

## UNIT4- MULTIPATH MITIGATION TECHNIQUES

### 4.1 EQUALIZATION

Mobile communication systems require signal processing techniques that improve the link performance in mobile radio environments.

If the modulation bandwidth exceeds the coherence bandwidth of the radio channel, ISI occurs and modulation pulses are spread in time.

Equalization compensates for inter symbol interference (ISI) created by multipath within time dispersive channels.

An equalizer within a receiver compensates for the average range of expected channel amplitude and delay characteristics.

Equalizers must be adaptive since the channel is generally unknown and time varying.

#### Fundamentals of Equalization

Inter symbol interference (ISI)

- caused by multipath propagation (time dispersion);
- cause bit errors at the receiver;
- The major obstacle to high speed data transmission over mobile radio channels.

#### **Adaptive Equalization and Operating modes of an adaptive equalizer**

##### **Training (first stage)**

A known fixed-length training sequence is sent by the transmitter so that the receiver's equalizer may average to a proper setting.

The training sequence is designed to permit an equalizer at the receiver to acquire the proper filter coefficients in the worst possible channel conditions

The training sequence is typically a pseudorandom binary signal or a fixed, prescribed bit pattern.

The time span over which an equalizer converges is a function of

- the equalizer algorithm
- the equalizer structure
- The time rate of change of the multipath radio channel.

Equalizers require periodic retraining in order to maintain effective ISI cancellation.

### Tracking (second stage)

Immediately following the training sequence, the user data is sent.

As user data are received, the adaptive algorithm of the equalizer tracks the changing channel and adjusts its filter characteristics over time.

Commonly used in digital communication systems where user data is segmented into short time blocks.

TDMA wireless systems are particularly well suited for equalizers .TDMA systems send data in fixed-length time blocks, and the training sequence is usually sent at the beginning of a block.

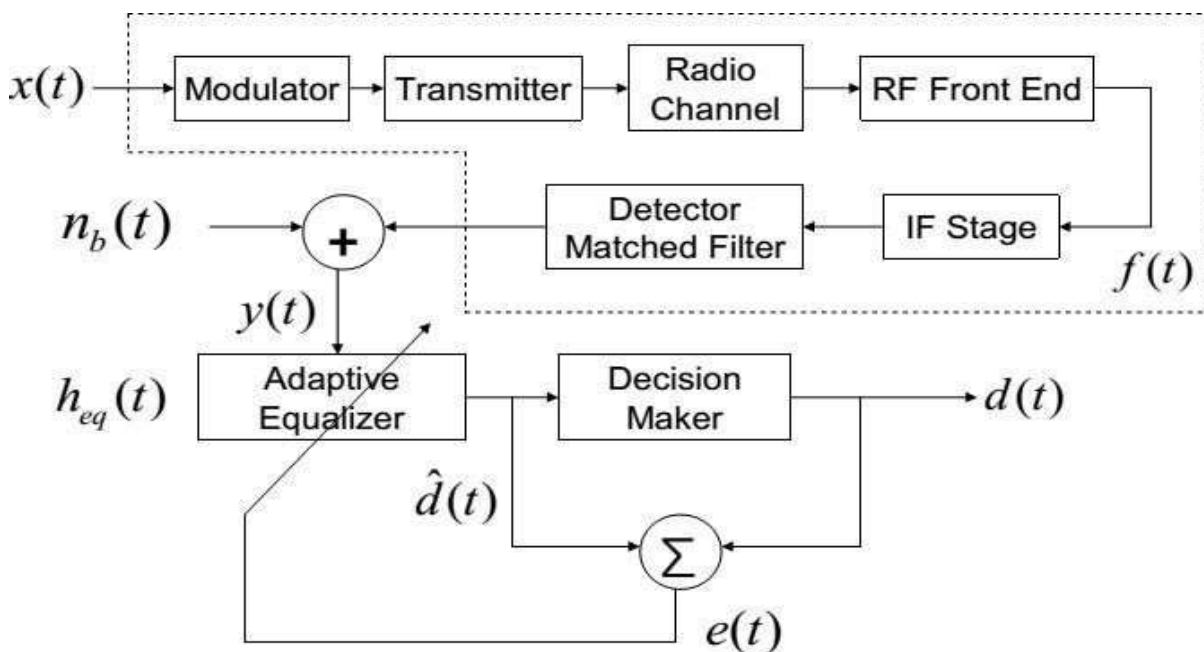
Each time a new data block is received, the equalizer is retrained using the same training sequence.

### Communication system with an adaptive equalizer

Equalizer can be implemented at baseband or at IF in a receiver.

Since the baseband complex envelope expression can be used to represent band pass waveforms and, thus, the channel response, demodulated signal, and adaptive equalizer algorithms are usually simulated and implemented at base band .

Block diagram of a simplified communications system using an adaptive equalizer at the receiver is shown in figure 4.1.1.



**Fig4.1.1: Adaptive Equalizer**

[Source: "Wireless communications" by Theodore S. Rappaport, Page-302]

If  $x(t)$  is the original information signal, and  $f(t)$  is the combined complex baseband impulse response of the transmitter, channel, and the RF/IF sections of the receiver, the signal received by the equalizer may be expressed as

$$y(t) = x(t) \otimes f^*(t) + n_b(t)$$

Where  $f^*(t)$  is the complex conjugate of  $f(t)$ ,  $n_b(t)$  is the baseband noise at the input of the equalizer, and  $\otimes$  denotes the convolution operation. If the impulse response of the equalizer is  $h_e(t)$ , then the output of the equalizer is

$$\begin{aligned} \hat{d}(t) &= x(t) \otimes f^*(t) \otimes h_{eq}(t) + n_b(t) \otimes h_{eq}(t) \\ &= x(t) \otimes g(t) + n_b(t) \otimes h_{eq}(t) \end{aligned}$$

Where  $g(t)$  is the combined impulse response of the transmitter, channel, RF/IF sections of the receiver, and the equalizer. The complex baseband impulse response of a transversal filter equalizer is given by

$$h_{eq}(t) = \sum_n c_n \delta(t - nT)$$

Where  $c_n$  are the complex filter coefficients of the equalizer. The desired output of the equalizer is  $x(t)$ , the original source data. Assume that  $n_b(t) = 0$ . Then, in order to force  $\hat{d}(t) = x(t)$  in equation (above),  $g(t)$  must be equal to

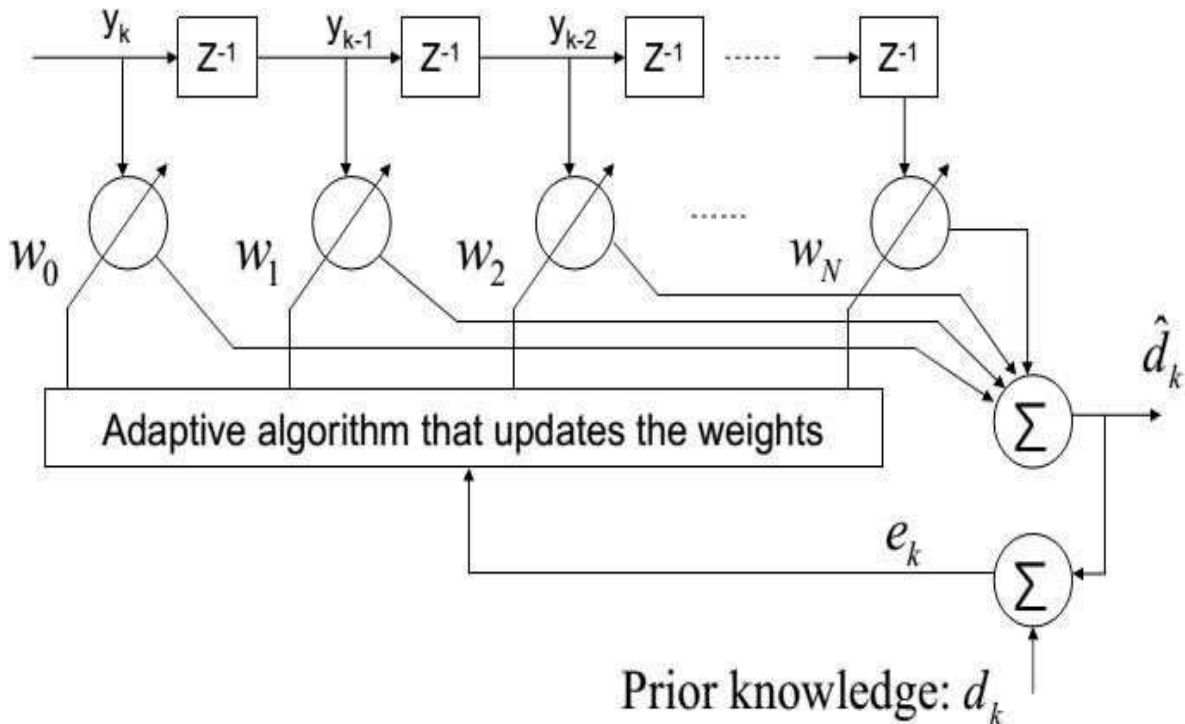
$$g(t) = f^*(t) \otimes h_{eq}(t) = \delta(t)$$

In the frequency domain, the above equation can be expressed as

$$H_{eq}(f) F^*(-f) = 1$$

Where  $H_e(f)$  and  $F(-f)$  are Fourier transforms of  $h_e(t)$  and  $f(t)$ , respectively.

### A Generic Adaptive Equalizer



**Fig4.1.2: A Generic Equalizer**

[Source: "Wireless communications" by Theodore S. Rappaport, Page-303]

The value of  $Y_k$  depends upon the instantaneous state of the radio channel and the specific value of the noise (as shown in figure 4.1.2).  $Y_k$  is a random process.

The adaptive equalizer structure is called a transversal filter, and it has  $N$  delay elements,  $N + 1$  taps, and  $N + 1$  tunable complex multipliers, called weights.

These weights are updated continuously by the adaptive algorithm either on a sample by sample basis or on a block by block basis.

The adaptive algorithm is controlled by the error signal  $e_k$ .

$e_k$  is derived by comparing the output of the equalizer with some signal which is either an exact scaled replica of the transmitted signal  $w_k$ . or which represents a known property of the transmitted signal.

A cost function is used, the cost function is minimized by using  $e_k$ , and the weights are updated iteratively.

For example, the least mean squares (LMS) algorithm can serve as a cost function. Iterative operation based on LMS algorithm.

New weights = Previous weights + (constant) x (Previous error) x (Current input vector), Where

Previous error = Previous desired output — Previous actual output

This process is repeated rapidly in a programming loop while the equalizer attempts to converge upon reaching convergence, the adaptive algorithm freezes the filter weights until the error signal exceeds an acceptable level or until a new training sequence is sent.

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## **EQUALIZERS IN A COMMUNICATION RECEIVER**

In communication systems the instantaneous combined frequency response will not always be flat, resulting in some finite prediction error.

Because the noise  $n_b(t)$  is present, an equalizer is unable to achieve perfect performance.

Thus there is always some residual ISI and some small tracking error.

Noise makes equation hard to realize in practice. Therefore, the instantaneous combined frequency response will not always be flat, resulting in some finite prediction error.

Because adaptive equalizers are implemented using digital logic, it is most convenient to represent all time signals in discrete form.

Let  $T$  represent some increment of time between successive observations of signal states.

Letting  $t = nT$  where  $n$  is an integer that represents time  $= nT$ , time waveforms may be equivalently expressed as a sequence on  $n$  in the discrete domain. Using this notation, equation may be expressed as

$$\hat{d}(n) = x(n) \otimes g(n) + n_b(n) \otimes h_{eq}(n)$$

The prediction error is

$$e(n) = d(n) - \hat{d}(n) = d(n) - [x(n) \otimes g(n) + n_b(n) \otimes h_{eq}(n)]$$

The mean squared error  $E[|e(n)|^2]$  is one of the most important measures of how well an equalizer works.  $E[|e(n)|^2]$  is the expected value (ensemble average) of the squared prediction error  $e(n)$  but time averaging can be used if  $e(n)$  is ergodic.

Equalization techniques can be subdivided into two categories:

Linear equalization

The output of the decision maker is not used in the feedback path to adapt the equalizer.

Nonlinear equalization

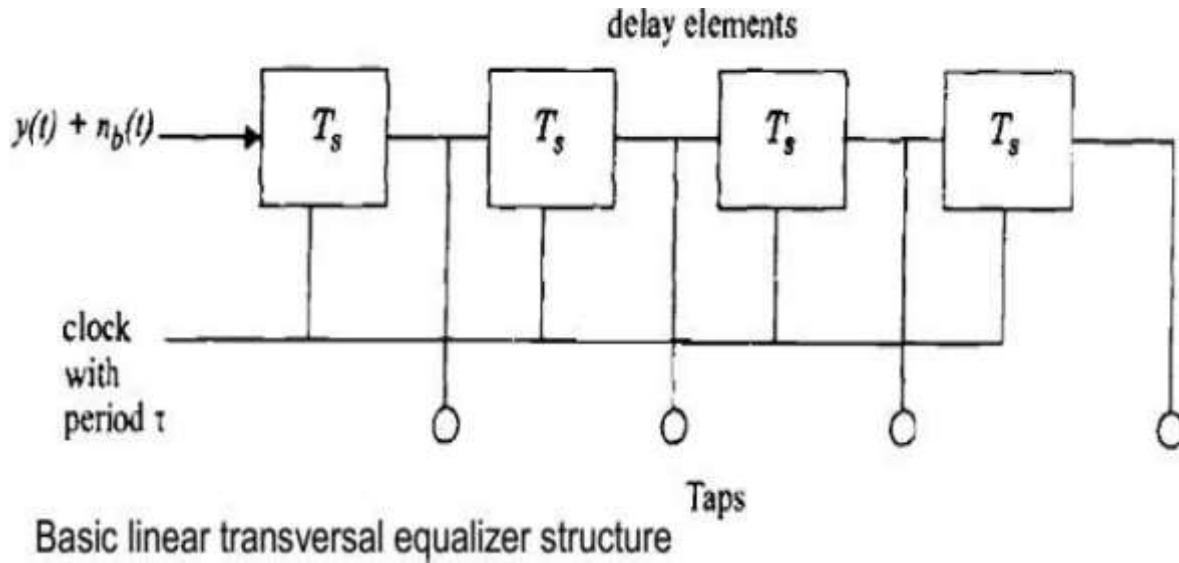
The output of the decision maker is used in the feedback path to adapt the equalizer. Many filter structures are used to implement linear and nonlinear equalizers

### **Classification of equalizers**

If the equalizer output is not used in the feedback path to adapt the equalizer, the equalization is linear.

If the equalizer output is fed back to change the subsequent outputs of the equalizer, the equalization is nonlinear.

**Linear transversal equalizer (LTE)** is made up of tapped delay lines as shown in figure 4.1.3, with the tapings spaced a symbol period ( $T_s$ ) a part .The transfer function can be written as a function of the delay operator or assuming that the delay elements have unity gain and delay  $T_s$ .

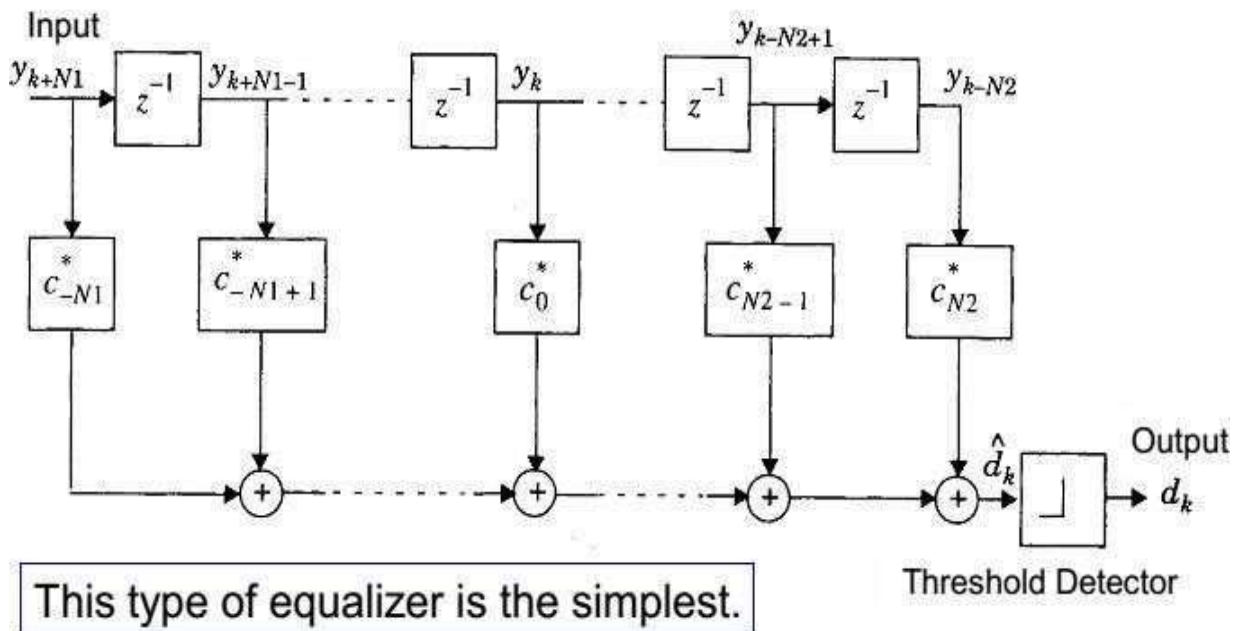


**Fig4.1.3: Basic structure of transversal equalizer**

[Source : "Wireless communications "by Theodore S. Rappaport,Page-309]

Structure of linear transversal equalizer

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**Fig4.1.4: Equalizer**

[Source: "Wireless communications" by Theodore S. Rappaport, Page-311]

Linear equalizer is also called Transversal filter as shown in figure 4.1.4.

In linear equalizer, the current and past values of the received signal are linearly weighted by the filter coefficient and summed to produce the output, as shown in figure. If the delays and the tap gains are analog, the continuous output of the equalizer is sampled at the symbol rate and the samples are applied to the decision device.

The implementation is, usually carried out in the digital domain where the samples of the received signal are stored in a shift register.

The output of this filter before a decision is made (threshold decision) is

$$\hat{d}_k = \sum_{n=-N_1}^{N_2} C_n^* y_{k-n}$$

$C_n^*$  represents the complex filter coefficients or tap weights,

$\hat{d}_k$  is the output at time index  $k$ ,  $y_n$  is the input received signal at time  $t_0 + iT$ ,  $t_0$  is the equalizer starting time and  $N = N_1 + N_2 + 1$  is the number of taps.

The values  $N_1$  and  $N_2$  denote the number of taps used in the forward and reverse portions of the equalizer.

The minimum mean squared error  $E[|e(n)|^2]$  that a linear transversal equalizer can achieve is

$$E[|e(n)|^2] = \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{N_0}{|F(e^{j\omega T})|^2 + N_0} d\omega$$

$F(e^{j\omega T})$  is the frequency response of the channel, and  $N_0$  is the noise

Spectral density.

The linear equalizer can also be implemented as a lattice filter, whose structure is shown in Figure 4.1.5.

The input signal  $Y_k$  is transformed into a set of  $N$  intermediate forward and backward error signals,  $f_n(k)$  and  $b_n(k)$  respectively, which are used as inputs to the tap multipliers and are used to calculate the updated coefficients



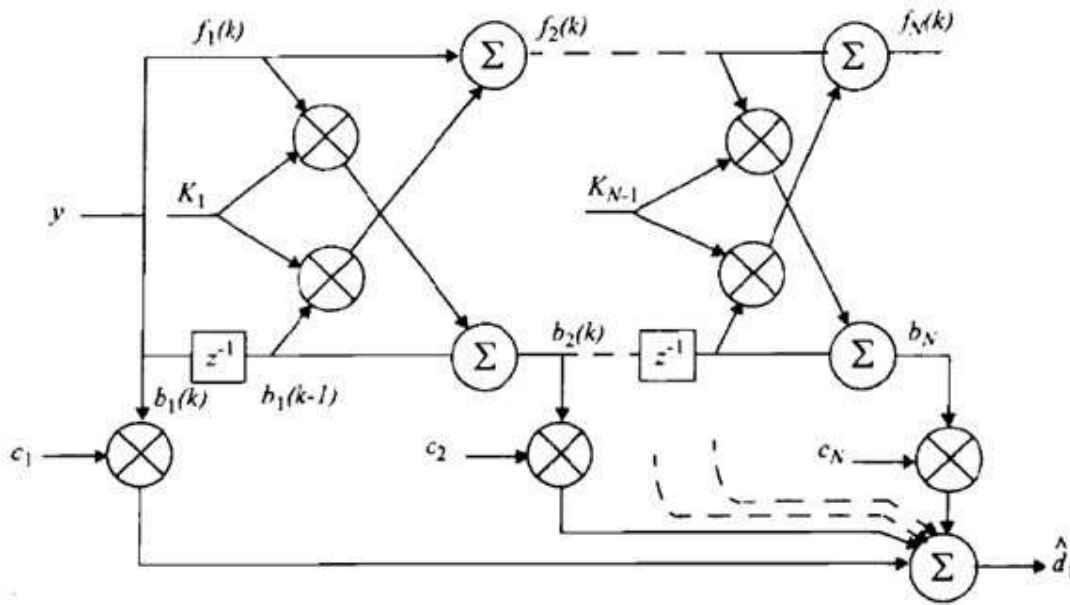


Fig4.1.5: Lattice Equalizer

[Source : "Wireless communications" by Theodore S. Rappaport, Page-312]

Each stage of the lattice is then characterized by the following recursive equations.

$$f_1(k) = b_1(k) = y(k)$$

$$f_n(k) = y(k) - \sum_{i=1}^n K_i y(k-i) = f_{n-1}(k) + K_{n-1}(k) b_{n-1}(k-1)$$

$$b_n(k) = y(k-n) - \sum_{i=1}^n K_i y(k-n+i) = b_{n-1}(k-1) + K_{n-1}(k) f_{n-1}(k)$$

Where  $K_n(k)$  is the reflection coefficient for the  $n$  the stage of the lattice. The backward error signals, are then used as inputs to the tap weights, and the output of the equalizer is given by

$$\hat{d}_k = \sum_{n=1}^N c_n(k) b_n(k)$$

**Two main advantages** of the lattice equalizer is its numerical stability and faster convergence. The unique structure of the lattice filter allows the dynamic assignment of the most effective length of the lattice equalizer. Hence, if the channel is not very time dispersive, only a fraction of the stages are used. When the channel becomes more time dispersive, the length of the equalizer can be increased by the algorithm without stopping the operation of the equalizer.

### **4.3 ALGORITHMS FOR ADAPTIVE EQUALIZATION**

Equalizer requires a specific algorithm to update the coefficients and track the channel variations.

Since it compensates for an unknown and time-varying channel

This section outlines three of the basic algorithms for adaptive equalization.

Though the algorithms detailed in this section are derived for the linear, transversal equalizer, they can be extended to other equalizer structures, including nonlinear equalizers.

#### **Factors determining the performance of an algorithm:**

##### **Rate of convergence (fast or slow?)**

Defined as the number of iterations required for the algorithm, in response to stationary inputs, to converge close enough to the optimum solution.

A fast rate of convergence allows the algorithm to adapt rapidly to a stationary environment of unknown statistics.

Furthermore, it enables the algorithm to track statistical variations when operating in a non-stationary environment.

##### **Miss adjustment (precise or not?)**

Provides a quantitative measure of the amount by which the final value of the mean square error, averaged over an ensemble of adaptive filters, deviates from the optimal minimum mean square error.

**Computational complexity** — this is the number of operations required to make one complete iteration of the algorithm.

**Numerical properties** — When an algorithm is implemented numerically, inaccuracies are produced due to round-off noise and representation errors in the computer. These kinds of errors influence the stability of the algorithm.

#### **Algorithms for Adaptive Equalization**

Zero Forcing Algorithm (ZF), Least Mean Square Algorithm (LMS)

##### **1. Zero Forcing Algorithm**

In a zero forcing equalizer, the equalizer coefficients are chosen to force the samples of the combined channel and equalizer impulse response to zero at all but one of the  $NT$  spaced sample points in the tapped delay line filter.

By letting the number of coefficients increase without bound, an infinite length equalizer with zero ISI at the output can be obtained.

When each of the delay elements provide a time delay equal to the symbol duration  $T$ , the frequency response  $H_e(f)$  of the equalizer is periodic with a period equal to the symbol rate  $1/T$ .

The combined response of the channel with the equalizer must satisfy Nyquist's first criterion

$$H_{ch}(f)H_{eq}(f) = 1, |f| < 1/2T$$

Where  $H_{och}(f)$  is the folded frequency response of the channel.

Thus, an infinite length, zero, ISI equalizer is simply an inverse filter which inverts the folded frequency response of the channel. This infinite length equalizer is usually implemented by a truncated length version.

The zero forcing equalizer has the disadvantage that the inverse filter may excessively amplify noise at frequencies where the folded channel spectrum has high attenuation.

The ZF equalizer thus neglects the effect of noise altogether, and is not often used for wireless links.

## 2. Least Mean Square Algorithm

LMS equalizer is used to minimize the mean square error (MSE) between the desired equalizer output and the actual equalizer output.

The prediction error is given by

$$e_k = d_k - \hat{d}_k = x_k - \hat{d}_k$$

Another equal for prediction error is

$$e_k = x_k - \mathbf{y}_k^T \mathbf{w}_k = x_k - \mathbf{w}_k^T \mathbf{y}_k$$

The mean square error  $|e_k|^2$  at time instant  $k$ , is given by

$$\xi = E[e_k^* e_k]$$

For a specific channel condition, the prediction error  $e_k$  is dependent on the tap gain vector  $\mathbf{W}_N$ , so the MSE of an equalizer is a function of  $\mathbf{W}_N$ .

Let the cost function  $J(\mathbf{w})$  denote the mean squared error as a function of tap gain vector  $\mathbf{w}_N$ .

In order to minimize the MSE, set as

$$\frac{\partial J(\mathbf{w}_N)}{\partial \mathbf{w}_N} = -2\mathbf{p}_N + 2\mathbf{R}_{NN}\mathbf{w}_N = 0$$

$$\mathbf{R}_{NN}\hat{\mathbf{w}}_N = \mathbf{p}_N$$

And is called the normal equation, since the error is minimized and is made orthogonal (normal) to the projection related to the desired signal  $\mathbf{w}_k$ .

Then the MMSE of the equalizer is

$$J_{opt} = J(\hat{\mathbf{w}}_N) = E[x_k x_k^*] - \mathbf{p}_N^T \hat{\mathbf{w}}_N$$

Where

$$\hat{\mathbf{w}} = \mathbf{R}_{NN}^{-1} \mathbf{p}_N$$

The LMS algorithm is the simplest equalization algorithm and requires only  $2N + 1$  operations per iteration.

Advantages: Low computational complexity, Simple program

Disadvantages: Poor tracking, slow convergence

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## **4.4 DIVERSITY**

### **Fundamentals of Diversity Techniques**

Diversity exploits the random nature of radio propagation by finding independent signal paths for communication, so as to boost the instantaneous SNR at the receiver.

Diversity is a powerful communication receiver technique that provides wireless link improvement at relatively low cost. Requires no training

A diversity scheme is a method that is used to develop information from several signals transmitted over independent fading paths.

Small Scale fading causes deep and rapid amplitude fluctuations as mobile moves over a very small distances.

If we space 2 antennas at 0.5 m, one may receive a null while the other receives a strong signal. By selecting the best signal at all times, a receiver can mitigate or reduce small-scale fading. This concept is called Antenna Diversity.

In virtually all applications, diversity decisions are made by the receiver, and are unknown to the transmitter.

Two types of diversity

Microscopic diversity - small scale fading

macroscopic diversity - large scale fading

#### **Microscopic diversity**

Small-scale fades: deep and rapid amplitude fluctuations over distances of just a few wavelengths. Caused by multiple reflections from the surroundings in the vicinity of the mobile. Results in a Rayleigh fading distribution of signal strength over small distances.

Microscopic diversity techniques can exploit the rapidly changing signal.

**For example**, use two antennas at the receiver (separated by a fraction of a meter), one may receive a null while the other receives a strong signal.

By selecting the best signal at all times, a receiver can mitigate small-scale fading effects called antenna diversity or space diversity.

Example: Rake receiver

#### **Macroscopic diversity**

Macroscopic diversity is also useful at the base station receiver.

By using base station antennas that are sufficiently separated in space, the base station is able to improve the reverse link by selecting the antenna with the strongest signal from the mobile.

Used to combat slow fading (shadowing)

Samples: Base-station handoff in cellular networks.

### Derivation of Selection Diversity improvement

Consider M independent Rayleigh fading channels available at a receiver as in figure 4.4.1. Each channel is called a diversity branch.

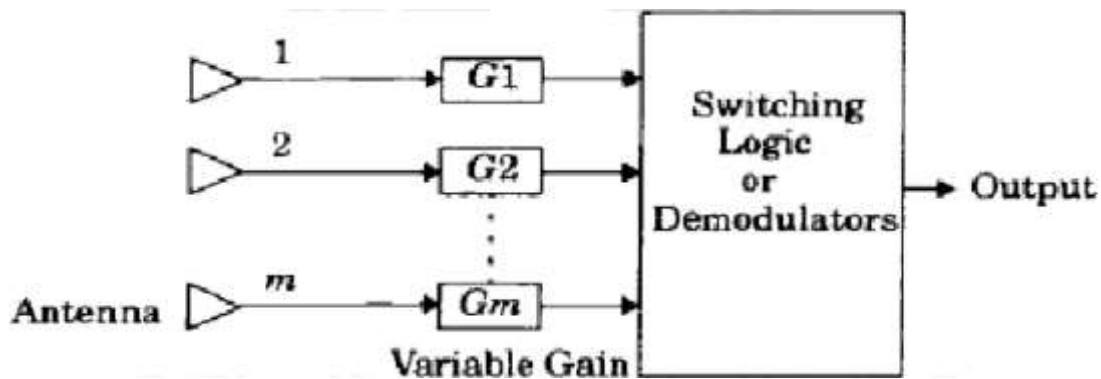


Fig4.4.1: Space diversity

[Source: "Wireless communications" by Theodore S. Rappaport, Page-330]

Assume that each branch has the same average SNR given by

$$\text{SNR} = \Gamma = \frac{E_b}{N_0} \alpha^2$$

$$\overline{\alpha^2} = 1$$

$$p(\gamma_i) = \frac{1}{\Gamma} e^{-\frac{\gamma_i}{\Gamma}} \quad \gamma_i \geq 0$$

Assume, If each branch has an instantaneous SNR=  $\gamma_i$ ,

Where  $\Gamma$  is the mean SNR of each branch.

The probability that a single branch has SNR less than some threshold is

$$Pr\{\gamma_i \leq \gamma\} = \int_0^{\gamma} p(\gamma_i) d\gamma_i = \int_0^{\gamma} \frac{1}{\Gamma} e^{-\frac{\gamma_i}{\Gamma}} d\gamma_i$$

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### **Practical Space Diversity Considerations**

Space diversity (also known as antenna diversity), is one of the most popular forms of diversity used in wireless systems.

The signals received from spatially separated antennas on the mobile would have essentially uncorrelated envelopes for antenna separations of one half wavelength or more.

Space diversity can be used at either the mobile or base station, or both.

Since the important caterers are generally on the ground in the vicinity of the mobile, when base station diversity is used, the antennas must be spaced considerably far apart to achieve decorrelation (several tens of wavelengths).

#### **Space diversity reception methods:**

##### **Selection diversity.**

It is the simplest diversity technique.

Selection diversity offers an average improvement in the link margin without requiring additional transmitter power or sophisticated receiver circuitry.

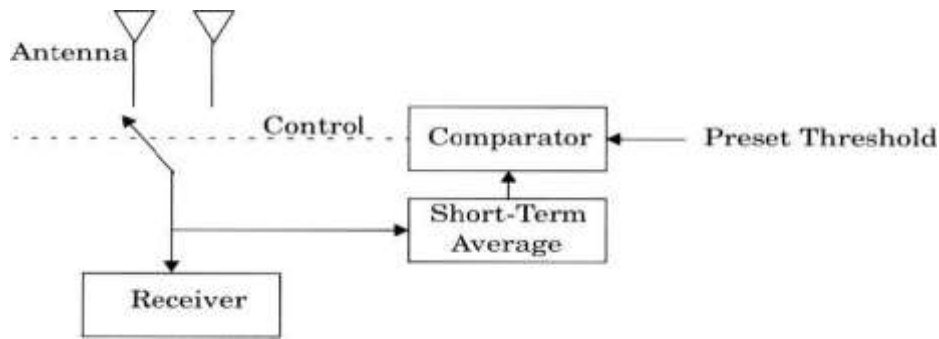
Selection diversity is easy to implement because all that is needed is a side monitoring station and an antenna switch at the receiver.

It is not an optimal diversity technique because it does not use all of the possible branches simultaneously.

##### **Feedback or Scanning diversity**

Scanning all the signals in a fixed sequence until the one with SNR more than a predetermined threshold is identified. Also called switch and stay combining.

Basic form of scanning diversity (Fig: 4.4.2).



**Fig4.4.2: Basic form of scanning diversity**

[Source: "Wireless communications" by Theodore S. Rappaport, Page-331]

### Maximal Ratio Combining

In this method proposed by Kahn, the signals from all of the  $M$  branches are weighted according to their individual signal voltage to noise power ratios and then summed.

Combining all the signals in a co-phased and weighted manner so as to have the highest achievable SNR at the receiver at all times.

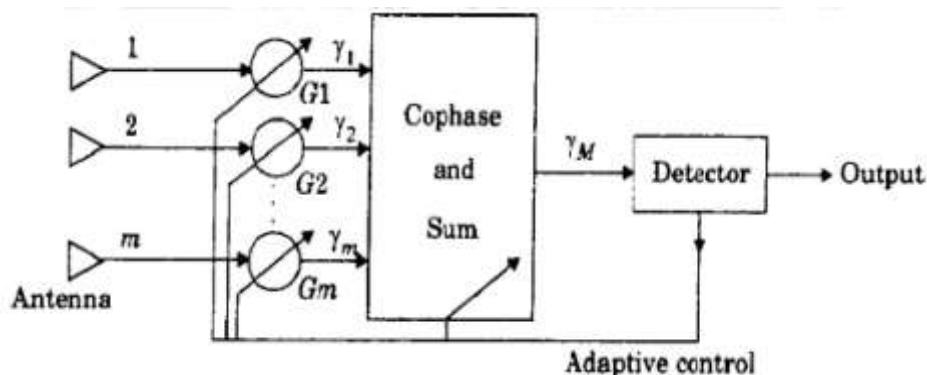
Consider  $M$  branches which are maximal ratio combined in a co phased and weighted manner in order to achieve high SNR.(Fig: 4.4.3)

Here the individual signals must be co-phased before being summed.

Maximal ratio combining produces an output SNR equal to the sum of the individual SNRs.

Thus, it has the advantage of producing an output with an acceptable SNR even when none of the individual signals are themselves acceptable.

This technique gives the best statistical reduction of fading of any known linear diversity combiner. Modern DSP techniques and digital receivers are now making this optimal form of diversity practical.



**Fig4.4.3: Maximal ratio combiner**

[Source: "Wireless communications" by Theodore S. Rappaport, Page-332]



### **Equal Gain Combining**

Combining all the signals in a co-phased manner with unity weights for all signal levels. The possibility of producing an acceptable signal from a number of unacceptable inputs is still retained, and the performance is only marginally inferior to maximal ratio combining and superior to selection diversity.

### **Polarization Diversity**

At the base station, space diversity is considerably less practical.

Polarization diversity only provides two diversity branches, but allows the antenna elements to be co-located. Measured horizontal and vertical polarization paths between a mobile and a base station are reported to be uncorrelated.

The correlation for the signals in each polarization is caused by multiple reflections between mobile and base station antennas.

The reflection coefficient for each polarization is different, which results in different amplitudes and phases for each, or at least some, of the reflections.

After sufficient random reflections, the polarization state of the signal will be independent of the transmitted polarization.

In practice, there is some dependence of the received polarization on the transmitted polarization. Transmits information on more than one carrier frequency.

Frequencies separated by more than the coherence bandwidth of the channel will not experience the same fades.

**Frequency diversity** is employed in microwave LOS links.

Frequency diversity transmits information on more than one carrier frequency.

Due to tropospheric propagation and resulting refraction, deep fading sometimes occurs. In practice, 1: N protection switching is provided by a radio licensee, wherein one frequency is nominally idle but is available on a stand-by basis to provide frequency diversity switching for any one of the N other carriers (frequencies) being used on the same link, each carrying independent traffic.

When diversity is needed, the appropriate traffic is simply switched to the backup frequency.

**Disadvantage:** not only requires spare bandwidth but also requires that there be as many receivers as there are channels used for the frequency diversity.

For critical traffic, the expense may be justified.

New OFDM modulation and access techniques exploit frequency diversity by providing simultaneous modulation signals with error control coding across a large bandwidth.

If a particular frequency undergoes a fade, the composite signal will still be demodulated.

### **Time Diversity**

Time diversity repeatedly transmits information at time spacing's that exceed the coherence time of the channel

Multiple repetitions of the signal will be received with independent fading conditions.

One modern implementation of time diversity involves the use of the RAKE receiver for spread spectrum CDMA, where the multipath channel provides redundancy in the transmitted message.

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## **4.5 ERROR PROBABILITY IN FADING CHANNELS WITH DIVERSITY RECEPTION**

Here we determine the Symbol Error Rate (**SER**) in fading channels when diversity is used at the receiver (RX).

Consider the flat-fading channels and computing the statistics of the received power and the BER.

In dispersive channels, we can analyze how diversity can mitigate the detrimental effects of dispersive channels on simple RXs.

### **Error Probability in Flat-Fading Channels**

#### **Classical Computation Method**

Here we can compute the error probability of diversity systems by averaging the conditional error probability (conditioned on a certain SNR) over the distribution of the SNR

$$\overline{SER} = \int_0^{\infty} pdf_{\gamma}(\gamma) SER(\gamma) d\gamma$$

**As an example**, let us compute the performance of BPSK with  $N_r$  diversity branches with MRC.

The SER of BPSK in AWGN is

$$SER(\gamma) = Q(\sqrt{2\gamma})$$

We obtain an equation that can be evaluated analytically

$$\overline{SER} = \left(\frac{1-b}{2}\right)^{N_r} \sum_{n=0}^{N_r-1} \binom{N_r-1+n}{n} \left(\frac{1+b}{2}\right)^n$$

Where b is defined as

$$b = \sqrt{\frac{\bar{\gamma}}{1 + \bar{\gamma}}}$$

For large values of  $\bar{\gamma}$  this can be approximated as

$$\overline{SER} = \left(\frac{1}{4\bar{\gamma}}\right)^{N_r} \binom{2N_r - 1}{N_r}$$

From this, we can see that (with  $N_r$  diversity antennas) the BER decreases with the  $N_r$ -th power of the SNR.

### Symbol Error Rate in Frequency-Selective Fading Channels

Here we determine the SER in channels that suffer from time dispersion and frequency dispersion.

We assume here FSK with differential phase detection.

For binary FSK with selection diversity

$$\overline{SER} = \frac{1}{2} - \frac{1}{2} \sum_{n=1}^{N_r} \binom{N_r}{n} (-1)^{n+1} \frac{b_0 \operatorname{Im}\{\rho_{XY}\}}{\sqrt{(\operatorname{Im}\{\rho_{XY}\})^2 + n(1 - |\rho_{XY}|^2)}}$$

Where  $b_0$  is the transmitted bit. This can be approximated as

$$\overline{SER} = \frac{(2N_r - 1)!!}{2} \left( \frac{1 - |\rho_{XY}|^2}{2(\operatorname{Im}\{\rho_{XY}\})^2} \right)^{N_r}$$

Where  $(2N_r - 1)!! = 1 \cdot 3 \cdot 5 \dots (2N_r - 1)$ .

For binary FSK with MRC:

$$\overline{SER} = \frac{1}{2} - \frac{1}{2} \frac{b_0 \operatorname{Im}\{\rho_{XY}\}}{\sqrt{1 - (\operatorname{Re}\{\rho_{XY}\})^2}} \sum_{n=0}^{N_r-1} \frac{(2n - 1)!!}{(2n)!!} \left( 1 - \frac{(\operatorname{Im}\{\rho_{XY}\})^2}{1 - (\operatorname{Re}\{\rho_{XY}\})^2} \right)^n$$

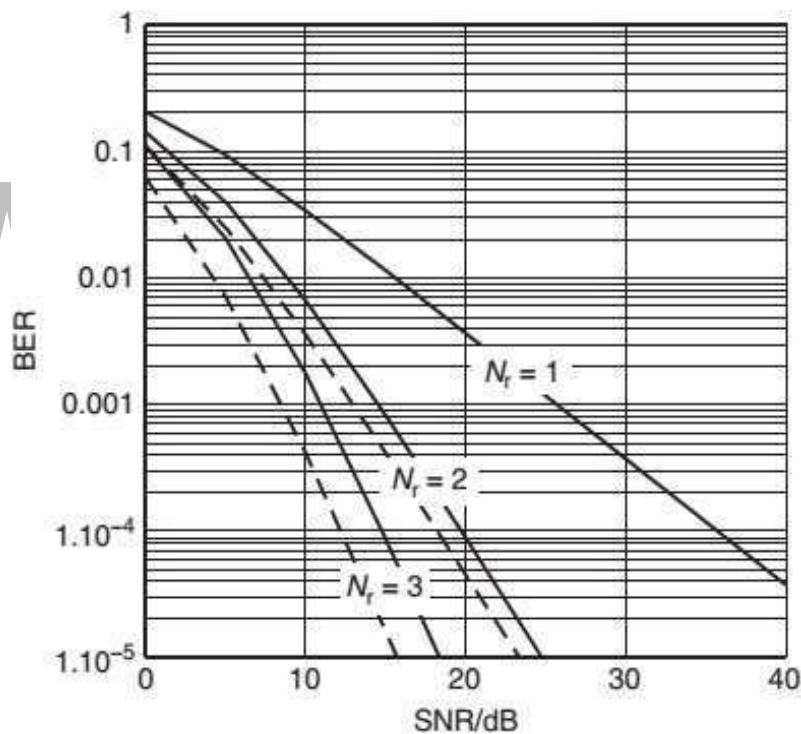
Which can be approximated as

$$\overline{SER} = \frac{(2N_r - 1)!!}{2(N_r!)} \left( \frac{1 - |\rho_{XY}|^2}{2(\text{Im}\{\rho_{XY}\})^2} \right)^{N_r}$$

This formulation shows that MRC improves the SER by a factor  $N_r!$  compared with selection diversity.

A further important consequence is that the errors due to delay dispersion and random Frequency modulation (FM) are decreased in the same way as errors due to noise.

The SER with diversity is approximately the  $N_r$ -the power of the SER without diversity (Fig. 4.5.1). i.e. Bit error rate of minimum shift keying (MSK) with received-signal-strength- indication driven selection diversity (solid) and maximum ratio combining (dashed) as a function of the signal-to-noise ratio with  $N_r$  diversity antennas.



**Fig4.5.1. SER with diversity**

[Source: "Wireless communications" by Andreas F.Molisch, Page-271]

## 4.2 NON LINEAR EQUALIZATION

Linear equalizers do not perform well on channels which have deep spectral nulls in the pass band. In an attempt to compensate for the distortion, the linear equalizer places too much gain in the vicinity of the spectral null, thereby enhancing the noise present in those frequencies.

Nonlinear equalizers are used in applications where the channel distortion is too severe for a linear equalizer to handle.

Types of nonlinear equalizer. Decision

Feedback Equalization (DFE)

Maximum Likelihood Sequence Estimation (MLSE)

### Decision Feedback Equalization (DFE)

Basic idea: Once an information symbol  $s$  has been detected, the ISI that it induces on future symbols can be estimated and subtracted out before detection of subsequent symbols.

**DFE can be realized in either the direct transversal form or as a lattice filter.**

The LTE form consists of a feed forward filter (FFF) and a feedback filter (FBF) as shown in figure 4.2.1.

The FBF is driven by decisions on the output of the detector, and its coefficients can be adjusted to cancel the ISI on the current symbol from past detected symbols.

The equalizer has  $N_1 + N_2 + 1$  taps in FFF and  $N_3$  taps in FBF.

The output of DFE is

$$\hat{d}_k = \sum_{n=-N_1}^{N_2} c_n^* y_{k-n} + \sum_{i=1}^{N_3} F_i d_{k-i}$$

Where  $c_n^*$ , and  $in$ , are tap gains and the inputs, respectively, to the forward filter,  $F_i^*$  are tap gains for the feedback filter, and  $d_i$  ( $i < k$ ) is the previous decision made on the detected signal.

That is, once  $d_0$  is obtained using equation (above),  $d_0$  is decided from it. Then,  $d_0$  along with previous decisions  $d_{-1}$ ,  $d_{k-2}$  are fed back into the equalizer, and  $d_{k+1}$  is obtained using the above equation.

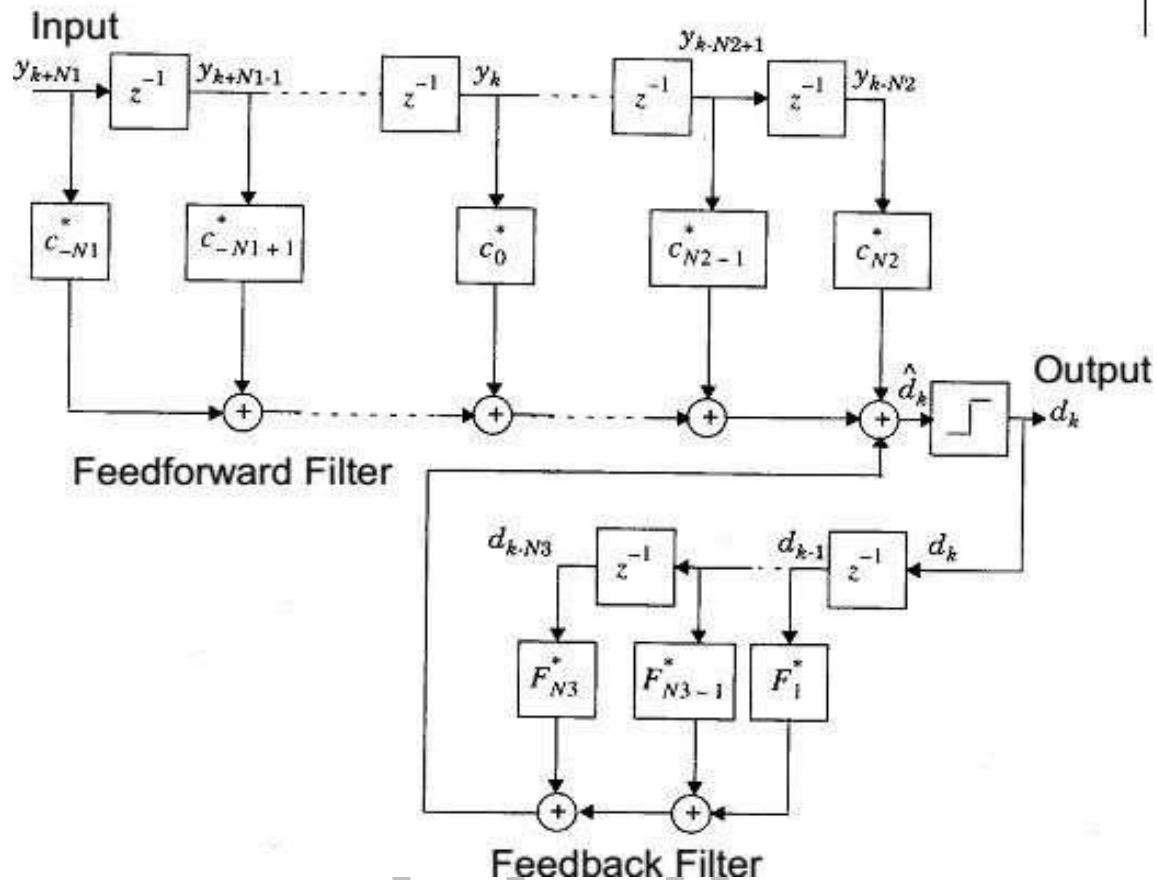


Fig4.2.1: Decision Feedback Equalizer

[Source : "Wireless communications" by Theodore S. Rappaport, Page-314]

The minimum mean square error of DFE

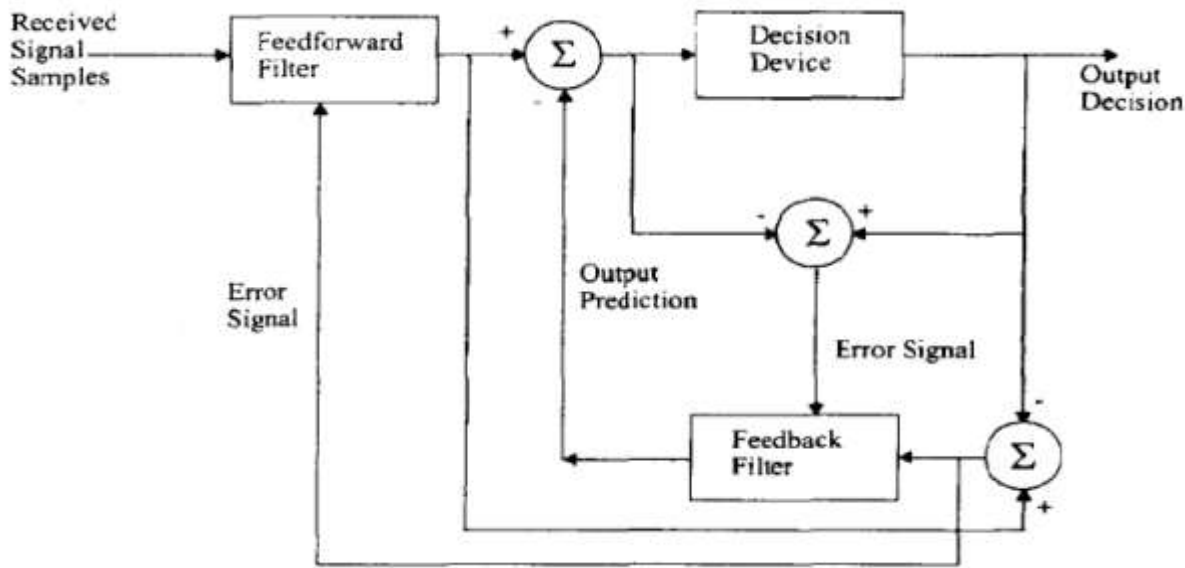
$$E[|e(n)|^2]_{min} = \exp \left\{ \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \ln \left[ \frac{N_0}{|F(e^{j\omega T})|^2 + N_0} \right] d\omega \right\}$$

It can be seen that the minimum MSE for a DFE is always smaller than that of an LTE.

An LTE is well behaved when the channel spectrum is comparatively flat. A DFE is more appropriate for severely distorted wireless channels. If the channel is severely distorted or exhibits nulls in the spectrum, the performance of an LTE deteriorates and the mean squared error of a DFE is much better than a LTE.

Also, an LTE has difficulty in equalizing a non-minimum phase channel, where the strongest energy arrives after the first arriving signal component. Thus, a DFE is more appropriate for severely distorted wireless channels.

Another type of DFE proposed by Belfour and Park is called a predictive DFE, and is shown in Figure 4.2.2.



**Fig4.2.2: Predictive Equalizer**

[Source: "Wireless communications" by Theodore S. Rappaport, Page-315]

It consists of a **feed forward filter (FFF)** as in the DFE.

Here, the feedback filter (FBF) is driven by an input sequence formed by the difference of the output of the detector and the output of the feed forward filter. Hence, the FBF here is called a noise predictor because it predicts the noise and the residual contained in the signal at the FFF output and subtracts from it the detector output after some feedback delay.

The predictive DFE performs as well as the conventional DFE as the limit

In the number of taps in the FFF and the FBF approach infinity. The FBF in the predictive DFE can be realized as a lattice structure. The RLS lattice algorithm can be used in this case to yield fast convergence.

### **Maximum Likelihood Sequence Estimation (MLSE) Equalizer**

The MSE-based linear equalizers are optimum with respect to the criterion of minimum probability of symbol error when the channel does not introduce any amplitude distortion.

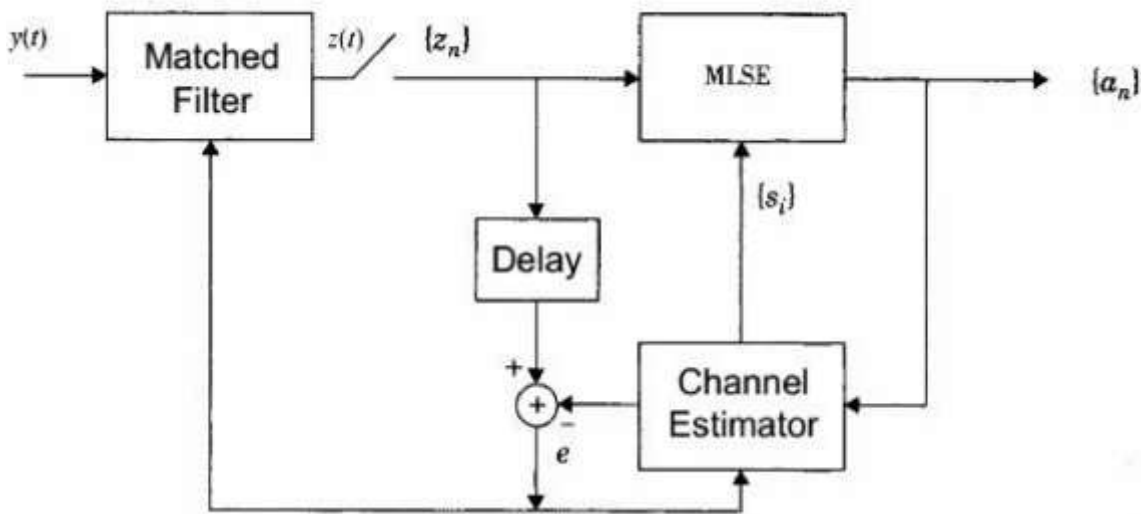
This is precisely the condition in which an equalizer is needed for a mobile communications link.

**MLSE uses** various forms of the classical maximum likelihood receiver structure.

The MLSE tests all possible data sequences (rather than decoding each received symbol by itself), and chooses the data sequence with the maximum probability as the output. A



Drawback: An MLSE usually has a large computational Requirement especially when the delay spread of the channel is large.



**Fig 4.2.3: MLSE Structure**

[Source: "Wireless communications" by Theodore S. Rappaport, Page-316]

The MLSE can be viewed as a problem in estimating the state of a discrete time finite state machine. The channel has  $M$  states, where  $M$  is the size of the symbol alphabet of the modulation. An  $M$ -state trellis is used by the receiver to model the channel over time.

The Viterbi algorithm then tracks the state of the channel by the paths through the trellis.

The block diagram of a MLSE receiver based on the DFE is shown in Figure 316.

The MLSE is optimal in the sense that it minimizes the probability of a sequence error. The MLSE requires knowledge of the channel characteristics in order to compute the metrics for making decisions.

The MLSE also requires knowledge of the statistical distribution of the noise corrupting the signal. Thus, the probability distribution of the noise determines the form of the metric for optimum demodulation of the received signal. The matched filter operates on the continuous time signal, whereas the MLSE and channel estimator rely on discretized (nonlinear) samples.

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## **4.6 RAKE RECEIVER**

**RAKE receiver** is used in CDMA-based Code Division Multiple Access systems and can combine multipath components, which are time-delayed versions of the original signal transmission.

Combining is done in order to improve the signal to noise ratio at the receiver.

RAKE receiver attempts to collect the time-shifted versions of the original signal by providing a separate correlation receiver for each of the multipath signals. This can be done due to multipath components are practically uncorrelated from another when their relative propagation delay exceeds a chip period.

**The basic idea of a RAKE receiver** was first proposed by Price and Green (1956).

Due to reflections from obstacles a radio channel can consist of many copies of originally transmitted signals having different amplitudes, phases, and delays.

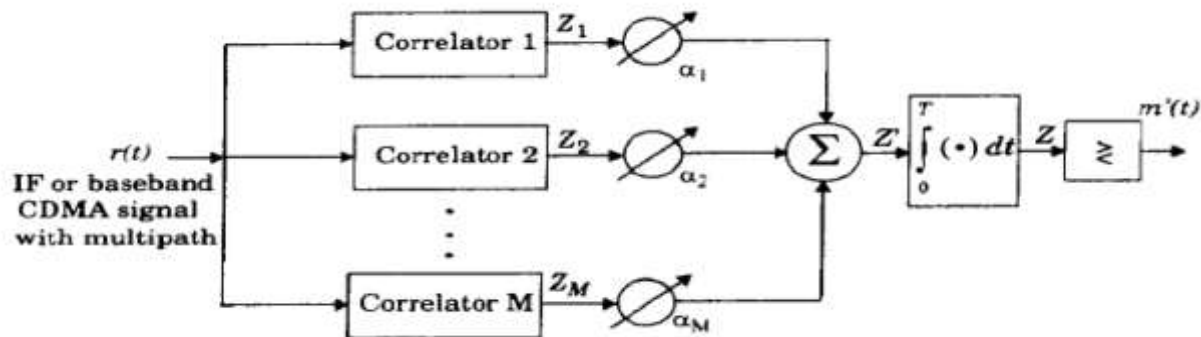
If the signal components arrive more than duration of one chip apart from each other, a RAKE receiver can be used to resolve and combine them.

The RAKE receiver uses a multipath diversity principle. Multipath can occur in radio channel in various ways such as, reflection and diffraction from buildings, and scattering from trees.

### **M-finger RAKE Receiver**

A RAKE receiver utilizes (shown in figure 4.6.1) multiple correlates to separately detect M strongest multipath components. The outputs of each correlate are weighted to provide better estimate of the transmitted signal than is provided by a single component.

Demodulation and bit decisions are then based on the weighted outputs of the M correlates.



**Fig4.6.1: RAKE Receiver Implementation**

Source: "Wireless communications" by Theodore S. Rappaport, Page-336]

Each correlate detects a time-shifted version of the original CDMA transmission, and each finger of the RAKE correlates to a portion of the signal, which is delayed by at least one chip in time from the other fingers.

Assume  $M$  correlates are used in a CDMA receiver to capture  $M$  strongest multipath components. A weighting network is used to provide a linear combination of the correlate output for bit decision. Correlate 1 is synchronized to the strongest multipath  $m_1$ . Multipath component  $m_2$  arrived  $t_1$  later than  $m_1$  but has low correlation with  $m_1$ .

The RAKE receiver uses several baseband correlates to individually process several signal multipath components. The correlate outputs are combined to achieve improved communications reliability and performance.

Bit decisions based only a single correlation may produce a large bit error rate as the multipath component processed in that correlate can be corrupted by fading.

In a RAKE receiver, if the output from one correlate is corrupted by fading, the others may not be, and the corrupted signal may be discounted through the weighting process.

The  $M$  decision statistics are weighted to form an overall decision statistics.

The outputs of the  $M$  correlates are denoted as  $Z_1, Z_2, \dots$  and  $Z_M$ . They are weighted by  $\alpha_1, \alpha_2, \dots$  and  $\alpha_M$ , respectively.

The weighting coefficients are based on the power or the SNR (Signal-to Noise Ratio) from each correlate output.

**If the power or SNR** is small out of a particular correlate, it will be assigned a small weighting factor,  $\alpha$ .

$$Z' = \sum_{m=1}^M \alpha_m Z_m$$

The weighting coefficients,  $\alpha_m$  are normalized to the output signal power of the correlate in such a way that the coefficients sum to unity.

$$\alpha_m = \frac{Z_m^2}{\sum_{m=1}^M Z_m^2}$$

Due to Multiple Access Interference (MAI), RAKE fingers with strong multipath amplitudes will not necessarily provide strong output after correlation.

Choosing weighting coefficients based on the actual outputs of the correlate yields better RAKE performance.

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