4.4 BESSEL'S DIFFERENTIAL EQUATION AND BESSEL FUNCTION & TM AND TE WAVES IN CIRCULAR WAVE GUIDES:

A circular waveguide is a hollow metallic tube with circular cross section for propagating the electromagnetic waves by continuous reflections from the surfaces or walls of the guide.

The circular waveguides are avoided because of the following reasons:

a) The frequency difference between the lowest frequency on the dominant mode and the next mode is smaller than in a rectangular waveguide, with b/a= 0.5

b) The circular symmetry of the waveguide may reflect on the possibility of the wave not maintaining its polarization throughout the length of the guide.

c) For the same operating frequency, circular waveguide is bigger in size than a rectangular waveguide.

The possible TM modes in a circular waveguide are: TM01, TM02, TM11, TM12.

The root values for the TM modes are:

- (ha)01 = 2.405 for TM01
- (ha)02 = 5.53 for TM02
- (ha)11 = 3.85 for TM11
- (ha)12 = 7.02 for TM12

The *dominant mode* for a circular waveguide is defined as the lowest order mode having the lowest root value.

The possible TE modes in a circular waveguide are: TE01, TE02, TE11, and TE12. The root values for the TE modes are:

- (ha)01 = 3.85 for TE01
- (ha)02 = 7.02 for TE02
- (ha)11 = 1.841 for TE11

• $(ha)_{12} = 5.53$ for TE_{12}

The dominant mode for TE waves in a circular waveguide is the TE11.v. Because it has the lowest root value of 1.841.

Since the root value of TE_{11} is lower than TM_{01}, TE_{11} is the dominant or the lowest order mode for a circular waveguide.

RECTANGULAR AND CIRCULAR CAVITY RESONATORS:

Resonator is a tuned circuit which resonates at a particular frequency at which the energy stored in the electric field is equal to the energy stored in the magnetic field.

Resonant frequency of microwave resonator is the frequency at which the energy in the resonator attains maximum value. i.e., twice the electric energy or magnetic energy.

At low frequencies up to VHF (300 MHz), the resonator is made up of the reactive elements or the lumped elements like the capacitance and the inductance.

The inductance and the capacitance values are too small as the frequency is increased beyond the VHF range and hence difficult to realize.

Transmission line resonator can be built using distributed elements like sections of coaxial lines. The coaxial lines are either opened or shunted at the end sections thus confining the electromagnetic energy within the section and acts as the resonant circuit having a natural resonant frequency.

At very high frequencies transmission line resonator does not give very high quality factor Q due to skin effect and radiation loss. So, transmission line resonator is not used as microwave resonator.

The performance parameters of microwave resonator are:

(i) Resonant frequency

(ii) Quality factor

(iii) Input impedance

Quality Factor of a Resonator.

• The quality factor Q is a measure of frequency selectivity of the resonator.

• It is defined as Q = 2 x Maximum energy stored / Energy dissipated per cycle = W / P

Where,

a. W is the maximum stored energy

b. P is the average power loss

The methods used for constructing a resonator:

The resonators are built by,

a) Using lumped elements like L and C

b) Using distributed elements like sections of coaxial lines

c) Using rectangular or circular waveguide

there are two types of cavity resonators.

a) Rectangular cavity resonator

b) Circular cavity resonator

Rectangular or circular cavities can be used as microwave resonators because they have natural resonant frequency and behave like a LCR circuit.

Cavity resonator can be represented by a LCR circuit as:

• The electromagnetic energy is stored in the entire volume of the cavity in the form of electric and magnetic fields.

• The presence of electric field gives rise to a capacitance value and the presence of magnetic field gives rise to a inductance value and the finite

Conductivity in the walls gives rise to loss along the walls giving rise to a resistance value.

• Thus the cavity resonator can be represented by an equivalent LCR circuit and have a natural resonant frequency.

* Cavity resonators are formed by placing the perfectly conducting sheets on the rectangular or circular waveguide on the two end sections and hence all the sides are surrounded by the conducting walls thus forming a cavity.

* The electromagnetic energy is confined within this metallic enclosure and they acts as resonant circuits.

4.2 GENERAL WAVE BEHAVIOUR ALONG UNIFORM GUIDING STRUCTURES (or) APPLICATION OD MAXWELL'S EQUATIONS TO THE RECTANGULAR WAVEGUIDE:

In rectangular waveguide, the propagation of energy takes place in the Z-direction, with the length of the guide infinite in the Z-direction.

The field components of electric field and magnetic field are obtained by solving Maxwell's equation and wave equations applying appropriate boundary conditions.

The general equations for field components is determined from Maxwell's curl equations.

$\nabla \times H = j\omega\, \varepsilon E$ …….(1)

$\nabla \times E = -j\omega\, \mu H$ …….(2)

Expanding equation (1),

$$\nabla \times H = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ H & H_y & H \end{vmatrix} = j\omega\, [E_x \hat{x} + E_y \hat{y} + E_z \hat{z}]$$

Equating x, y, z components,

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\, \varepsilon\, E_x \qquad \text{…… (3a)}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\, \varepsilon\, E_y \qquad \text{……… (3b)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\, \varepsilon\, E_z \qquad \text{………. (3c)}$$

Similarly Expanding equation (2),

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\,[H_x \vec{a_x} + H_y \vec{a_y} + H_z \vec{a_z}]$$

Equating x, y, z components ,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad \ldots\ldots (3d)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad \ldots\ldots\ldots (3e)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad \ldots\ldots\ldots. (3f)$$

The wave equations are written as,

For non-conducting in medium

$$\nabla^2 E = -\omega^2 \mu \varepsilon E \quad \ldots\ldots. (4)$$

$$\nabla^2 H = -\omega^2 \mu \varepsilon H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(5)$$

It can be written as,

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2 \mu \varepsilon H \quad \ldots\ldots (6)$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \varepsilon E \quad \ldots\ldots (7)$$

, $H_y = H^o_y e^{-\gamma z}$ …….

(8) Diff w.r.to ‘z’

$$\frac{\partial H_y}{\partial z} = H^o_y e^{-\gamma z}(-\gamma)$$

$$\frac{\partial H_y}{\partial z} = -\gamma H^o_y e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma H_y \quad \ldots\ldots.. (9)$$

$$\frac{\partial H_x}{\partial z} = -\gamma H_x \quad \ldots\ldots\ldots. (10)$$

And also let,

$E_y = E^o_y e^{-\gamma z}$ …….(11)

Diff w.r.to ‘z’

$$\frac{\partial E_y}{\partial z} = E_y^o e^{-\gamma z}(-\gamma)$$

$$\frac{\partial E_y}{\partial z} = -\gamma E_y^o e^{-\gamma z}$$

$$\frac{\partial E_y}{\partial z} = -\gamma E_y \quad \text{........ (12)}$$

$$\frac{\partial E_x}{\partial z} = -\gamma E_x \quad \text{.......... (13)}$$

Sub the equ (9), (10), (12), (13) in equal (3),

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega \varepsilon E_x \quad \text{......... (14a)}$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega \varepsilon E_y$$

$$\gamma H_x + \frac{\partial H_z}{\partial x} = -j\omega \varepsilon E_y \quad \text{........ (14b)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega \varepsilon E_z \quad \text{........ (14c)}$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega \mu H_x \quad \text{.......(14d)}$$

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \quad \text{......... (14e)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega \mu H_z \quad \text{.......... (14f)}$$

The wave equations (6) and (7) can also be written as,

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu \varepsilon H_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z \quad \text{...... (15)}$$

Solving Equations (14a) and (14d),

From (14d),

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega \mu H_x$$

$$H_x = \frac{1}{-j\omega\mu}\left[\frac{\partial E_z}{\partial y} + \gamma E_y\right]$$

Sub the H_x value in equ (14b),

From (14b),

$$\gamma H_x + \frac{\partial H_z}{\partial x} = - j\omega \varepsilon E_y$$

$$\gamma \left(\frac{1}{-j\omega\mu} \left[\frac{\partial E_z}{\partial y} + \gamma E_y\right]\right) + \frac{\partial H_z}{\partial x} = - j\omega \varepsilon E_y$$

$$\frac{\gamma}{-j\omega\mu}\frac{\partial E_z}{\partial y} - \frac{\gamma^2 E_y}{j\omega\mu} + \frac{\partial H_z}{\partial x} = - j\omega \varepsilon E_y$$

$$\frac{\partial H_z}{\partial x} - \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial y} = E_y \left[\frac{\gamma^2}{j\omega\mu} - j\omega\varepsilon\right]$$

Multiply throughout by $j\omega\mu$,

$$j\omega\mu \frac{\partial H_z}{\partial x} - \gamma \frac{\partial E_z}{\partial y} = E_y \left[\gamma^2 + \omega^2 \mu \varepsilon\right]$$

$$\gamma^2 + \omega^2 \mu \varepsilon = h^2$$

$$j\omega\mu \frac{\partial H_z}{\partial x} - \gamma \frac{\partial E_z}{\partial y} = E_y h^2$$

$$E_y = \frac{1}{h^2}\left[j\omega\mu \frac{\partial H_z}{\partial x} - \gamma \frac{\partial E_z}{\partial y}\right]$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} \quad \text{...... (16a)}$$

Similarly,

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \quad \text{......... (16b)}$$

From equal (14a),

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j \quad x$$

$$E_x = \frac{1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial y} + \gamma H\right]$$

Sub the E_x value in equ (14e),

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\gamma \left(\frac{1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial y} + \gamma H_y\right]\right) + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} + \frac{\gamma^2}{j\omega\varepsilon} H_y + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} = H_y \left(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}\right)$$

Multiply throughout by $j\omega\varepsilon$

$$j\omega\varepsilon\frac{\partial E_z}{\partial x} + \gamma\frac{\partial Hz}{\partial y} = -H_y(\gamma^2 + \omega^2\mu\varepsilon)$$

$$j\omega\varepsilon\frac{\partial E_z}{\partial x} + \gamma\frac{\partial Hz}{\partial y} = -H_y h^2$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} \quad \text{....... (16c)}$$

Similarly,

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \quad \text{....... (16d)}$$

The above equations are in terms of E_z and H_z.

For wave propagation either E_z of H_z should exits.

If both E_z and H_z are zero, all the fields within guide will vanish.

Wave propagation within the guide is divided into two sets, TE waves and with $E_z = 0$ and TM waves with $H_z = 0$ shown in Fig 4.2.1.

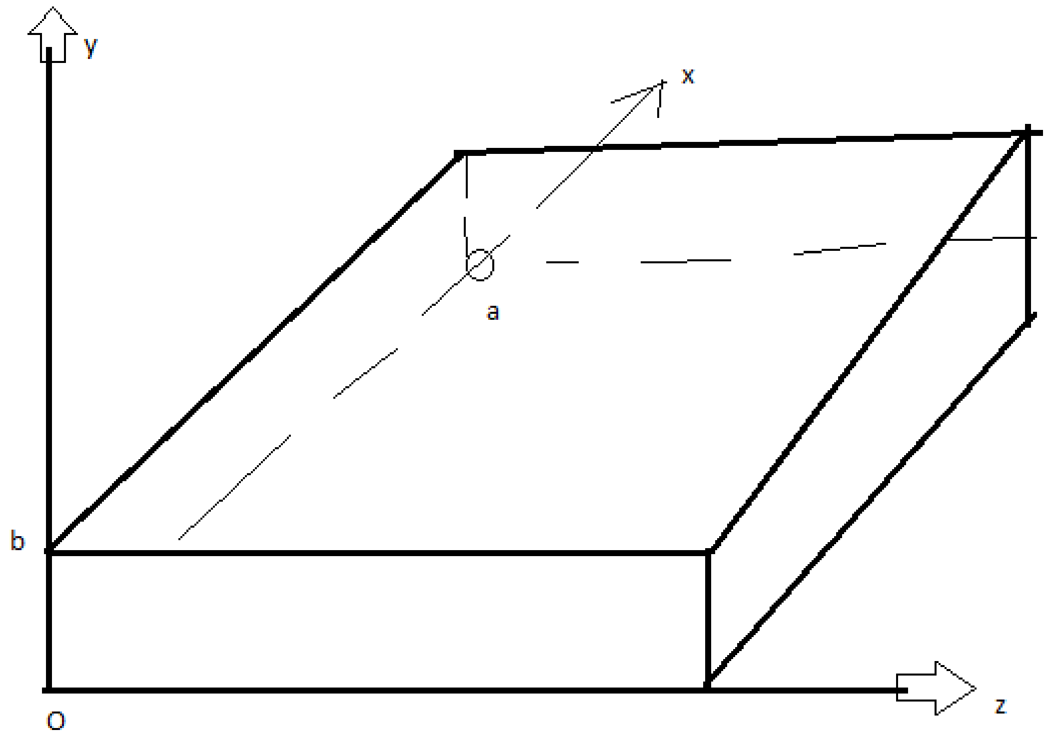

Fig: 4.2.1 Rectangular waveguide

FIELD COMPONENTS OF TRANSVERSE MAGNETIC WAVES IN RECTANGULAR WAVEGUIDE:

For TM waves, $H_z = 0$ and E_z is to be solved from eave equations.

Wave equation for E_z is given by,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon \mathrm{E} \qquad \ldots\ldots (1)$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon \mathrm{E} \qquad \ldots\ldots (2)$$

The wave equation is a partial differential equation that can be solved by the usual technique of assuming a product solution.

$E_z(x, y, z) = E_z^o(x, y)\, e^{-\gamma z}$

Let us assume a solution,

$E_z^o(x, y) = \mathrm{X}(x)\,\mathrm{Y}(y)$ ……

(3) Where X is the function of x alone.

Y is the function y alone.

Sub equal (3) in equal(2)

$Y \frac{d^2X}{dx^2} + X \frac{d^2Y}{dy^2} + \gamma^2 XY = -\omega^2 \mu \varepsilon XY$ …..(4)

$Y \frac{d^2X}{dx^2} + X \frac{d^2Y}{dy^2} + [\gamma^2 + \omega^2 \mu \varepsilon] XY = 0$

$\gamma^2 + \omega^2 \mu \varepsilon = h^2$

$Y \frac{d^2X}{dx^2} + X \frac{d^2Y}{dy^2} + h^2 XY = 0$

Dividing by XY,

$\frac{1}{X} \frac{d^2X}{dx^2} + \frac{1}{Y} \frac{d^2Y}{dy^2} + h^2 = 0$

$\frac{1}{X} \frac{d^2X}{dx^2} + h^2 = -\frac{1}{Y} \frac{d^2Y}{dy^2}$ …… (5)

This expression equates a function of x alone to a function of y alone and the only way for the above equation to be true is to have each of these functions equal to some constant A^2.

$\frac{1}{X} \frac{d^2X}{dx^2} + h^2 = A^2$ ……. (6)

$\frac{1}{X} \frac{d^2X}{dx^2} + h^2 - A^2 = 0$

$B^2 = h^2 - A^2$

$\frac{1}{X} \frac{d^2X}{dx^2} + B^2 = 0$ …… (7)

$-\frac{1}{Y} \frac{d^2Y}{dy^2} = A^2$ ……. (8)

A solution of equation (7) is of the form

$X = C_1 \text{Cos Box} + C_2 \text{Sin Box}$

Where $B^2 = h^2 - A^2$

The solution of equation (8) is of the form

$Y = C_3 \text{ Cos Ay} + C_4 \text{Sin Ay}$ ……

(9) Wit,

$E_z{}^o (x, y) = XY$

$E_z{}^o = (C_1 \text{Cos Box} + C_2 \text{sin Box}) (C_3 \text{Cos Ay} + C_4 \text{sin Ay} E_z{}^o = C_1 \text{Cos Ay}$

$E_z{}^o = C_2 C_3 \sin$ Box Cos Ay + $C_2 C_4 \sin$ Bx sin Ay ……

(12) Sub y=o in equal (8)

$E_z{}^o = C_2 C_3 \sin$ Bx Cos Ay = 0

X and B ≠ 0, either C_2 *or* C_3 has to zero. If C_2 = 0, then the equal (12) become zero.

Since $C_3 = 0$ ……

(13) Sub equ (13) in equ (12)

$E_z{}^o = C_2 C_4 \sin$ Bx sin Ay ……(14)

If x = a, $E_z{}^o$ = 0, sub in (14)

$E_z{}^o = C_2 C_4 \sin$ Ba sin Ay = 0

Since A ≠ 0

sin Ba = 0

Ba = mπ

B = $\frac{m\pi}{a}$ where m = 1,2,3,................................(15)

Sub equ (15) in equ (14)

$E_z{}^o = C_2 \; C_4 \sin (\frac{m\pi}{a}) x \sin Ay$ ……(16)

If y = a, $E_z{}^o$ = 0, sub in (16)

$E_z{}^o = C_2 \; C_4 \sin (\frac{m\pi}{a}) x \sin Ab = 0$

sin Ab = 0

Ab = π

A = $\frac{n\pi}{b}$ where n = 1,2,3,.................................(17)

Sub equ (17) in equ (16)

$E_z{}^o = C_2 \; C_4 \sin (\frac{m\pi}{a}) x \sin (\frac{n\pi}{b}) y$ ……(18)

C = $C_2 C_4$

$E_z{}^o = C \sin (\frac{m\pi}{b}) x \sin$

The general field components with $H_z = 0$ and $\gamma = j\beta$ is given by,

$$E_x = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial x} \quad \text{......(20a)}$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \quad \text{.......(20b)}$$

$$E_y = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial y} \quad \text{......(20c)}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \quad \text{......(20d)}$$

Using equ (19) and equ (20a), 20b, 20c, 20d,

$$E_x^{\,0} = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial x}$$

$$E_x^{\,0} = -\frac{j\beta}{h^2}\, C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$H_x^{\,o} = \frac{j\omega\,\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_x^{\,o} = \frac{j\omega\,\varepsilon}{h^2}\, C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$E_y^{\,0} = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial y}$$

$$E_y^{\,0} = -\frac{j\beta}{h^2}\, C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$H_y^{\,o} = \frac{-j\omega\,\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$H_y^{\,o} = \frac{-j\omega\,\varepsilon}{h^2}\, C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

Wkt,

$A = \frac{n\pi}{b}$ & $B = \frac{m\pi}{a}$

$E_z = E_z^{\,0}\, e^{-\gamma z}$

$E_z = E_z^{\,0}\, e^{-j\beta z}$

From equ $(19)_z = C \sin Bx \sin Ay\, e^{-j\beta} E_x = E_x^{\,0}\, e^{-j\beta z}$

$E_x = -\frac{j\beta}{h^2}\ BC \cos Bx \sin Ay\ e^{-j\beta z}$

$E_y = -\frac{j\beta}{h^2}\ \text{AC} \sin Bx \cos Ay\ e^{-j\beta z}$

$H_x = \frac{j\omega\ \varepsilon}{h^2}\ \text{AC} \sin Bx \cos Ay\ e^{-j\beta z}$

$H_y = \frac{j\omega\varepsilon}{h^2}\ \text{AC} \cos Bx \sin Ay\ e^{-j\beta z}$

CHARACTERISTICS OF TE AND TM WAVES IN RECTANGULAR WAVEGUIDE:

$A^2 + B^2 = h^2$

$\text{A} = \frac{n\pi}{b}$ & $\text{B} = \frac{m\pi}{a}$

$h^2 = \gamma^2 + \omega^2\mu\ \varepsilon$

a = width of guide along x

b = width of guide along y

m , n = integers

i) PROPAGATION CONSTANT AND CUT OFF FREQUENCY:

$h^2 = \gamma^2 + \omega^2\mu\ \varepsilon\ = A^2 + B^2$

$\gamma = \sqrt{h^2 - \omega^2\mu\varepsilon}$

$\gamma = \sqrt{(A^2 + B^2) - \omega^2\mu\ \varepsilon}$

$\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 - \omega^2\ \mu\ \varepsilon}$

This is the equation of propagation constant in a rectangular waveguide for TE and TM waves. Foe small frequencies γ = α , γ is real and there is no wave propagation.

As frequency increases and reaches a particular value f , γ becomes zero.

Then for all values of f greater than $f_c\gamma$ is imaginary, γ = jβ, wave propagation takes place.

At f = f , $\gamma = 0\omega_c{}^2\mu\ \varepsilon = h^2$

$$\omega_c^2 \mu \varepsilon = \left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c^2 = \frac{1}{\mu \varepsilon}\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]$$

(or)

$$\omega_c = \frac{1}{\sqrt{\mu \varepsilon}}\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$$

$$\omega_c = \frac{h}{\sqrt{\mu \varepsilon}}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu \varepsilon}}\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$$

The frequency f_c below which there is no wave propagation (or) thefrequency above which the wave propagation exits is called cut off frequency. The propagation constant can be given by,

$$\gamma = \sqrt{h^2 - \omega^2 \mu \varepsilon}$$

$$\gamma = h\sqrt{1 - \frac{\omega^2 \mu \varepsilon}{h^2}}$$

$$\gamma = h\sqrt{1 - \frac{\omega^2 \mu \varepsilon}{\omega_c^2 \mu \varepsilon}}$$

$$\gamma = h\sqrt{1 - \frac{f^2}{f_c^2}}$$

$$\gamma = \omega\sqrt{\mu \varepsilon}\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

ii) ATTENUATION CONSTANT:

When $\left(\frac{f}{f_c}\right)^2 < 1$ (i.e) $f < f_c$, $\gamma = real$, $\gamma = \alpha$ No wave propagation

$$\gamma = \alpha = h\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

$$\gamma = \alpha = \sqrt{h^2 - \omega^2 \mu \varepsilon}$$

iii) PHASE SHIFT:

$$\gamma = \sqrt{-(\omega^2 \mu \varepsilon - h^2)}$$

$$\gamma = j\beta = j\sqrt{(\omega^2\mu\,\varepsilon - h^2)}$$

$$\gamma = j\sqrt{\omega^2\mu\,\varepsilon - (A^2 + B^2)}$$

$$j\beta = j\sqrt{\omega^2\mu\,\varepsilon - \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$$

$$\beta = \sqrt{\omega^2\mu\,\varepsilon - \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$$

$$\gamma = j\beta$$

$$\gamma = j\sqrt{(\omega^2\mu\,\varepsilon - h^2)}$$

$$\gamma = j\sqrt{(\omega^2\mu\,\varepsilon - \omega_c^2\mu\ )}$$

$$\gamma = j\ \omega\sqrt{\mu\,\varepsilon}\sqrt{1-\left(\frac{\ }{\omega_c}\right)^2}$$

$$\gamma = j\ \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\ }{f_c}\right)^2}$$

iv) CUT- OFF WAVELENGTH:

It is the wavelength at cut – off frequency

$$\lambda_c = \frac{v}{f_c}$$

$$\lambda_c = \frac{velocity}{cut\text{-}off\ frequency}$$

$$\lambda_c = \frac{v}{\frac{1}{2\pi\sqrt{\mu\,\varepsilon}}\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$$

$$\lambda_c = \frac{v\ 2\pi\sqrt{\mu\,\varepsilon}}{\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$$

$$v = \frac{1}{\sqrt{\mu\,\varepsilon}}$$

$$\lambda_c = \frac{2\pi}{\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$$

$$\lambda_c = \frac{2\pi}{\pi\sqrt{\left[\left(\frac{n}{b}\right)^2 + \left(\frac{m}{a}\right)^2\right]}}$$

$$\lambda_c = \frac{2}{\sqrt{[(\frac{n}{b})^2 + (\frac{m}{a})^2]}}$$

v) GUIDED WAVELENGTH (λ_g):

$$\lambda_g = \frac{v}{f} = \frac{2\pi}{\beta}$$

$$\lambda_g = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \left[\left(\frac{n\pi}{b}\right)2 + \left(\frac{m\pi}{a}\right)2\right]}}$$

(Or)

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f}{fc}\right)^2}}$$

vi) PHASE VELOCITY(v_p):

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{\omega}{\sqrt{\omega^2\mu\,\varepsilon - \left[\left(\frac{n\pi}{b}\right)2 + \left(\frac{m\pi}{a}\right)2\right]}}$$

At $\omega = \omega_c$

$$v_p = \frac{\omega_c}{\omega_c\sqrt{\mu\,\varepsilon}\sqrt{1-\left(\frac{f}{fc}\right)^2}}$$

$$v_p = \frac{v}{\sqrt{1-\left(\frac{f}{fc}\right)^2}}$$

vi) GROUP VELOCITY:

$$v_g = \frac{d\omega}{d\beta}$$

$$\beta = \omega\sqrt{\mu\,\varepsilon}\sqrt{1 - 1 - \left(\frac{f}{fc}\right)^2}$$

$$\beta = \omega\sqrt{\mu\,\varepsilon}\sqrt{\omega^2 - {\omega_c}^2}$$

$$V_g = v\sqrt{1-\left(\frac{f}{fc}\right)^2}$$

4.1 GENERAL WAVE BEHAVIOUR ALONG UNIFORM PARALLEL PLANES (or) APPLICATION OF RESTICTIONS TO MAXWELL'S EQUATION (or) WAVES BETWEEN PARALLEL PLANES OF PERFECT CONDUCTORS:

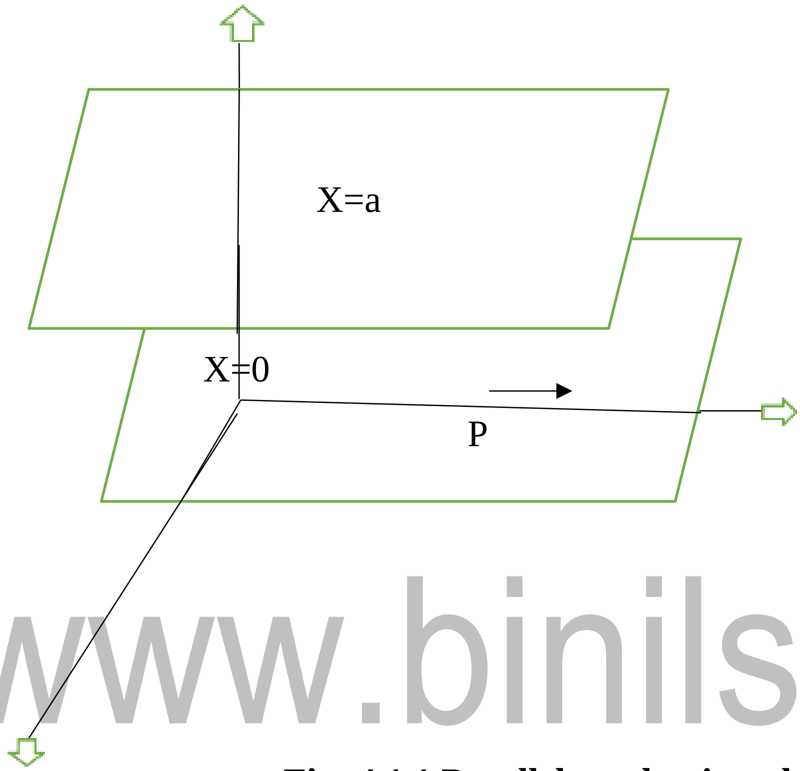

Fig: 4.1.1 Parallel conducting planes

In Fig 4.1.1 consider an electromagnetic wave propagate between a pair of parallel perfectly conducting planes of infinite incident in the plane of Y and Z direction the Maxwell equation for long conducting rectangular region is given by,

$\nabla \times H = j\omega \varepsilon E$ ……. (1)

$\nabla \times E = -j\omega \mu H$ ……. (2)

$\nabla^2 E = \gamma^2 E$ ……. (3)

$\nabla^2 H = \gamma^2 H$ ....(4)

Where,

$\gamma^2 = -\omega^2 \mu \varepsilon$

For non-conducting in medium

$$\nabla^2 E = -\omega^2 \mu \varepsilon E \quad \ldots\ldots. (5)$$

$$\nabla^2 H = -\omega^2 \mu \varepsilon H \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

It can be written as,

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \varepsilon E \quad \ldots\ldots (7)$$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2 \mu \varepsilon H \quad \ldots\ldots (8)$$

From the properties of vector algebra,

$$\nabla \times H = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \vec{a_x}\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] - \vec{a_y}\left[\frac{\partial Hz}{\partial x} - \frac{\partial H_x}{\partial z}\right] + \vec{a_z}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] \quad \ldots..(9)$$

Equal (1) can be written as,

$$\nabla \times H = j\omega\varepsilon[E_x \vec{a_x} + E_y \vec{a_y} + E_z \vec{a_z}]$$

$$\nabla \times H = j\omega\varepsilon E_x \vec{a_x} + j\omega\varepsilon E_y \vec{a_y} + j\omega\varepsilon E_z \vec{a_z} \quad \ldots\ldots. (10)$$

Equate equal (9) and (10),

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \quad \ldots\ldots (11)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \quad \ldots\ldots\ldots (12)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad \ldots\ldots\ldots.($$

$$\nabla \times E = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \vec{a_x}\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] - \vec{a_y}\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] + \vec{a_z}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right] \quad .....(14)$$

Equal (2) can be written as,

$$\nabla \times E = -j\omega\mu\,[H_x \vec{a_x} + H_y \vec{a_y} + H_z \vec{a_z}]$$

$$\nabla \times E = -j\omega\mu\, H_x \vec{a_x} + j\omega\varepsilon\, H_y \vec{a_y} + j\omega\varepsilon\, H_z \vec{a_z} \quad .... (15)$$

Equate equal (14) & (15)

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu E_x \quad ...... (16)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu E_y \quad .... (17)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu E_z \quad .... (18)$$

It is assumed that the propagation is in z direction.

The radiation of component in this z-direction may be expressed in terms of $e^{-\gamma z}$

Where γ is propagation constant,

$\gamma = \alpha + j\beta$

If $\alpha = 0$ waves propagate without attenuation.

If γ = real then $\beta = 0$, there is no wave propagation

Let, $H_y = H^o_y e^{-\gamma z}$

Diff w.r.to 'z'

$$\frac{\partial H_y}{\partial z} = H^o_y\, e^{-\gamma z}(-\gamma)$$

$$\frac{\partial H_y}{\partial z} = -\gamma\, H^o_y\, e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma\, H_y \quad ....(20)$$

$$\frac{\partial H_x}{\partial z} = -\gamma\, H_x \quad ....(21)$$

And also let,

$$E_y = E^o_y\, e^{-\gamma z} \quad ....(22)$$

Diff w.r.to 'z'

$\frac{\partial E_y}{\partial z} = E_y^o \, e^{-\gamma z} \, (-\gamma)$

$\frac{\partial E_y}{\partial z} = -\gamma \, E_y^o \, e^{-\gamma z}$

$\frac{\partial E_y}{\partial z} = -\gamma E_y$ …….. (23)

$\frac{\partial E_x}{\partial z} = -\gamma E_x$ ………. (24)

There is no attenuation in y direction. Hence the derivative of y is zero.

Let E = $E_o \, e^{-\gamma z}$

Diff w. r. to 'z'

$\frac{\partial E}{\partial z} = E_o \; e^{-\gamma z} \, (-\gamma)$

Again diff w. r. to 'z'

$\frac{\partial^2 E}{\partial z^2} = E_o \; e^{-\gamma z} \, (-\gamma) \, (-\gamma)$

$\frac{\partial^2 E}{\partial z^2} = E_o \; e^{-\gamma z} \, \gamma^2$

$\frac{\partial^2 E}{\partial z^2} = \gamma^2 \mathrm{E}$

From equal (7),

$\frac{\partial^2 E}{\partial x^2} + 0 + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \, \varepsilon \, \mathrm{E}$

$\frac{\partial^2 E}{\partial x^2} + \gamma^2 \mathrm{E} = -\omega^2 \mu \varepsilon \, \mathrm{E}$ …… (25)

From equal (8),

$\frac{\partial^2 H}{\partial x^2} + 0 + \frac{\partial^2 H}{\partial z^2} = -\omega^2 \mu \, \varepsilon \, \mathrm{H}$

$\frac{\partial^2 H}{\partial x^2} + \gamma^2 \mathrm{H} = -\omega^2 \mu \varepsilon \, \mathrm{H}$ …… (26)

Sub equal (20) & (21) in (11), (12) & (13)

From equal (11),

$-(-\gamma \, H_y) = \mathrm{j}\omega \, \varepsilon \, E_x$

$\gamma \, H_{y} = \mathrm{j}\omega \, \varepsilon \; E_x$ ………

(27) From equal (12),

$$\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\, \varepsilon\, E_y \qquad \text{........ (28)}$$

From equal (13),

$$\frac{\partial H_y}{\partial x} = j\omega\, \varepsilon\, E_z \qquad \text{........ (29)}$$

Sub equal (23) & (24) in (16), (17) & (18)

From equal (16),

$$-(-\gamma E_y) = -j\omega\, \mu\, H_x$$

$$\gamma E_y = -j\omega\, \mu\, H_x \qquad \text{.......}$$

(30) From equal (17),

$$(-\gamma E_x) - \frac{\partial E_z}{\partial x} = -j\omega\mu\, H_y$$

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\, H_y \qquad \text{......... (31)}$$

From equal (18),

$$\frac{\partial E_y}{\partial x} = -j\omega\, \mu\, H_z \qquad \text{.......... (32)}$$

From equal (30),

$$H_x = \frac{-\gamma E_y}{j\omega\, \mu} \qquad \text{........ (33)}$$

From equal (28),

$$E_y = \frac{-1}{j\omega\, \varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right) \qquad \text{...... (34)}$$

Sub equal (34) in equal (33)

$$H_x = \frac{-\gamma}{j\omega\,\mu}\left(\frac{-1}{j\omega\,\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)\right)$$

$$H_x = \frac{\gamma}{j^2\omega^2\mu\,\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)$$

$[\, j^2 = -1]$

$$H_x = \frac{-\gamma}{\omega^2\mu\,\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{-\gamma^2}{\omega^2\mu\,\varepsilon} H_x - \frac{\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x + \frac{\gamma^2}{\omega^2\mu\,\varepsilon} H_x = \frac{-\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x\left(1+\frac{\gamma^2}{\omega^2\mu\,\varepsilon}\right)=\frac{-\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x=\frac{\frac{-\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}}{\left(1+\frac{\gamma^2}{\omega^2\mu\,\varepsilon}\right)}$$

$$H_x=\frac{\frac{-\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}}{\left(\frac{\omega^2\mu\,\varepsilon+\gamma^2}{\omega^2\mu\,\varepsilon}\right)}$$

$$H_x=\left(\frac{-\gamma}{\omega^2\mu\,\varepsilon+\gamma^2}\right)\frac{\partial H_z}{\partial x}$$

It is given that,

$$\omega^2\mu\,\varepsilon+\gamma^2=h^2$$

$$H_x=\left(\frac{-\gamma}{h^2}\right)\frac{\partial H_z}{\partial x}$$

$$H_x=\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \qquad \text{........ (35)}$$

To find H_y, we need to solve equal (27) & (31)

from equal (27),

$$\gamma\,H_y=\mathrm{j}\omega\,\varepsilon\,E_x$$

$$H_y=\frac{\mathrm{j}\omega\,\varepsilon\,E_x}{\gamma} \qquad \text{....... (36)}$$

From equal (31),

$$\gamma\,E_x+\frac{\partial E_z}{\partial x}=\mathrm{j}\omega\mu\,H_y$$

$$\gamma\,E_x=\mathrm{j}\omega\mu\,H_y-\frac{\partial E_z}{\partial x}$$

$$E_x=\frac{1}{\gamma}\left(\mathrm{j}\omega\mu\,H_y-\frac{\partial E_z}{\partial x}\right) \qquad \text{...... (37)}$$

Sub equal (37) in equ (36),

$$H_y=\frac{\mathrm{j}\omega\varepsilon}{\gamma}\frac{1}{\gamma}\left(\mathrm{j}\omega\mu\,H_y-\frac{\partial E_z}{\partial x}\right)$$

$$H_y=\frac{\mathrm{j}\omega\varepsilon}{\gamma^2}(\mathrm{j}\omega\mu\,H_y)-\frac{\mathrm{j}\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y=\frac{-\omega^2\mu\,\varepsilon H_y}{\gamma^2}-\frac{\mathrm{j}\omega\,\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y + \frac{\omega^2 \mu\ \varepsilon H_y}{\gamma^2} = \frac{-j\omega\ \varepsilon}{\gamma^2}\ \frac{\partial E_z}{\partial x}$$

$$H_y\ (1 + \frac{\omega^2 \mu\ \varepsilon}{\gamma^2}) = \frac{-j\omega \varepsilon}{\gamma^2}\ \frac{\partial E_z}{\partial x}$$

$$H_y = \frac{\frac{-j\omega\ \varepsilon}{\gamma^2} \frac{\partial E_z}{\partial x}}{\frac{\gamma^2 + \omega^2 \mu\ \varepsilon}{\gamma^2}}$$

$$H_y = \frac{-j\omega\ \varepsilon}{\gamma^2 + \omega^2 \mu \varepsilon}\ \frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\ \varepsilon}{h^2} \frac{\partial E_z}{\partial x} \qquad \ldots\ldots (38)$$

To find E_x,

Solve equal (27) & (31),

From equal (27),

$$\gamma\ H_y = j\omega\ \varepsilon\ E_x$$

$$H_y = \frac{j\omega\ \varepsilon\ E_x}{\gamma} \qquad \ldots\ldots (39)$$

From equal (31),

$$\gamma\ E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\ H_y$$

Sub equal (39) in equal (31)

$$\gamma\ E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\ (\frac{j\omega\ \varepsilon\ E_x}{\gamma})$$

$$\gamma\ E_x + \frac{\partial E_z}{\partial x} = \frac{-\omega^2 \mu\ \varepsilon E_x}{\gamma}$$

$$\gamma\ E_x + \frac{\omega^2 \mu\ \varepsilon E_x}{\gamma} = -\ \frac{\partial E_z}{\partial x}$$

$$E_x\ (\gamma + \frac{\omega^2 \mu\ \varepsilon}{\gamma}) = -\ \frac{\partial Ez}{\partial x}$$

$$E_x = \frac{-\frac{\partial E_z}{\partial x}}{\gamma + \frac{\omega^2 \mu\ \varepsilon}{\gamma}}$$

$$E = \gamma - \frac{\partial E_z}{\partial}$$

$$E_x = \frac{-\frac{\partial E_z}{\partial x}}{\frac{h^2}{\gamma}}$$

$$E_x = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right) \qquad \ldots\ldots..(40)$$

To find E_y:

Solve equ (28) & (30),

From equ (30),

$\gamma\, E_y$ = - jω µ H_x

$$H_x = \frac{-\gamma\, E_y}{j\omega\, \mu} \qquad \ldots\ldots.(41)$$

Sub equ (41) in equ (28),

$$-\gamma\, H_x - \frac{\partial H_z}{\partial x} = j\omega\, \varepsilon\, E_y$$

$$-\gamma\left(\frac{-\gamma\, E_y}{j\omega\, \mu}\right) - \frac{\partial H_z}{\partial x} = j\omega\, \varepsilon\, E_y$$

$$\frac{\gamma^2\, E_y}{j\omega\, \mu} - \frac{\partial H_z}{\partial x} = j\omega\, \varepsilon\, E_y$$

$$\frac{\gamma^2\, E_y}{j\omega\, \mu} - j\omega\, \varepsilon\, E_y = \frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{2\, j\omega}{\mu} - j\omega\right] = \frac{\partial Hz}{\partial x}$$

$$E_y\left[\frac{\gamma^2+\omega^2 \mu\varepsilon}{j\omega\, \mu}\right] = \frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{h^2}{j\omega\, \mu}\right] = \frac{\partial H_z}{\partial x}$$

$$E_y = \frac{\partial H_z}{\partial x}\left[\frac{j\omega\mu}{h^2}\right] \qquad \ldots\ldots..(42)$$

The various components of electric and magnetic field strength in equ (35), (38), (40), (42) is expressed interms of E_z & H .

There will be z component either in E or H otherwise all the components should be zero.In general both the E_z & H_z may nor present at the same time the solutions are divided into two cases.

Case (i):

If E_z is present and H_z= 0 , then the wave is called **transverse magnetic wave or TM wave or E wave** because the magnetic field strength is completely transverse to the direction of propagation z.

Case (ii):

If H_z is present and E_z= 0 , then the wave is called **transverse electric wave or TE wave or H wave,** because the electric field strength is completely transverse to the direction of propagation .

Case (iii):

Transverse Magnetic Waves or TEM waves are waves that contain neither E_z or H_z. Both the electric field and magnetic field components are transverse to the direction of propagation, z-direction.

TRANSMISSION OF TRANSVERSE ELECTRIC WAVES BETWEEN PARALLEL PLANES [E_z= 0]

The general field equations of equation (35), (38), (40), (42) for E_z= 0 is given By,

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} = 0$$

$$E_x = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right) = 0$$

$$E_y = \frac{\partial H_z}{\partial x}\left[\frac{j\omega\mu}{h^2}\right]$$

The field components E_x and H_y are zero.

The field components $H_{x,}$ E_y and H_z are too determined.

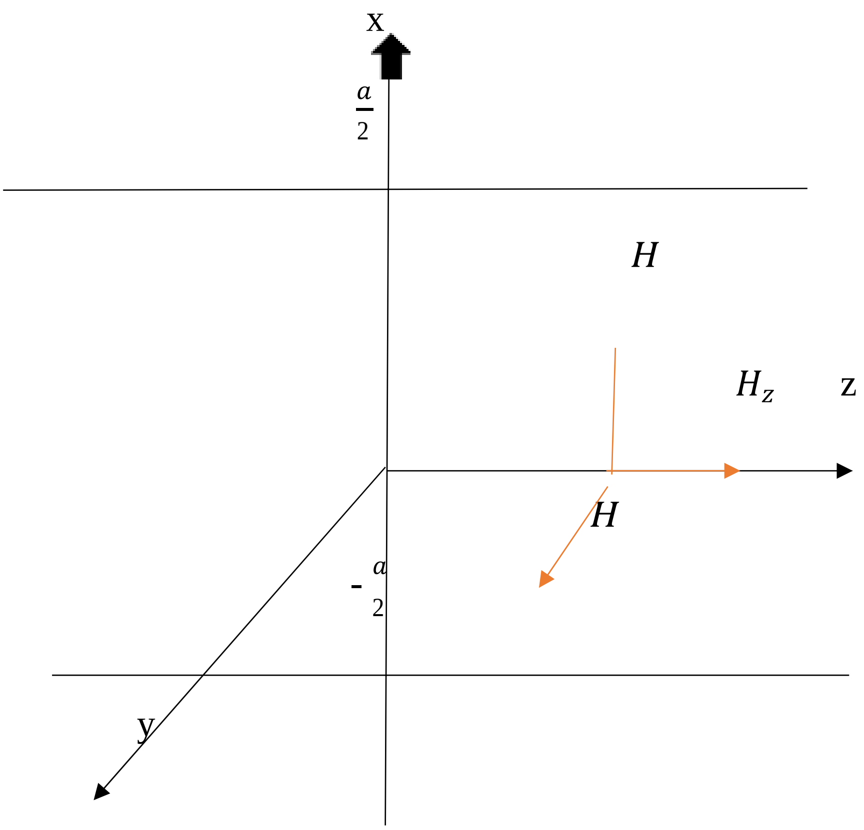

Fig: 4.1.2 Fields in TE waves (H-waves)

In the above Fig 4.1.2, $E_x = E_z = 0$ and the electric field E_y is made wholly transverse to the direction of propagation z.

The magnetic field components H_x and H_z, but H_y=0. The wave is called as

Transverse electric wave or H-wave.

The wave equation for the field component E_y can be written as,

from equal (25),

$$\frac{\partial^2 E}{\partial x^2} + \gamma^2 E = -\omega^2 \mu\, \varepsilon\, E$$

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2\ E_y = \varpi^2 \mu\, \varepsilon\, E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2\ E_y + \omega^2 \mu\, \varepsilon\, E_y = 0$$

$$\frac{\partial^2 E_y}{\partial x^2} + (\gamma^2 + \omega^2\ \mu\, \varepsilon)\, E_y = 0$$

$$\omega^2 \mu\, \varepsilon + \gamma^2 = h^2$$

$$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0 \qquad \ldots\ldots. (1)$$

Let $E_y = E_{yo} e^{-\gamma z}$

Equal (1) is a second order differential equation and the solution of this equation is given by,

$E_{yo} = C_1 \sin hex + C_2 \text{Cos hex}$ …….

(2) Where C_1 and C_2 are arbitrary constants.

If E_y is expressed in time and direction $E_y = E_{yo}\, e^{-\gamma z}$, then solution becomes

$E_y = [C_1 \text{Sinhex} + C_2 \text{Cash}] e^{-\gamma z}$ …… (3)

The tangential component of E is zero at the surface of the conductors for all values of Z.

I. $E_y = 0$ at x = 0

ii. $E_y = 0$ at x = a

These are the boundary conditions to be applied.

Applying the boundary conditions E_y = 0 at x = 0 in equal

(3) 0 = [C_1 sin h (0) + C_2 Cash (0)] $e^{-\gamma z} C_2 = 0$ …… (4)

Sub equal (4) in equal (3),

$E_y = C_1 \sin hex\, e^{-\gamma z}$ …… (5)

Applying the boundary conditions $E_y = 0$ at x = a in equal

(5) $0 = C_1 \sin ha\, e^{-\gamma z}$

Sin ha = 0

Ha = $\sin^{-1} 0$

Ha = mπ

h = $\frac{m\pi}{a}$ where m = 1, 2, 3 …….

Sub 'h' value in equal (5),

$E_y = C_1 \text{ Sin}(\frac{M\pi}{a})x\, e^{-\gamma z}$ …… (7)

Sub E_y in equal (42),

$$E_y = \frac{\partial H_z}{\partial x}\left[\frac{j\omega\mu}{h^2}\right]$$

$$\frac{\partial H_z}{\partial x} = E_y \cdot \frac{h^2}{j\omega\mu}$$

$$H_z = \int E_y \cdot \frac{h^2}{j\omega\mu} .dx$$

$$H_z = \int E_y \cdot \frac{\left(\frac{m\pi}{a}\right)^2}{j\omega\mu} .dx$$

$$H_z = \left(\frac{m\pi}{a}\right)^2 \cdot \frac{1}{j\omega\mu} \int E_y .dx$$

$$H_z = \left(\frac{m\pi}{a}\right)^2 \cdot \frac{1}{j\omega\mu} \int C_1 \sin\left(\frac{m\pi}{a}\right) x\, e^{-\gamma z} .dx$$

$$\int \sin ax = \frac{-\cos ax}{a}$$

$$H_z = \left(\frac{m\pi}{a}\right)^2 \cdot \frac{-1}{j\omega\mu} \cdot C_1 \frac{\cos\left(\frac{m\pi}{a}\right)x}{\left(\frac{m\pi}{a}\right)} \cdot e^{-\gamma z}$$

$$H_z = \frac{-1}{j\omega\mu}\left(\frac{m\pi}{a}\right) C_1 \cos\left(\frac{m\pi}{a}\right) x\, e^{-\gamma z} \quad \text{......(8)}$$

Sub equ (8) in equ (35).

Cos ax= (-sin ax) a

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial}{\partial x}\left(\frac{-1}{j\omega\mu}\left(\frac{m\pi}{a}\right) C_1 cos\left(\frac{m\pi}{a}\right) x\, e^{-\gamma z}\right)$$

$$H_x = \frac{-\gamma}{\left(\frac{m\pi}{a}\right)^2}\frac{-1}{j\omega\mu}\left(\frac{m\pi}{a}\right) C_1\left(-sin\left(\frac{m\pi}{a}\right)x\right).\frac{m\pi}{a}e^{-\gamma z}$$

$$H_x = \frac{-\gamma}{j\omega\mu}C_1 \; sin\left(\frac{m\pi}{a}\right)xe^{-\gamma z}$$

Each value of m specifies a particular field of configuration or mode and is designated as TE_{mo} mode.

The second subscript refers to another factor which varies with y, which is found in rectangular waveguides.

The smallest value of m=1, because m=0 makes all fields identically zero.

Therefore lowest order mode is TE_{10}. This is also called as the dominant mode in TE waves.

The propagation constant γ = α +jβ. If the wave propagates without attenuation , α = 0 then γ = jβ.

Sub γ = jβ in equation (7), (8), (9),

$$E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x\, e^{-j\beta z}$$

$$H_z = \frac{-1}{j\omega\mu}\left(\frac{m\pi}{a}\right) C_1 cos\left(\frac{m\pi}{a}\right) x\, e^{-j\beta z}$$

$$H_x = \frac{-j\beta}{j\omega\mu}C_1\; sin\left(\frac{m\pi}{a}\right)xe^{-j\beta z}$$

$$H_x = \frac{-\beta}{\omega\mu}C_1\; sin\left(\frac{m\pi}{a}\right)xe^{-j\beta z}$$

The above equations represent the field strength of TE waves between parallel conducting planes.

TRANSMISSION OF TRANSVERSE ELECTROMAGNETIC WAVE BETWEEN PARALLEL PLANES (TEM WAVES)

Consider the electric field is totally along the x-axis (i.e., $E_x = E_y = 0$) and the magnetic field along the y-axis. (i.e., $H_x = H_y = 0$) shown in Fig 4.1.3.

Both the electric and magnetic field components are transverse to the direction propagation on z, and the wave is said **transverse electromagnetic wave or principal wave.**

TEM wave is a **special case of transverse magnetic wave** in which the electric field E_z along the direction of propagation is zero.

The condition on E_z is obtained if m is made zero in TE waves.

TEM is also called as **Principal wave.**

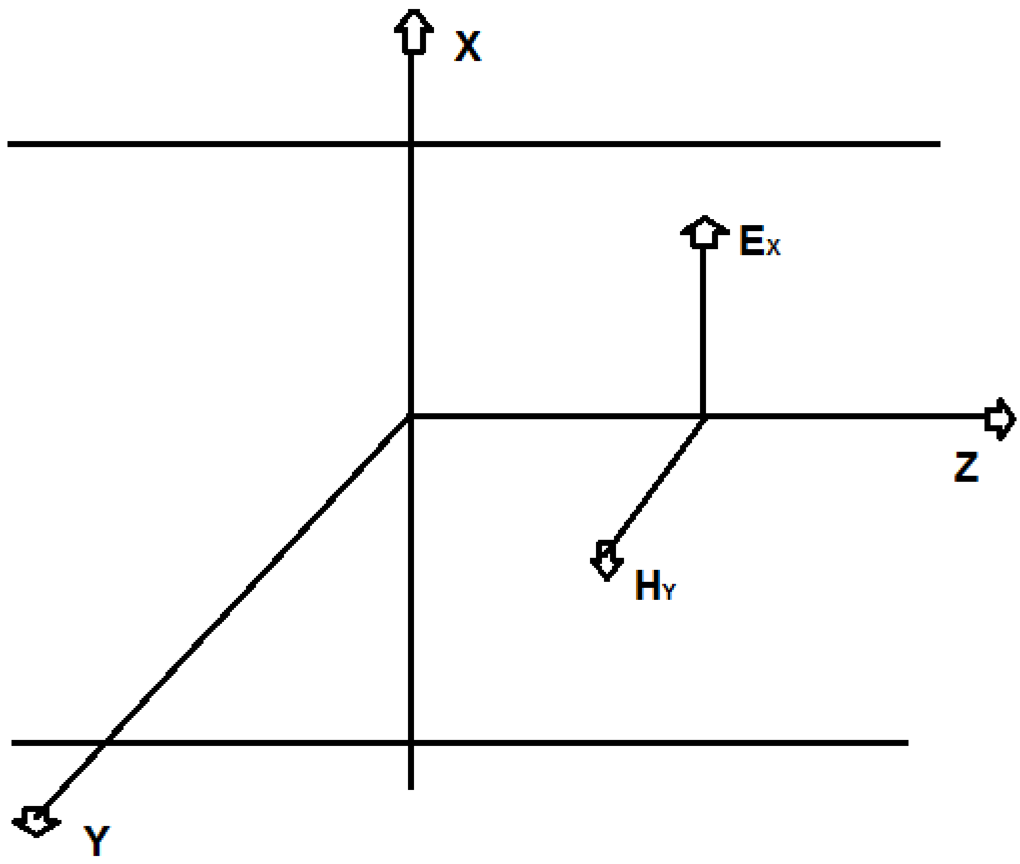

Fig: 4.1.3 Transverse Electromagnetic field vectors

Accordingly the TEM wave becomes a TM waves with m=0, the field equations of TM waves from equation are:

$H_y = C_4 \text{ Cos } (\frac{m\pi}{a}) \, x \, e^{-j\beta z}$

$E_x = \frac{\beta}{\omega\varepsilon} C_4 \text{ Cos } (\frac{m\pi}{a}) \, x \, e^{-j\beta z}$

$E_y = \frac{jm\pi}{\omega\varepsilon a} C_4 \text{ Cos } \frac{m\pi}{}) \, x e^{-j\beta z}$ (Putting m=0 in the above equations of TM waves, **the field equations of TEM waves are obtained**

$H_y = C_4 x e^{-j\beta z}$ ……(1)

$E_x = \frac{\beta}{\omega\varepsilon} C_4 \, e^{-j\beta z}$ …… (3)

$E_y = 0$ …… (4)

These fields are not only transverse, but they are constant in amplitude across a cross section normal to the direction of propagation.

Characteristics of TEM waves:

For m = 0 and dielectric is air.

i. Propagation Constant

$\gamma = \sqrt{(\frac{m\pi}{a})^2 - \omega^2 \mu_o \varepsilon_o}$

$\gamma = \sqrt{-\omega^2 \mu_o \varepsilon_o}$

$\gamma = j\,\omega\sqrt{\mu_o \varepsilon_o}$

$\gamma = \alpha + j\beta$

$\gamma = j\,\omega\sqrt{\mu_o \varepsilon_o}$ …… (4)

Equating real and imaginary parts,

$\alpha = 0$

$\beta = \omega\sqrt{\mu_o \varepsilon_o}$ …..(5)

ii. Guided Wavelength

$\lambda_g = \frac{2\pi}{\beta}$

$\lambda_g = \frac{2\pi}{\omega\sqrt{\mu_o \varepsilon_o}}$

$\omega = 2\pi f$

$v_o = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$

$\lambda_g = \frac{2\pi v_o}{2\pi f} = \lambda$ = Wavelength of free space …..(6)

iii. Velocity of Propagation

$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_o \varepsilon_o}} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = C$ (7)

Velocity of TEM is independent of frequency and has a familiar free space value, C = 3x10^8 m/s.

iv. From equal (7), cut off frequency is given by,

$f_c = \frac{m}{2a\sqrt{\mu_o \varepsilon_o}}$

For m = 0

$f_c = 0$ …….(8)

Cut off frequency of the TEM waves is zero, indicating all the frequencies down to zero can propagate along the guide.

v. The ratio of the amplitudes of E to H between planes is defined as **Characteristic wave impedance** given by

$\frac{E_x}{H_y} = \frac{\beta}{\omega\varepsilon} = \frac{\omega\sqrt{\mu_o \varepsilon_o}}{\omega\varepsilon_o} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = \eta$ ……. (9)

Ƞ is the intrinsic impedance of the dielectric medium existing between the planes.

$E_x = \eta H_y$ ……… (8)

vi. The total power propagating in the Z-direction is calculated using Pointing theorem

$\gamma = \iint E \times H \, dx \, dy$

$P = \int_{x=-\frac{a}{2}}^{x=+\frac{a}{2}} \int_{y=0}^{1} \left(\frac{E_x}{\sqrt{2}}\right)\left(\frac{H_y}{\sqrt{2}}\right) dx\, dy$ for 1 meter width along y direction

$$P=\frac{1}{2}E_x H_y\,[x]_{-\frac{a}{2}}^{+\frac{a}{2}}\,[y]_0^1 \qquad E_x=\eta H_y$$

$$P=\frac{1}{2}E_x H_y\,a$$

$$P=\frac{1}{2}(\eta H_y)\,H_y\,a$$

$P = \frac{1}{2}\eta\, a\, H_y^2$ watts / meter of width. …… (9)

CHARACTERISTICS OF TE AND TM WAVES:

The characteristics of TE and TM waves cab be studied by analyzing propagation constant γ.

$h^2 = \omega^2\mu\,\varepsilon + \gamma^2$

$\gamma^2 = h^2 - \omega^2\mu\,\varepsilon$

$\gamma = \sqrt{h^2 - \omega^2\mu\,\varepsilon}$ …… (1)

i. Cut-off frequency (f_c):

Sub h = $\frac{m\pi}{a}$ in equal (1),

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon} = \alpha + j\beta \;\ldots\ldots\ldots\ldots (2)$$

When $\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2$. (i.e) at higher frequencies, γ becomes imaginary equal to jβ. Phase change for the wave occurs and hence the wave propagates at lower frequencies, $\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2$ so that , γ becomes real equal to the attenuation constant 'α' and 'β' is zero. The wave completely attenuated and no propagation takes place.

As the frequency is decreased a critical frequency ω_c is reached when $\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2$ The frequency at which wave motion ceases or the frequency above which wave motion exits is called the cutoff frequency of the guide.

The system acts as a high pass filter with a cutoff frequency 'f' and is defined as the frequency at which the attenuation condition changes to the propagation condition.

At $f = f_c$, $\gamma = 0$,

From equal (2),

$\omega_c = 2\pi f_c$

$$\sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \mu \varepsilon} = 0$$

$$\omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c^2 = \frac{1}{\mu \varepsilon}\left(\frac{m\pi}{a}\right)^2$$

$$\omega_c = \sqrt{\frac{1}{\mu \varepsilon}}\left(\frac{m\pi}{a}\right)$$

$$f_c = \frac{1}{2\pi\sqrt{\mu \varepsilon}} \cdot \frac{m\pi}{a}$$

$$f_c = \frac{m}{2a\sqrt{\mu \varepsilon}}$$

Cutoff frequency is defined as the frequency at which propagation constant changes from being real to imaginary.

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \varepsilon}$$

$$\gamma = \frac{m\pi}{a}\sqrt{1 - \frac{\omega^2 \mu \varepsilon}{\left(\frac{m\pi}{a}\right)^2}}$$

$$\omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\gamma = \frac{m\pi}{a}\sqrt{1 - \frac{\omega^2 \mu \varepsilon}{\omega_c^2 \mu \varepsilon}}$$

$$\omega_c = 2\pi f_c$$

$$\omega = 2\pi f$$

$$\gamma = \frac{m\pi}{a}\sqrt{1 - \frac{f^2}{f_c^2}} \qquad \ldots\ldots\ldots(4)$$

$$\frac{m\pi}{a} = \omega_c\sqrt{\mu\,\varepsilon}$$

$$\gamma = \omega_c\sqrt{\mu\,\varepsilon}\sqrt{1 - \frac{f^2}{f_c^2}} \qquad \ldots\ldots(5)$$

For frequencies below cutoff where $f < f_c$ and γ is real, $\gamma = \alpha$, $\beta = 0$.

At frequencies above cutoff, $f > f_c$, γ is imaginary and $\alpha = 0$. Thus propagation will occur and

$\gamma = j\beta$

From equal (4),

$$j\beta = \frac{m\pi}{a}\sqrt{1 - \frac{f^2}{f_c^2}}$$

$$j\beta = \frac{m\pi}{a}\sqrt{-1\left(\frac{f^2}{f_c^2} - 1\right)}$$

$$j\beta = j\frac{m\pi}{a}\sqrt{\left(\frac{f^2}{f_c^2} - 1\right)}$$

$$j\beta = j\omega_c\sqrt{\mu\,\varepsilon}\sqrt{\left(\frac{f^2}{f_c^2} - 1\right)}$$

$$\beta = \omega_c\sqrt{\mu\,\varepsilon}\sqrt{\left(\frac{f^2}{f_c^2} - 1\right)}$$

$$\beta = \omega_c\sqrt{\mu\,\varepsilon}\sqrt{\left(\frac{f^2 - f_c^2}{f_c^2}\right)}$$

$$\beta = \frac{\omega_c\sqrt{\mu\,\varepsilon}}{f_c}\sqrt{(f^2 - f_c^2)}$$

$$\beta = \frac{2\pi f_c\sqrt{\mu\,\varepsilon}}{f_c}\sqrt{(f^2 - f_c^2)}$$

$\beta = 2\pi\sqrt{\mu\varepsilon}\sqrt{(f^2 - f_c^2)}$ ……. (7)

(Or)

$$\gamma = j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\,\varepsilon}$$

$$j\beta = \sqrt{-\left[\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2\right]}$$

$$j\beta = j\sqrt{\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\beta = \sqrt{\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

From equal (3),

Cut off frequency $f_c = \frac{m}{2a\sqrt{\mu\,\varepsilon}}$

$$f_c = \frac{m\,v}{2a.}$$

$$v = \frac{1}{\sqrt{\mu\,\varepsilon}}$$

v is the velocity of propagation = 3 x 10^8 *m/s*

ii. Wavelength (λ) / Guided Wavelength (λ_g):

The distance travelled by a wave to undergo a phase shift of 2π radians is called wavelength. It is the wavelength in the direction of propagation and hence also called as guided wavelength.

$$\lambda = \frac{2\pi}{\beta} = \lambda_g$$

$$\lambda_g = \frac{2\pi}{\sqrt{\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2}} \quad \text{……(9)}$$

iii. Cut off Wavelength (λ_c):

Wavelength at cutoff frequency is called as cutoff wavelength.

$$\lambda_c = \frac{v}{f_c}$$

$$\lambda_c = \frac{v}{\frac{M v}{2a.}}$$

$$\lambda_c = \frac{2a}{m} \qquad \ldots\ldots (10)$$

From equal (9),

$$\lambda_g = \frac{2\pi}{\sqrt{\omega^2 \mu\, \varepsilon - (\frac{M\pi}{a})^2}}, \qquad \text{at cutoff } (\frac{M\pi}{a})^2 = \omega_c^{\ 2} \mu\, \varepsilon$$

$$\lambda_g = \frac{2\pi}{\sqrt{\omega^2 \mu\, \varepsilon - \omega_c^2 \mu\, \varepsilon}}$$

$$\lambda_g = \frac{2\pi}{\omega\sqrt{\mu\, \varepsilon}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

$$\omega_c = 2\pi f_c$$

$$\omega = 2\pi f$$

$$\lambda_g = \frac{2\pi}{2\pi f \sqrt{\mu\, \varepsilon}\sqrt{1 - \frac{2\pi f_c^2}{2\pi f^2}}}$$

$$\lambda_g = \frac{1}{f\sqrt{\mu \varepsilon}\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$v = \frac{1}{\sqrt{\mu\, \varepsilon}}$$

$$\lambda_g = \frac{v}{f\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$\lambda = \frac{v}{f}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$f = \frac{v}{\lambda}$$

$$f = \frac{v}{}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\frac{v}{\lambda_c}}{\frac{v}{\lambda}}\right)^2}}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Squaring on both sides,

$$\lambda_g{}^2 = \frac{\lambda^2}{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$$

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \frac{\lambda^2}{\lambda_g{}^2}$$

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$1 = \left(\frac{\lambda}{\lambda_g}\right)^2 + \left(\frac{\lambda}{\lambda_c}\right)^2$$

$$1 = \lambda^2\left[\frac{1}{\lambda_g{}^2} + \frac{1}{\lambda_c{}^2}\right]$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g{}^2} + \frac{1}{\lambda_c{}^2}$$

λ –Free space wavelength

λ_c – Cutoff wavelength

λ_g – Guide wavelength