MA3151 MATRIX AND CALCULUS

IMPORTANT QUESTIONS AND QUESTION BANK

UNIT-1 MATRICES

2-Marks

1. Find the characteristic equation of $A = \begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$
2. Find the eigen values of A^2 if $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$
3. If the eigen values of the matrix A Of order 3X3 are 2,3 and 1, then the find the determinant of A
4. If the sum of 2 eigen values and the trace of a 3X3 matrix are equal, find the value of |A|
5. Prove that sum of eigen values of a matrix is equal to its trace.
6. Find the sum of eigen values of 2A, if A = $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$
7. Find the sum and product of the eigen values of A = $\begin{pmatrix} 2 & -2 & 2 \\ -2 & -1 & -1 \\ 2 & -1 & -1 \end{pmatrix}$
8. State Cayley-Hamilton theorem
9. Define index, signature and rank
10. Write any 2 applications of Cayley Hamilton theorem.

13-Marks,

1. Test for the consistency of the following system of equations and solve them, if consistent $3x + y + z = 8,\ -x + y - 2z = -5,\ x + y + z = 6, -2x + 2y - 3z = -7$.
2. Examine the consistency of the equations $x + y + z = 3,\ 2x - y + 3z = 4,\ 5x - y + 7z = 11$.
3. Investigate for the value of λ, μ the system of equations $x + y + z = 6, x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$ have (i) Unique solution, (ii) Infinitely many Solution (iii) no solution.
4. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$
5. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$
6. Obtain the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$.

7. Obtain the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$
8. Find the characteristic equation of the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ and also find A^4
9. Verify Cayley-Hamilton theorem and hence find A1 of $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$
10. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ and also find A^{-1}
11. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ in to canonical form by orthogonal reduction
12. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & -2 & 1 \\ 5 & -1 & -5 \end{pmatrix}$ and also find A^{-1}
13. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and also find A^T
14. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & -2 & -3 \\ -1 & -2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$ and also find A^4
15. Determine the nature quadratic form $2xy - 2yz + 2xz$ by reduce into canonical form by orthogonal transformation

UNIT-II DIFFERENTIAL CALCULUS

2-Marks

1. Find the critical points of $y = 5x^2 - 6x$
2. Find the critical number of the function $f(x) = 2x^3 - 3x^2 - 36x$
3. Find the Taylor's series expansion of the function $f(x) = \sin x$ about the point $x = \frac{\pi}{2}$
4. Verify Lagrange's law for the function $f(x) = \frac{1}{x}, [1,2]$
5. Estimate $\frac{d}{dx}((sinx)^{cosx})$

6. Estimate $y' if\ x^3 + 3 = 6xy$
7. Where the function $f(x) = |x|$ is differentiable?
8. Point out $\frac{dy}{dx}, if\ y = In|\cos(In\ x)|$.
9. Using Rolle's theorem find the value of c for the function $f(x) = \sqrt{1-x^2}$, -1≤ $x \leq 1$
10. State Rolle's theorem.

13-Marks

1. Find $\frac{dy}{dx}$ if $y = x^2 e^{2x}(x^2+1)^4$.
2. Discuss the curve y= $x^4 - 4x^3$ with respect to concavity points of inflection and local maxima and minima.
3. Find $y' for\ cos(xy) = 1 + siny$
4. Verify Rolle's theorem for the following $f(x) = x(x-1)(x-2), x \in [0,2]$
5. Find $\frac{dy}{dx}$ for the following functions $e^x + e^y = e^{x+y}$
6. Find $y"\ if\ x^4 + y^4 = 16$
7. Find the Taylor's series expansion $f(x) = \tan^{-1} x\ \ about\ x = 0$
8. Examine the local extreme of $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$. Also discuss the concavity and find the inflection points.
9. Use second derivative test to examine the relative maxima for $f(x) = x(12-2x)^2$.
10. Find the tangent line to the equation $x^2 = x^2 = 6xy\ at\ the\ point\ (3,3)$ and at what point the tangent line is horizontal in the first quadrant.
11. Find $\frac{dy}{dx}$, when $y = \frac{a\ cosx + b\ sinx}{b\ cosx - asinx}$.
12. Find the point on the parabola $y^2 = 2x$ that is close to the point (1,4)
13. Find the equation of tangent at a point (a, b) to the curve $xy = c^2$.
14. At what point on the curve $x^2 - y^2 = 2,\ the$ slopes of tangents are equal to 2.
15. A cylindrical hole 4mm in diameter and 12mm deep in a metal block is rebored to increase the diameter to 4.12mm. Estimate the amount of metal removed.

UNIT-III FUNCTIONS OF SEVERAL VARIABLES

2-Marks

1. If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.
2. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3ax^2y$

3. If $x^y + y^x = 1$ then find $\frac{dy}{dx}$
4. Find the values of $\frac{du}{dt}$, given $u = x^2+y^2, x = at^2, y = 2at$
5. If $u = x^3y^2 + x^2y^3$ where $x = at^2$ and $y = 2at, then\ find\ \frac{du}{dt}$
6. Find $\frac{du}{dx} if\ u = \frac{x}{y}, where\ ex = e^t, y = \log t$
7. Find $\frac{\partial r}{\partial x}, if\ x = r\cos\theta\ and\ y = r\sin\theta$
8. Find the Taylor series expansion of x^y near the point (1,1) up to first term.
9. Expand $xy + 2x - 3y + 2\ in\ powers\ of\ (x-1)\&(y+2),$ using Taylor's theorem up to first degree form.
10. State the sufficient condition for $f(x,y)$ to be extremum at a point.

13-Marks

1. Divide the number 24 into the three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.
2. The temperature at a point (x, y, z) in a space given by $T = kxyz^2$. Where k is constant. Find the height temperature on the surface of the sphere$x^2 + y^2 + z^2 = a^2$
3. Find the dimension of the rectangular box without a top of maximum capacity, whose surface area 108 sq.cm
4. Find the shortest and longest distance from the point (1, 2, -1) to the sphere $x^2 + y^2 + z^2 = 24$
5. Discuss the maxima and minima of $f(x,y) = x^3y^2(1 - x - y)$.
6. Find the maximum value of $x^m y^n z^p\ when\ x + y + z = a$.
7. Find the Taylors series expansion of $e^x\ siny\ \ at\ the\ point\ (-1, \frac{\pi}{2})$ up to second degree terms.
8. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.
9. Expand e^{xy} in powers of $(x-1)\ and\ (y-1)$ upto second degree term by Taylors series.
10. Examine $f(x,y) = x^3 + 3xy^2 - 15x^2 + 72x$ for extreme values.
11. Expand $\tan^{-1}\frac{y}{x}$ in the (1, 1) as Taylors series up to second degree term.
12. If $u = f(2x - 3y, 3y - 4z, 4z - 2x), then$, find $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$.
13. Find the extreme values of $f(x,y) = x^3 + y^3 - 3x - 12y = 20$.
14. Expand $e^x \log(1 + y)$ in powers of x & y upto terms of 3rd degree using Taylors series.
15. Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$.

UNIT-IV INTEDRAL CALCULUS

2-Marks

1. State the fundamental theorem of calculus.
2. Prove that the following integral by interpreting each in terms of areas $\int_a^b x dx = \frac{b^2-a^2}{2}$
3. Evaluate $\int_0^1 \sqrt{1-x^2}\ dx$ in terms of areas?
4. If f is continuous and $\int_0^4 f(x)\ dx = 10, find\ \int_0^2 f(2x)\, dx$.
5. Evaluate the integral $\int_a^b x dx$ by using Riemann sum method
6. Calculate $\int \frac{x^3}{\sqrt{4+x^2}} dx$
7. Calculate $\int \sqrt{1+x^2 x^5}\ dx$
8. Find $\int \sqrt{2x+1}\ dx$.
9. Find $\int \frac{x}{\sqrt{1-4x^2}}\ dx$
10. Find $\int_2^5 \frac{dx}{\sqrt{x-2}}$.

13-Marks

1. Evaluate $\int \frac{(lnx)^2}{x^2} dx$
2. Calculate $\int \frac{1}{\sqrt{a^2-x^2}} dx$ by using trigonometric substitution.
3. Find $\int x^3 \sqrt{9-x^2}\ dx$ by trigonometric substitution
4. Evaluate $\int_0^{\frac{\pi}{2}} \frac{sinxcosx}{cos^2x+3cosx+2} dx$
5. Evaluate $\int e^{ax} \cos bx\ dx, a > 0$ using integration by parts
6. Evaluate $\int e^{ax}\ sinbx\ dx, a > 0$ using integration by parts
7. Find $\int \frac{sec^2x}{tan^2x+3tsnx+2}\ dx$
8. Evaluate $\int x \tan^{-1} x\ dx$.
9. Calculate by partial fraction $\int \frac{x^2+1}{(x-3)(x-2)^2} dx$.
10. Evaluate $\int \frac{tanx}{secx+cosx}\ dx$
11. Evaluate $\int \frac{xe^{2x}}{(1+2x)^2} dx$
12. Calculate using the partial fraction $\int \frac{10}{(x-1)(x^2+9)}\ dx$
13. Evaluate $\int sin^6 x cos^3 x\ dx$
14. The region enclosed by the circle $x^2+y^2=a^2$ is divided into two segments by the line x=h. Find the area of smaller segment

15. Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.

UNIT-V MULTIPLE INTEGRALS

2-Marks

1. Evaluate $\int_2^3 \int_1^2 \frac{dxdy}{xy}$
2. Evaluate $\int_0^{\pi} \int_0^{\sin\theta} rdrd\theta$
3. Find the area bounded by the lines $x = 0, y = 0\ amd\ \ y = x$
4. Evaluate $\int_0^{\pi} \int_0^{a} rdrd\theta$
5. Evaluate $\int_0^5 \int_0^2 (\,x^2 + y^2)\ dxdy$
6. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dydx$
7. Evaluate $\int_0^{\pi} \int_0^5 r^4\ sin\theta drd\theta$
8. Change the order of integration $\int_0^1 \int_{y^2}^{y} f(x,y)dxdy$
9. Change the order of integration $\int_0^{\infty} \int_x^{\infty} f(x\,,y)dxdy$
10. Evaluate $\int\int dxdy\ over\ the\ region\ bounded\ by\ x = 0,\ \ x = 2, and\ y = 0, y = 2$

13-Marks

1. Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$
2. Using double integral find the area of the Ellipse $\frac{x^2}{a^2}$+ $\frac{y^2}{b^2}$ =1
3. Find the area bounded by parabola $y = x^2\ and\ straight\ line\ 2x - y + 3 = 0$
4. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
5. Find the volume of finite region of space (tetra-hardon) bounded by the planes $x = 0, y = 0, z = 0\ and\ 2x + 3y + 4z = 12$
6. Find the volume of the sphere bounded by $x^2 + y^2 + z^2 = a^2$
7. Find the area of which is inside the circle $r = 3a\ cos\theta$ and outside the cardioid $r = (1 + cos\theta)$
8. Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2)dxdydz$
9. Find the volume bounded by the cylinder$x^2 + y^2 = 1\ and\ the\ planes\ x + y + z = 3, z = 0$
10. Find the value of $\iiint xyz\ dxdydz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \le a^2$
11. Find the areas common to the cardioids $r = a(1 + cos\theta)\ and\ \ r = a(1 - cos\theta)$

12. Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
13. By change the order of integration and evaluate $\int_0^2\int_{x^2}^{2-x} xy\, dydx$
14. Find the areas included between the curves $y^2=4x\ and\ x^2=4y$
15. Evaluate $\int_1^e\int_1^{logy}\int_1^{ex} logz\, dzdydx$