

5.4 CHANNEL STATE INFORMATION AND CAPACITY

Algorithms for MIMO transmission can be categorized by the amount of CSI (Channel State Information) that they require.

We distinguish the following cases:

Full CSI at the TX (CSIT) and full CSI at the RX (CSIR): In this ideal case, both the TX and the RX have full and perfect knowledge of the channel. This case obviously results in the highest possible capacity.

Average CSIT and full CSIR: in this case, the RX has full information of the instantaneous channel state, but the TX knows only the average CSI – e.g., the correlation matrix of H or the angular power spectrum.

No CSIT and full CSIR: this is the case that can be achieved most easily, without any feedback or calibration.

The TX simply does not use any CSI, while the RX learns the instantaneous channel state from a training sequence or using blind estimation.

Noisy CSI: when we assume “full CSI” at the RX, this implies that the RX has learned the channel state perfectly.

However, any received training sequence will be affected by additive noise as well as quantization noise. It is thus more realistic to assume a “mismatched RX,” where the RX processes the signal based on the observed channel H_{obs} , while in reality the signals pass through channel H_{true} .

No CSIT and no CSIR: it is remarkable that channel capacity is also high when neither the TX nor the RX have CSI.

Example. use a generalization of differential modulation.

Channel State Information at the Transmitter and Full CSI at the Receiver

When the RX knows the channel perfectly, but no CSI is available at the TX, it is optimum to assign equal transmit power to all TX antennas, $P_{i,k} = P/N$, and use uncorrelated data streams.

Capacity thus takes on the form:

It is noted that (for sufficiently large N_s) the capacity of a MIMO system increases linearly with $\min(N_t, N_r)$, irrespective of whether the channel is known at the TX or not.

Assume that $N_t = N_r = N$:

1. **All transfer functions are identical** – i.e., $h_{1,1} = h_{1,2} = \dots = h_{N,n}$.

This case occurs when all antenna elements are spaced very closely together, and all waves are coming from similar directions.

In such a case, the rank of the channel matrix is unity. Then, capacity is

We see that, here the SNR is increased by a factor of N compared with the single antenna case, due to beam forming gain at the RX.

However, this only leads to a logarithmic increase in capacity with the number of antennas.

2. **All transfer functions are different** such that the channel matrix is full rank, and has N eigenvalues of equal magnitude.

This case can occur when the antenna elements are spaced far apart and are arranged in a special way.

In this case, capacity is

$$C_{\text{MIMO}} = N \log_2(1 + \bar{\gamma})$$

and, thus, increases linearly with the number of antenna elements.

3. Parallel transmission channels – e.g., parallel cables.

In this case, capacity also increases linearly with the number of antenna elements. However, the SNR per channel decreases with N , so that total capacity is

$$C_{\text{MIMO}} = N \log_2 \left(1 + \frac{\bar{\gamma}}{N} \right)$$

Figure 5.4.1 shows the capacity as a function of N for different values of SNR.

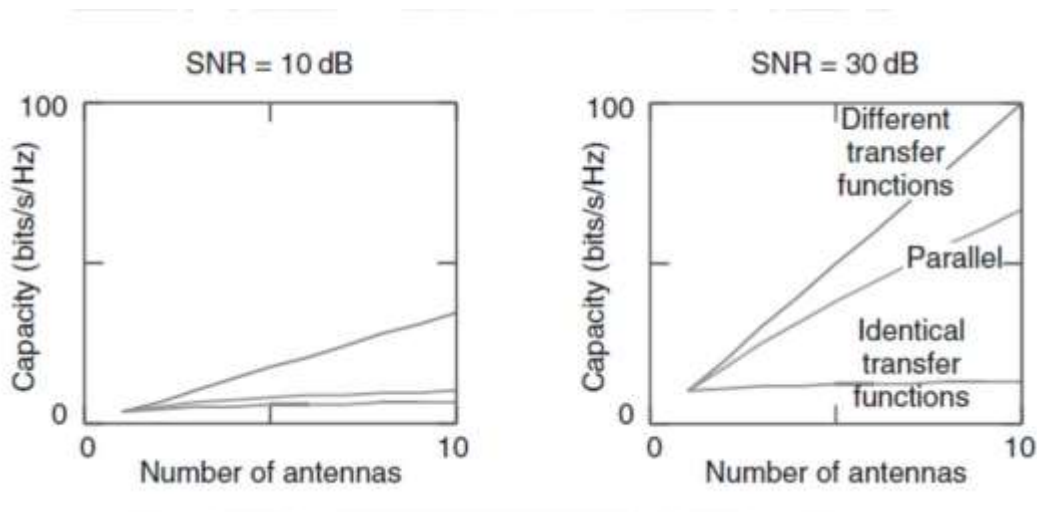


Fig 5.4.1: Capacity of MIMO in additive white Gaussian noise channels
[Source: "Wireless communications" by Andreas F. Molisch, Page-470]

Full Channel State Information at the Transmitter and Full CSI at the Receiver

Let us consider the case where both the RX and TX know the channel perfectly.

In such a case, it can be more advantageous to distribute power not uniformly between the different transmit antennas (or Eigen modes) but rather assign it based on the channel state.

In other words, we are faced with the problem of optimally allocating power to several parallel channels, each of which has a different SNR.

Capacity in Non fading Channels

Here we derive the capacity equation for MIMO systems in nonfading channels, often known as “Foschini’s equation”.

Consider the capacity equation for “normal” (single-antenna) Additive White Gaussian Noise (AWGN) channels. As Shannon showed, the information-theoretic (erotic) capacity of such a channel is

$$C_{\text{shannon}} = \log_2 (1 + \gamma \cdot |H|^2)$$

Where γ is the SNR at the RX, and H is the normalized transfer function from the TX to the RX (the transfer function is just a scalar number).

The key statement of this equation is that capacity increases only logarithmically with the SNR, so that boosting the transmit power is a highly ineffective way of increasing capacity

Consider now the MIMO case, where the channel is represented by matrix.

Let us then consider a singular value decomposition of the channel:

$$\mathbf{H} = \mathbf{W}\mathbf{\Sigma}\mathbf{U}^\dagger$$

where $\mathbf{\Sigma}$ is a diagonal matrix containing singular values, and \mathbf{W} and \mathbf{U}^\dagger are unitary matrices composed of the left and right singular vectors, respectively. The received signal is given as

$$\begin{aligned} \mathbf{r} &= \mathbf{H}\mathbf{s} + \mathbf{n} \\ &= \mathbf{W}\mathbf{\Sigma}\mathbf{U}^\dagger\mathbf{s} + \mathbf{n} \end{aligned}$$

Then, multiplication of the transmit data vector by matrix \mathbf{U} and the received signal vector by \mathbf{W}^\dagger dingo analyzes the channel

$$\begin{aligned} \mathbf{W}^\dagger\mathbf{r} &= \mathbf{W}^\dagger\mathbf{W}\mathbf{\Sigma}\mathbf{U}^\dagger\mathbf{U}\tilde{\mathbf{s}} + \mathbf{W}^\dagger\mathbf{n} \\ \tilde{\mathbf{r}} &= \mathbf{\Sigma}\tilde{\mathbf{s}} + \tilde{\mathbf{n}} \end{aligned}$$

Note that – because \mathbf{U} and \mathbf{W} are unitary matrices – \mathbf{n} has the same statistical properties as \mathbf{n} – i.e., it is independent identically distributed (iid) white Gaussian noise.

The matrix Σ is a diagonal matrix with R_H nonzero entries.

σ_k , where R_H is the rank of \mathbf{H} (and thus defined as the number of nonzero singular values), and σ_k is the k th singular value of \mathbf{H} . We have therefore R_H parallel channels (Eigen modes of the channel), and it is clear that the capacity of parallel channels just adds up.

The capacity of channel \mathbf{H} is thus given by the sum of the capacities of the Eigen modes (or antennas) of the channel:

$$C = \sum_{k=1}^{R_H} \log_2 \left[1 + \frac{P_k}{\sigma_n^2} \sigma_k^2 \right]$$

Where σ_n^2 is noise variance, and P_k is the power allocated to the k the Eigen mode. We assume that $\sum P_k = P$ is independent of the number of antennas.

The distribution of power among the different Eigen modes (or antennas) depends on the amount of CSIT.

Assume that the RX has perfect CSI. Therefore the capacity increases linearly with $\min(N_t, N_r, N_s)$.

Capacity in Flat-Fading Channels

Capacity is considered based on the following concepts:

1. *Erotic (Shannon) capacity: this is the expected value* of the capacity, taken over all realizations of the channel. This quantity assumes an infinitely long code that extends over all the different channel realizations.
2. Outage capacity: this is the minimum transmission rate that is achieved over a certain fraction of the time – e.g., 90% or 95%. We assume that data are encoded with a near-Shannon-limit achieving code that extends over a period that is much shorter than the channel coherence time.

Thus, each channel realization can be associated with a (Shannon) capacity value. Capacity thus becomes a random variable (rave) with an associated cumulative distribution function (cafe).

No Channel State Information at the Transmitter and Perfect CSI at the Receiver

Now, what is the capacity that we can achieve in a fading channel without CSI? Figure 5.4.2 shows the result for some interesting systems at a 21-dB SNR.

The (1, 1) curve describes a Single Input Single Output (SISO) system. We find that the median capacity is on the order of 6 bit/s/Hz, but the 5% outage capacity is considerably lower (on the order of 3 bit/s/Hz).

When using a (1, 8) system – i.e., 1 transmit antenna and 8 receive antennas – the mean capacity does not increase that significantly – from 6 to 10 bit/s/Hz. However, the 5% outage capacity increases significantly from 3 to 9 bit/s/Hz. The reason for this is the much higher resistance to fading that such a diversity system has.

However, when going to a (8, 8) system – i.e., a system with 8 transmit and 8 receive Antennas – both capacities increase dramatically: the mean capacity is on the order of 46 bit/s/Hz, and the 5% outage probability is more than 40 bit/s/Hz.

The exact expression for the ergodic capacity is

Where $m = \min(N_t, N_r)$ and $n = \max(N_t, N_r)$ and $L_k^{n-N}(\cdot)$ are associated Laguerre polynomials.

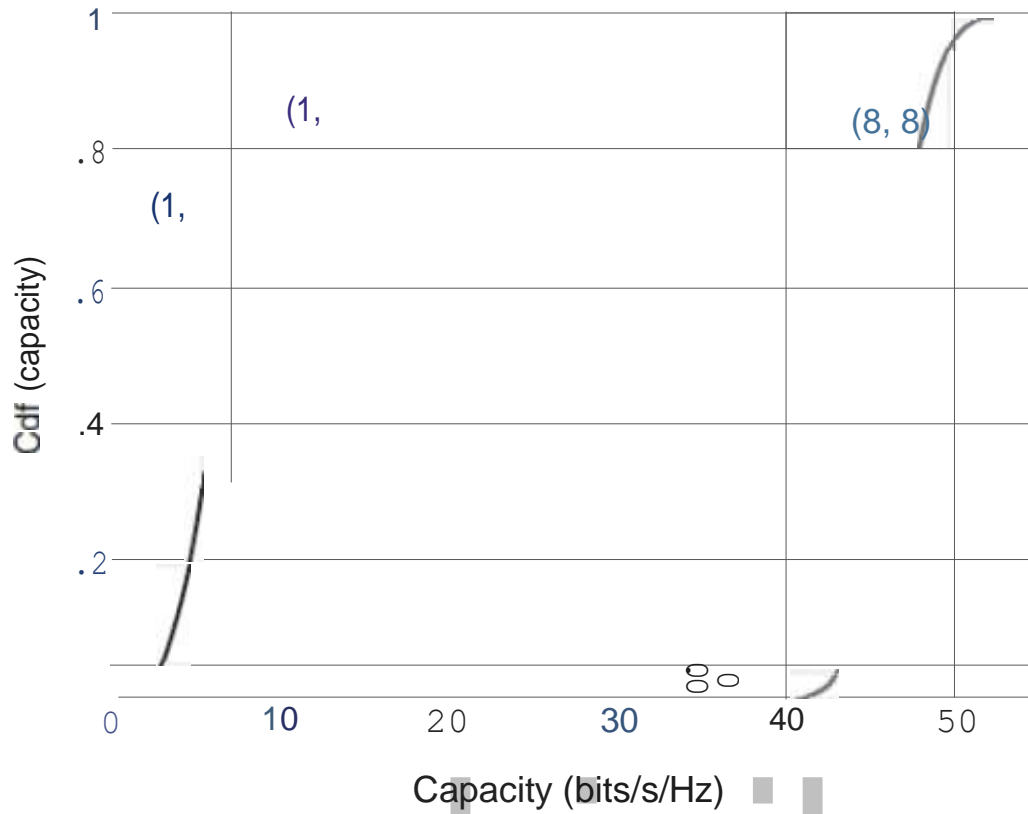


Fig 5.4.2: Cumulative distribution function of capacity for 1×1 , 1×8 , and the 8×8 optimum scheme.
[Source: "Wireless communications" by Andreas F. Molisch, Page-471]

UNIT 5- MULTIPLE ANTENNA TECHNIQUES

5.1 MIMO SYSTEMS

MIMO systems are systems with Multiple Element Antennas (MEAs) at both link ends.

In a MIMO system, same data is transmitted through multiple antennas over the same path in the same bandwidth. Because of this each signal reaches the receiving antenna through a different path, resulting in more reliable data. The data rate also increases by a factor determined by the number of transmit and receive antennas.

Wi-Fi, LTE; Long Term Evolution, and many other radio, wireless and RF technologies are using the new MIMO wireless technology to provide increased link capacity and spectral efficiency combined with improved link reliability using what were previously seen as interference paths.

The MEAs (The multiple element Antennas) of a MIMO system can be used for four different purposes:

(i) Beam forming, (ii) Diversity, (iii) Interference suppression, and (iv) Spatial multiplexing (transmission of several data streams in parallel).

The first three concepts are the same as for smart antennas. Having multiple antennas at both link ends leads to some interesting new technical possibilities, but does not change the fundamental effects of this approach.

Spatial multiplexing, is a new concept, and has thus drawn the greatest attention. It allows direct improvement of capacity by simultaneous transmission of multiple data streams.

One of the core ideas behind MIMO wireless systems is the space-time signal processing in which time (the natural dimension of digital communication data) is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas, i.e. the use of multiple antennas located at different points.

Accordingly MIMO wireless systems can be viewed as a logical extension to the smart antennas that have been used for many years to improve wireless.

It is found between a transmitter and a receiver, the signal can take many paths. Additionally by moving the antennas even a small distance the paths used will change. The variety of paths available occurs as a result of the number of objects that appear to the side or even in the direct path between the transmitter and receiver.

Previously these multiple paths only served to introduce interference. By using MIMO, these additional paths can be used to advantage. They can be used to provide additional robustness to the radio link by improving the signal to noise ratio, or by increasing the link data capacity.

The two main formats for MIMO are given below:

Spatial diversity: Spatial diversity is often refers to transmit and receive diversity. These two methodologies are used to provide improvements in the signal to noise ratio and they are characterised by improving the reliability of the system with respect to the various forms of fading.

Spatial multiplexing: This form of MIMO is used to provide additional data capacity by utilising the different paths to carry additional traffic, i.e. increasing the data throughput capability.

As a result of the use multiple antennas, MIMO wireless technology is able to considerably increase the capacity of a given channel while still obeying Shannon's law.

By increasing the number of receive and transmit antennas it is possible to linearly increase the throughput of the channel with every pair of antennas added to the system. This makes MIMO wireless technology one of the most important wireless techniques to be employed in recent years.

5.3 PRECODING

Precoding involves preprocessing of the transmit signal in an RF system. Precoding uses channel state information at the transmitter to improve performance and increase spectral efficiency.

It is used to implement the superposition of multiple beams, including several different data streams of information for spatial multiplexing. Precoding is the transmitter signal processing needed to affect the received signal's maximization to specific receivers and antennas while reducing the interference to all other receivers and receiving antennas.

Precoding and beam forming are used together in Wi-Fi, 4G, and 5G systems, and the words are sometimes used interchangeably, but they are not identical.

The word precoding refers more to a software implementation of communication theory, and beam forming refers more to the hardware implementation and the antennas in the system. And precoding generally refers to the transmitter side, while beam forming can be applied to both transmitters and receivers.

Precoding involves the individual control of the amplitudes and phases of the signals sent from the various transmit antennas. When precoding is implemented together with beam forming, it can better focus energy towards the intended receiver. Various aspects of beam forming and second-generation beam forming will be addressed in subsequent articles.

Precoding is used for various communications standards, including Wi-Fi, 4G, and 5G. Precoding assumes that channel state information (CSI) is known at the transmitter. Precoding starts with channel sounding that involves sending a coded message (called a sounding packet or a pilot signal) to the receiver.

Each of the users send back their individual CSIs to the transmitter. The users' CSIs are used to set the precoding (spatial mapping) matrix for subsequent data transmission.

Channel state information needed to characterize a MU-MIMO system. H is the spatial mapping matrix used for precoding as shown in figure 5.3.1.

In systems using time division multiplexing (the uplink and downlink transmissions are over the same subcarrier frequency), the over-the-air transmission channels between antennas and user terminals are the same in both directions, and they are called reciprocal

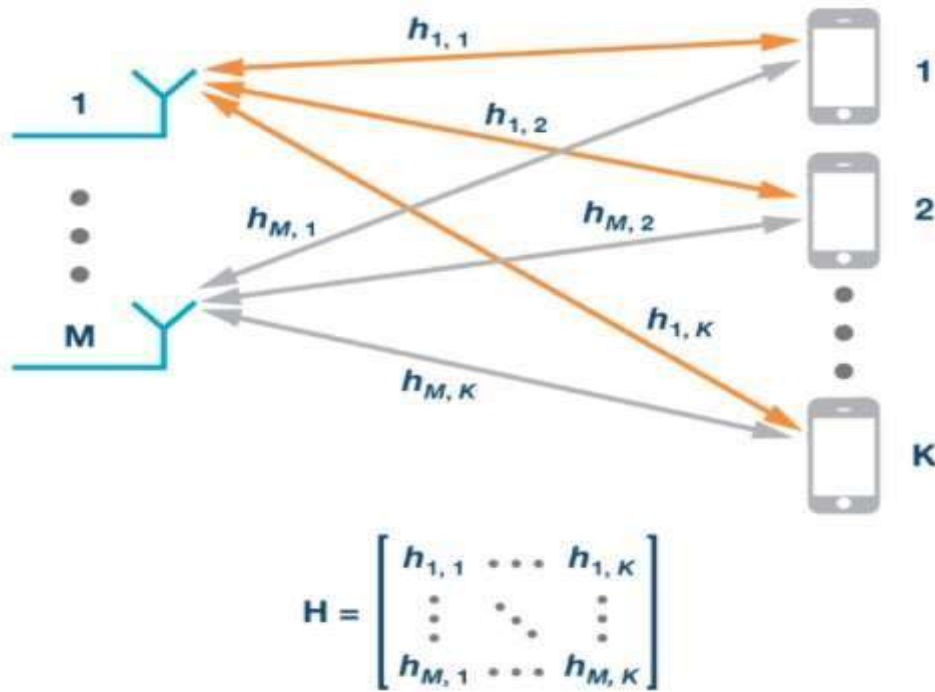


Fig5.3.1: Channel state information needed to characterize a MU-MIMO system. H is the spatial mapping matrix used for precoding.

[Source: <https://www.analogictips.com>]

Since the channels are reciprocal, CSIs only need to be determined for one direction. That enables the use of uplink-based determination of CSIs. An advantage of using uplink-based characterization is that the signal processing needed to determine the CSIs is performed at the base station and not in the user equipment.

Sounding is important in Wi-Fi and 4G installations, but it is critical to maximizing system performance for 5G systems. The 5G high-frequency band is particularly challenging in terms of coverage area limitations, signal attenuation, path and penetration losses, as well as scattering.

Multi-user multiple-input multiple-output (MU-MIMO) spatial multiplexing techniques improve the spectral efficiency of 5G networks. MU-MIMO enables multiple users in the same frequency-time block while reducing interference between users. The preceding challenges become even more complex when specific users and data packets have priority and deadlines for delivery; for example, emerging applications such as autonomous vehicles must have very low latency. Each packet of data has a delivery deadline that must be met.

BEAMFORMING

Beam forming is an active approach that combines antenna array elements to direct signals at specific angles and create constructive interference

Transmit and receive beam forming have been considered for cellular systems to improve the signal-to-noise ratio (SNR) or extend the coverage.

A better beam forming gain can be achieved if the number of antennas in an array is large. Beam forming can improve the spatial selectivity and efficiency of transmitters and receivers.

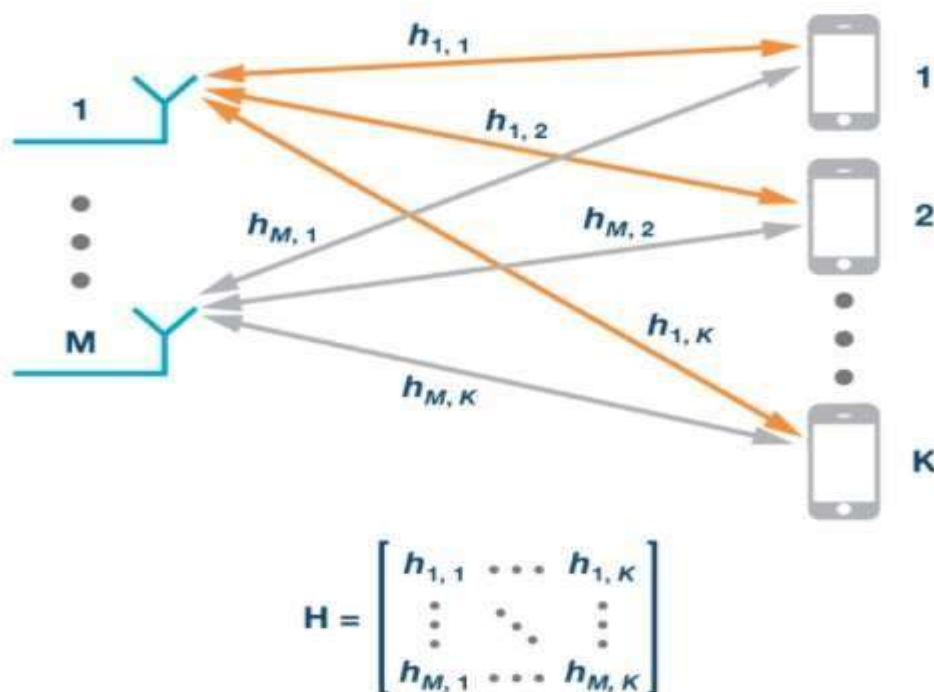
In the case of transmitters, beam forming combined with precoding is used in communications applications to affect maximization of the received signal to specific receivers and antennas while reducing the interference to all other receivers and receiving antennas.

In cellular systems, as base stations (BSs) can have a number of antennas, beam forming can be easily employed at BSs. In this case, transmit beam forming becomes downlink

Beam forming and receive beam forming becomes uplink beam forming.

Unfortunately, due to various problems, the number of antenna elements in an array for downlink beam forming is limited. has been considered seriously to extend the coverage.

How transmit beamforming works, as illustrated in figure 5.3.2.



Let h_1 and h_2 denote the channel coefficients from TX antennas 1 and 2, respectively. If TX antenna k transmits $w_k s$, where w_k is the weight and s is the signal to be transmitted, the received signal becomes

$$r = h_1 w_1 s + h_2 w_2 s + n = (h_1 w_1 + h_2 w_2) s + n;$$

where n is the background noise.

The SNR at the receiver becomes

$$\text{SNR} = \frac{|h_1 w_1 + h_2 w_2|^2 E_s}{N_0} \leq \frac{\|\mathbf{h}\|^2 \|\mathbf{w}\|^2 E_s}{N_0},$$

where the equality holds if $w_k \propto h_k^*$.

A full diversity order is equal to the number of antennas. The more transmit antennas, the better performance. However, the channel state information (CSI) is required for transmit beam forming to achieve the maximum SNR. If the transmitter does not know the CSI, the receiver has to feedback it to the transmitter.

With limited feedback, the number of antennas cannot be large, which has been one of the major drawbacks of transmit beam forming,

Beamforming for point-to-point MIMO .

It is often known as single-user beam forming

To maximize the SNR, the principle of matched filtering (MF) can be employed when CSI is available, which is known as maximal ratio transmission (MRT) scheme in the context of MIMO.

There are other approaches without CSI or partial CSI:

Blind beam forming (or long-term transmit beam forming) with statistical properties of channels ,

Semi- blind beam forming (with partial CSI).

Diversity beam forming (with channel coding): compared to blind beam forming, it can achieve a diversity gain.

Multiuser MIMO

There exists interference due to the presence of multiple signals to be transmitted to multiple users. Dirty paper coding (DPC) can achieve the channel capacity by suppressing known interference. However, its implementation is not easy.

Multiuser beam forming can provide a reasonable performance with low-complexity.

A better performance can be achieved with multiuser diversity & user selection

CSI at BS is required to mitigate the (intra-cell) interference:

- 1.No feedback in TDD: channel reciprocity can be used
- 2.Feedback in FDD: excessive overhead

In this approach, Resource allocation (including power control) becomes crucial.

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5.2 SPATIAL MULTIPLEXING

Spatial multiplexing uses MEAs (Multiple element antennas) at the TX (transmitter) for transmission of parallel data streams (see Figure 5.2.1).

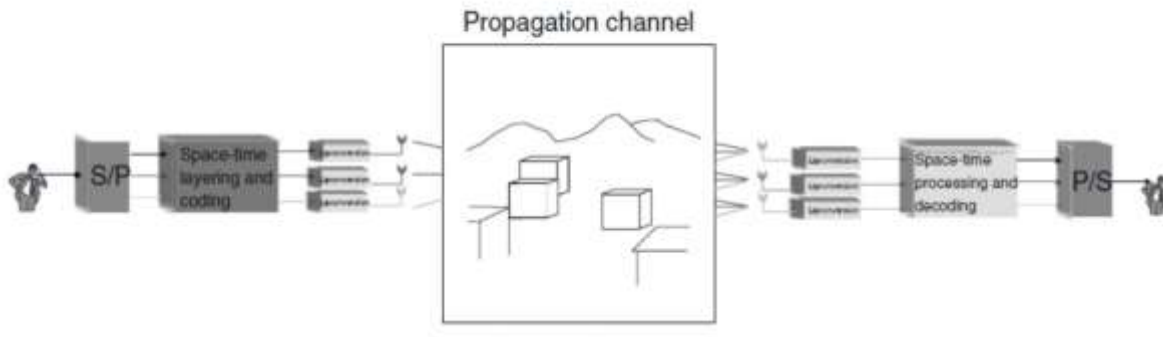


Fig 5.2.1: Spatial multiplexing

[Source: "Wireless communications" by Andreas F. Molisch, Page-465]

An original high-rate data stream is multiplexed into several parallel streams, each of which is sent from one transmit antenna element. The channel "mixes up" these data streams, so that each of the receive antenna elements sees a combination of them.

If the channel is well behaved, the received signals represent linearly independent combinations. In this case, appropriate signal processing at the RX (receiver) can separate the data streams.

A basic condition is that the number of receive antenna elements is at least as large as the number of transmit data streams. It is clear that this approach allows the data rate to be drastically increased – namely, by a factor of $\min(N_t, N_r)$.

For the case when the TX knows the channel, we can also develop another intuition (see Figure 5.2.2).

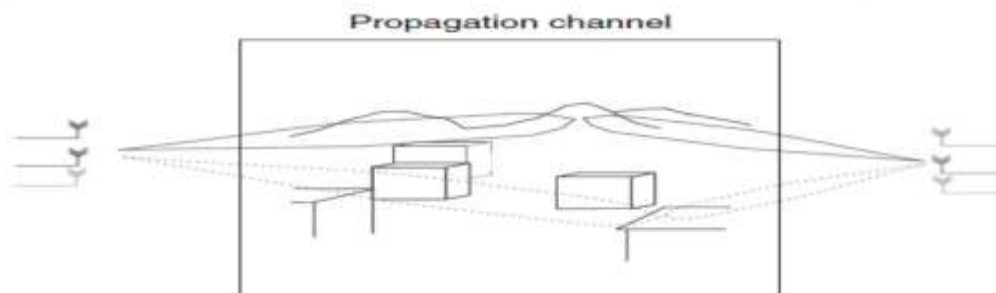


Fig 5.2.2: Transmission of different data streams via different interacting objects.

[Source: "Wireless communications" by Andreas F. Molisch, Page-466]

With N_t transmit antennas, we can form N_t different beams. We point all these beams at different Interacting Objects (IOs), and transmit different data streams over them.

At the RX, we can use N_r antenna elements to form N_r beams, and also point them at different IOs.

If all the beams can be kept orthogonal to each other, there is no interference between the data streams.

System Model

At the TX, the data stream enters an encoder, whose outputs are forwarded to N_t transmit antennas.

From the antennas, the signal is sent through the wireless propagation channel, which is assumed to be quasi-static and frequency-flat if not stated otherwise.

By quasi-static we mean that the coherence time of the channel is so long that “a large number” of bits can be transmitted within this time.

We denote the $N_r \times N_t$ matrix of the channel as

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1N_t} \\ h_{21} & h_{22} & \dots & h_{2N_t} \\ \vdots & \vdots & \dots & \vdots \\ h_{N_r1} & h_{N_r2} & \dots & h_{N_rN_t} \end{pmatrix}$$

Whose entries h_{ij} are complex channel gains (transfer functions) from the i th transmit to the j th receive antenna.

The received signal vector

contains the signals received by N_r antenna elements, where \mathbf{s} is the transmit signal vector and \mathbf{n} is the noise vector.

Block diagram of a multiple-input multiple-output system is shown in figure 5.2.3.

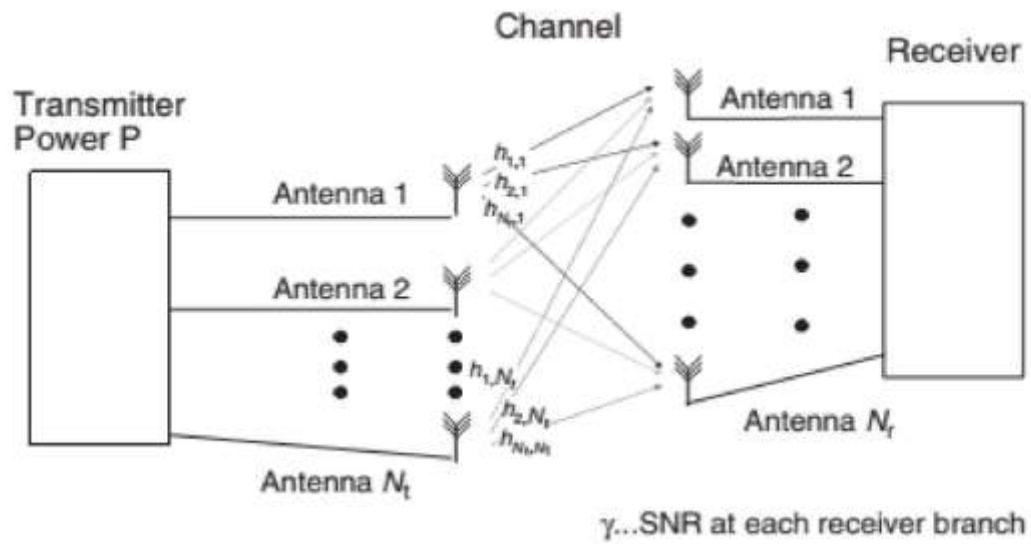


Fig 5.2.3: Block diagram of a multiple-input multiple-output system

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5.5 RECEIVER DIVERSITY

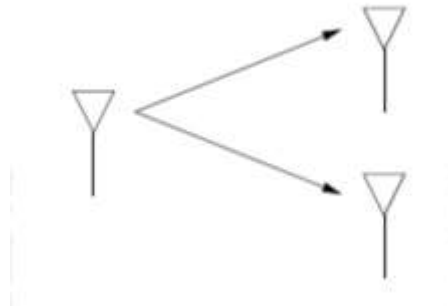


Fig 5.5.1: Receiver diversity.

[Source: "Fundamentals of Wireless Communication" by David Tso and Premed Viswanath, Page-89]

In a flat fading channel with 1 transmit antenna and L receive antennas as shown in figure 5.5.1., the channel model is as follows:

$$y_\ell[m] = h_\ell[m]x[m] + w_\ell[m] \quad \ell = 1, \dots, L$$

$$w_\ell[m] \sim \mathcal{CN}(0, N_0)$$

Where the noise $w_\ell[m]$ and independent across the antennas.

We can detect $x[1]$ based on $y_1[1], \dots, y_L[1]$. This is exactly the same detection problem as in the use of a repetition code over time, with L diversity branches now over space instead of over time.

If the antennas are spaced sufficiently far apart, then we can assume that the gains $h_\ell[1]$ are independent Rayleigh, and we get a diversity gain of L .

With receive diversity, there are actually two types of gain as we increase L . This can be seen by looking at the expression for the error probability of BPSK conditioned on the channel gains:

$$Q\left(\sqrt{2\|\mathbf{h}\|^2\text{SNR}}\right).$$

We can break up the total received SNR conditioned on the channel gains into a product of two terms:

$$\|\mathbf{h}\|^2 \text{SNR} = L \text{SNR} \cdot \frac{1}{L} \|\mathbf{h}\|^2.$$

The first term corresponds to a power gain (also called array gain): by having multiple receive antennas and coherent combining at the receiver, the effective total received signal power increases linearly with L : doubling L yields a 3 dB power gain.

The second term reflects the diversity gain: by averaging over multiple independent signal paths, the probability that the overall gain is small is decreased.

Note that if the channel gains h_{as} (1) are fully correlated across all branches, then we only get a power gain but no diversity gain as we increase L .

Transmitter Diversity

Now consider the case when there are L transmit antennas and 1 receive antenna, the MISO channel as shown in figure 5.5.2.

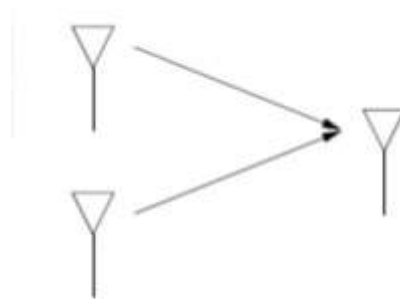


Fig 5.5.2: Transmitter diversity.

[Source: "Fundamentals of Wireless Communication" by David Tse and Pramod Viswanath, Page-89]

This is common in the downlink of a cellular system since it is often cheaper to have multiple antennas at the base station than to having multiple antennas at every handset. It is easy to get a diversity gain of

L: simply transmit the same symbol over the L different antennas during L symbol times. At any one time, only one antenna is turned on and the rest are silent.

This is simply a repetition code, and repetition codes are quite wasteful of degrees of freedom. More generally, any time diversity code of block length L can be used on this transmit diversity system: simply use one antenna at a time and transmit the coded symbols of the time diversity code successively over the different antennas. This provides a coding gain over the repetition code.

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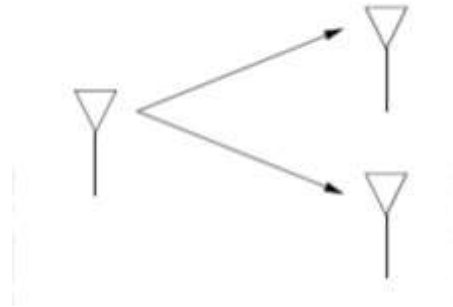


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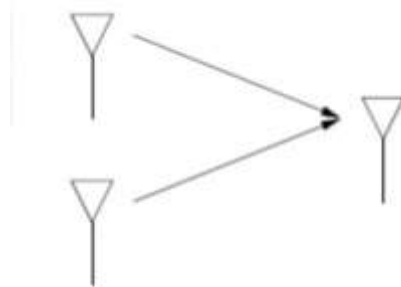


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