}$.
9. Find the Fourier Sine transform of e^{-ax}
10. Define self-reciprocal with respect to Fourier transform
11. Find the Fourier cosine transform of e^{-2x}
12. Give an example of a function which is self-reciprocal under Fourier sine & cosine Transforms
13. State Parseval's identity for Fourier Transform
14. Write down the Fourier cosine Transform pair of formulae
15. If F(s)= F[$f(x)$], then find F[$xf(x)$].

13Marks

1. Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$. By using the properties, find the Fourier transform of $e^{-2(x-3)^2}$.
2. Evaluate $\int_0^\infty \frac{dx}{(x^2+1)\,(x^2+4)}$ using Fourier transforms.
3. 1)Construct the Fourier sine transform $f(x) = \frac{e^{-ax}}{x}$

   2)Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| \ge 1 \end{cases}$ hence deduce $\int_0^\infty \frac{xcosx-sinx}{x^2} cos\left(\frac{x}{2}\right) dx$.

4. 1)Find the Fourier cosine transform of $f(x) = e^{-ax}$ and $g(x) = e^{-bx}$ using these transforms and Parseval's identity show that
$\int_0^\infty \frac{dt}{(a^2+t^2)\,(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$
2)Find the Fourier transform of $f(x) = cosx, 0 \le x \le 1$.
5. Find the Fourier transform of $f(x) where\ f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ and hence evaluate $\int_0^\infty \frac{sinx}{x}dx$.
6. Show that $\frac{1}{\sqrt{x}}$ is self-reciprocal under the Fourier cosine transform.
7. Find the Fourier cosine and sine transform of $e^{ax}, a > 0$ and hence deduce their inversion formulae.
8. Using Parseval's identity, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}, a > 0$.
9. Using Parseval's identities, prove that

$$\int_0^\infty \frac{dt}{(a^2+t^2)\,(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$$

$$\int_0^\infty \frac{t^2 dt}{(t^2+1)^2} = \frac{\pi}{4}$$

10. Find the infinite Fourier sine Transform of $\frac{1}{x}$.

UNIT-V Z - TRANSFORMS AND DIFFERENCE EQUATIONS

2Marks

1. The integers 0, 1, 1, 2, 3, 5, 8, … are said to form a Fibonacci sequence. Model the Fibonacci difference equation.[$no\ need\ to\ solve$]
2. State initial and final value theorems on Z-transforms.
3. Find the Z-transform of {n}.
4. What are the applications of Z- transforms?
5. Find the Z transform of $f(n) = (n+1)^2$
6. Find Z- transform of unit impulse sequence $\delta(n) = \begin{cases} 1, n = 0 \\ 0, n \ne 0 \end{cases}$

7. Show that $Z[a^n f(n)] = F\left(\frac{z}{a}\right)$ where $Z[f(n)] = F(z)$ is the Z-transform of $f(x)$.
8. Prove that $Z[f(n+1)] = zF(z) - zf(0)$
9. Find the Z transform of $\frac{z}{(z-1)(z-2)}$
10. Find $Z\left[\frac{1}{n!}\right]$
11. Find $Z\left[\frac{1}{n(n+1)}\right]$
12. Find $Z[n^2]$
13. Find $z^{-1}\left[\frac{z}{(z+1)^2}\right]$
14. Find $Z\left(\frac{a^n}{n!}\right)$
15. Find $Z(3^{n+2})$

<u>13Marks</u>

1. Find Z-transform of $\frac{2n+3}{(n+1)(n+2)}$.
2. Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4Z+1)}$. using convolution theorem for Z-Transform
3. 1)From the difference equation corresponding to the family of curves $y = ax + bx^2$.

   2)Find the Z transform of $u(n) = 3n - 4sin\left(\frac{n\pi}{4}\right) + 5a$, and $u(n) = cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$.
4. 1)Use convolution theorem to evaluate the inverse Z transform of $U(z) = \frac{z^2}{(z-a)(z-b)}$.

   2)Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with initial conditions $y_0 = y_1$=0, using Z transform.
5. Find the inverse Z-transform of

   1) $\frac{2z^2+3z}{(2z-1)(4Z+1)}$.

2) $\frac{2(z^2-5z+6.5)}{(z-2)(z-3)^2}$ for 2<|z|<3.

6. Using the Z-transform, solve

   1) $u_{n+2} + 4u_{n+1} + 3_{u_n} = 3^n$ with $u_0 = 0, u_1 = 1$

   2) $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$.

7. 1)Find Z $\{sinbt\}$ and hence find Z $\{e^{-at} \sin bt\}$.

   2)Find the inverse Z-transform of $\frac{8z^2}{(2z-1)\,(4Z+1).}$ using convolution theorem

8. 1)Using Z-transforms, solve the difference equation $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$ given $y_0 = y_1$=0, use partial fraction method to find the inverse Z-transform.

   2)using residue method, find $Z^{-1}\left\{\frac{z}{z^2+2z+2}\right\}$.

9. Find the inverse Z-transform of $\frac{(z^2+z)}{(z^2+1)(z-1)}$

10. Solve the equation using Z-Transform $y_{n+2} - 5y_{n+1} + 6y_n = 36$ given that y (0) =y (1) =0.