1.1 GENERAL THEORY OF TRANSMISSION LINES:

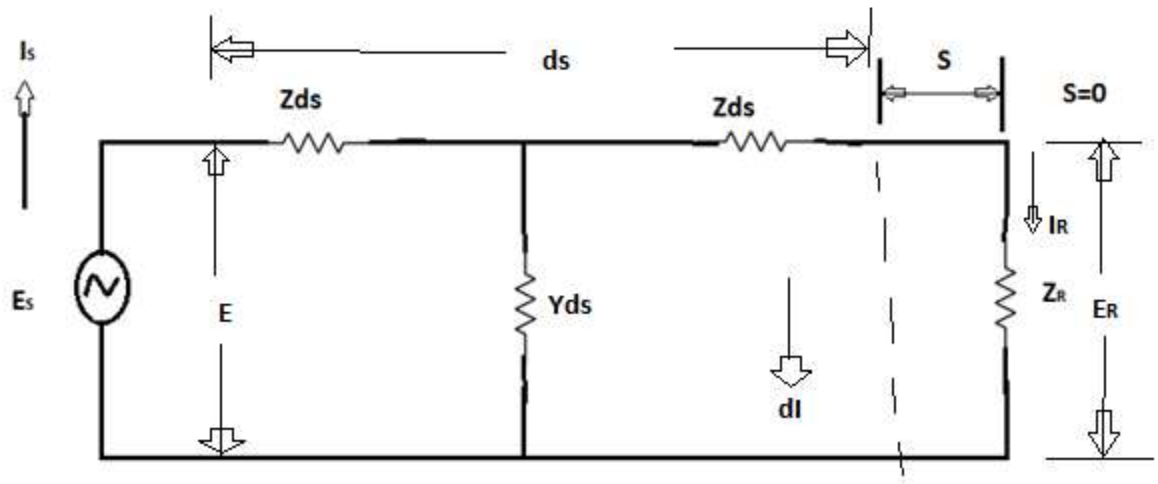

Fig: 1.1.1 a long line, with the elements of one of the infinitesimal sections shown.

From the Fig 1.1.1 consider an infinite length transmission line also an small element ds which located at a distance of s from the receiving end ( ). The sending end voltage and current are E_S and I_S. The receiving end voltage and current are E_R and I_R. The current and voltage at any point is E and I

The voltage in the length ds is DE.

DE = I Zeds

$\frac{dE}{ds}$= IZ ……… (1)

The current in the length ds is

did = E Y ds

$\frac{dI}{ds}$= EY...(2)

Diff equal (1) and (2)

From equal (1)……$\frac{d^2E}{ds^2} = Z\frac{dI}{ds}$

From equal (2)……$\frac{d^2I}{ds^2} = Y\frac{dE}{ds}$

Sub the values of $\frac{dI}{ds}$ and $\frac{dE}{ds}$ in above equ

$\frac{d^2E}{ds^2} = Z\,EY$

$\frac{d^2I}{ds^2} = Y\,IZ$

$\frac{d^2E}{ds^2} - Z\,EY = 0$ (3)

$\frac{d^2I}{ds^2} - Y\,IZ = 0$ (4)

These are the differential equations of transmission line.

From equal (3),

$(\frac{d^2}{ds^2} - Z\,Y)\,E = 0$

$(m^2 - Z\,Y)\,E = 0$

$(m^2 - Z\,Y) = 0$

$m^2 = Z\,Y$

$m = \pm\sqrt{ZY}$

The general solution can be written as $E = A\,e^{\sqrt{ZY}s} + B\,e^{-\sqrt{ZY}s}$ ………

(5) From equal (4),

$(\frac{d^2}{ds^2} - Z\,Y)\,I = 0$

$(m^2 - Z\,Y)\,I = 0$

$(m^2 - Z\,Y) = 0$

$m^2 = Z\,Y$

$m = \pm\sqrt{ZY}$

The general solution can be written as

$I = C\,e^{\sqrt{ZY}s} + D\,e^{-\sqrt{ZY}s}$ ……… (6)

where A, B, C, D are arbitrary constants

We need to find the values of A, B, C, D

By using the condition **s=0**

$I = I_R$

$E = E_R$

Use this condition in equal (5) and (6) From equal

(5), $E_R = A\, e^{\sqrt{(0)}} + B\, e^{-\sqrt{ZY}(0)}$

$E_R = A + B$

From equal (6), IR = C + D ……. (7)

Diff equal (5) and (6) with .r .top's'

From equal (5)

$\frac{dE}{ds} = A\sqrt{ZY}e^{\sqrt{ZY}s} + B(-\sqrt{ZY})\, e^{-\sqrt{ZY}s}$

[Using this formula, $e^{Fax} = a\, e^{Fax}$]

$\frac{dE}{ds} = A\sqrt{ZY}e^{\sqrt{ZY}s} - B\sqrt{ZY}e^{-\sqrt{ZY}s}$

$\frac{dE}{ds} = IZ$, sub in above equal

$IZ = A\sqrt{ZY}e^{\sqrt{ZY}s} - B\sqrt{ZY}e^{-\sqrt{ZY}s}$

$I = \frac{A\sqrt{ZY}\, e^{\sqrt{ZY}s} - B\sqrt{ZY}\, e^{-\sqrt{ZY}s}}{Z}$

$I = A\sqrt{\frac{Y}{Z}}e^{\sqrt{ZY}s} - B\sqrt{\frac{Y}{Z}}\, e^{-\sqrt{ZY}s}$ ……………………………(8)

$\left(\frac{\sqrt{ZY}}{Z} = \sqrt{\frac{Y}{Z}}\right)$

From equal (6),

$\frac{dI}{ds} = C\sqrt{ZY}e^{\sqrt{ZY}s} + D\sqrt{ZY}e^{-\sqrt{ZY}s}$

$E\, Y = C\sqrt{ZY}e^{\sqrt{ZY}s} + D\sqrt{ZY}e^{-\sqrt{ZY}s}$

$\left(\frac{dI}{ds} = E\, Y\right)$

$$E = \frac{C\sqrt{ZY}\,e^{\sqrt{ZY}s} - D\sqrt{ZY}\,e^{-\sqrt{ZY}s}}{Y}$$

$$E = C\sqrt{\frac{Z}{Y}}\,e^{\sqrt{ZY}s} - D\sqrt{\frac{Z}{Y}}\,e^{-\sqrt{ZY}s} \text{ ....(9)}$$

$$\left(\frac{\sqrt{ZY}}{Y} = \sqrt{\frac{Z}{Y}}\right)$$

At, s=0, E = E_R and I = I_R

Sub this values in (8) and (9)

from equal (8),

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \text{ .... (10)}$$

From equal (9),

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \text{ .... (11)}$$

From equal (7),

$$B = E_R - A$$

To find the value of a sub B value in equal (10)

$$I_R = A\sqrt{\frac{Y}{Z}} - (E_R - A)\sqrt{\frac{Y}{Z}}$$

$$I_R = A\sqrt{\frac{Y}{Z}} - E_R\sqrt{\frac{Y}{Z}} + A\sqrt{\frac{Y}{Z}}$$

$$I_R + E_R\sqrt{\frac{Y}{Z}} = 2A\sqrt{\frac{Y}{Z}}$$

$$2A\sqrt{\frac{Y}{Z}} = I_R + E_R\sqrt{\frac{Y}{Z}}$$

$$A = \frac{I_R + E_R\sqrt{\frac{Y}{Z}}}{2\sqrt{\frac{Y}{Z}}}$$

$A = \frac{I_R}{2\sqrt{\frac{Y}{Z}}} + \frac{E_R \sqrt{\frac{Y}{Z}}}{2\sqrt{\frac{Y}{Z}}}$

$A = \frac{I_R \sqrt{\frac{Z}{Y}}}{2} + \frac{E_R}{2}$

$(\sqrt{\frac{Z}{Y}} = Z_o)$

$A = \frac{I_R Z_o}{2} + \frac{E_R}{2}$

$A = I_R \frac{Z_o}{2} + \frac{E_R}{2}$

By using this formula, V = IR

E = IZ

$I = \frac{E}{Z}$

$I_R = \frac{ER}{Z_R}$

Sub I_R value in A,

$A = \frac{}{Z_R}\frac{}{2} + \frac{E_R}{2}$

$A = \frac{ER}{2} [1 + \frac{Z_o}{Z_R}]$ .... (12)

Sub equal (12) in (7),

$E_R = \frac{ER}{2} [1 + \frac{Z_o}{Z_R}] + B$

$B = E_R - \frac{E_R}{2}[1 + \frac{Z_o}{Z_R}]$

$B = E_R - \frac{E_R}{2} - \frac{Z_o}{Z_R} \cdot \frac{E_R}{2}$

$B = \frac{E_R}{2} - \frac{Z_o}{Z_R} \cdot \frac{E_R}{2}$

$B = \frac{ER}{2} [1 - \frac{Z_o}{Z_R}]$ .... (13)

From equal (7),

$I_R = C + D$

$d = I_R - C$

Sub the above value in (11),

$E_R = C\sqrt{\frac{Z}{Y}} (I_R - C)\sqrt{\frac{Z}{Y}}$

$E_R = C\sqrt{\frac{Z}{Y}} - I_R\sqrt{\frac{Z}{Y}} + C\sqrt{\frac{Z}{Y}}$

$E_R = 2C\sqrt{\frac{Z}{Y}} - I_R\sqrt{\frac{Z}{Y}}$

$2C\sqrt{\frac{Z}{Y}} = E_R + I_R\sqrt{\frac{Z}{Y}}$

$C = \frac{E_R + I_R\sqrt{\frac{Z}{Y}}}{2\sqrt{\frac{Z}{Y}}}$

$C = \frac{ER}{2\sqrt{\frac{Z}{Y}}} + \frac{I_R\sqrt{\frac{Z}{Y}}}{2\sqrt{\frac{Z}{Y}}}$

$C = \frac{ER}{2\sqrt{\frac{Z}{Y}}} + \frac{I_R}{2}$

$C = \frac{ER}{2Z_o} + \frac{I_R}{2}$

$(E_R = I_R Z_R)$

Sub E_R value in C,

$C = \frac{I_R Z_R}{2Z_o} + \frac{I_R}{2}$

$C = \frac{IR}{2}[1 + \frac{Z_R}{Z_0}]$ .... (14)

$D = I_R - C$

$D = I_R - \frac{IR}{2}[1 + \frac{Z_R}{Z_0}]$

$D = I_R - \frac{IR}{2} - \frac{I_R}{2}\frac{Z_R}{Z_0}$

$$D = \frac{I_R}{2} - \frac{I_R}{2}\frac{Z_R}{Z_0}$$

$$D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right] \qquad \text{............ (15)}$$

Sub the A, B, C, D values in (5) and (6),

from equal (5), $E = A\, e^{\sqrt{ZY}s} + B\, e^{-\sqrt{ZY}s}$

$$E = \frac{E_R}{2}\left[1 + \frac{Z_o}{Z_R}\right] e^{\sqrt{ZY}s} + \frac{E_R}{2}\left[1 - \frac{Z_o}{Z_R}\right] e^{-\sqrt{ZY}s} \text{...................... (16)}$$

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right] e^{\sqrt{ZY}s} + \frac{E_R}{2}\left[\frac{Z_R - Z_o}{Z_R}\right] e^{-\sqrt{ZY}s}$$

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right]\left[e^{\sqrt{ZY}S} + \left[\frac{Z_R - Z_o}{Z_R}\right]\left[\frac{Z_R}{Z_R + Z_o}\right] e^{-\sqrt{ZY}S}\right]$$

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right]\left[e^{\sqrt{ZY}S} + \left[\frac{Z_R - Z_o}{Z_R + Z_o}\right] e^{-\sqrt{ZY}S}\right] \qquad \text{....... (17)}$$

From equal (6), $I = C\, e^{\sqrt{ZY}s} + D e^{-\sqrt{ZY}s}$

$$I = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right] e^{\sqrt{ZY}s} + \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right] e^{-\sqrt{ZY}s} \text{.. (18)}$$

$$I = \frac{I_R}{2}\left[\frac{Z_0 + Z_R}{Z_0}\right] e^{\sqrt{ZY}s} + \frac{I_R}{2}\left[\frac{Z_0 - Z_R}{Z_0}\right] e^{-\sqrt{ZY}s}$$

$$I = \frac{I_R}{2}\left[\frac{Z_0 + Z_R}{Z_0}\right]\left[e^{\sqrt{ZY}S} + \frac{I_R}{2}\left[\frac{Z_0 - Z_R}{Z_0}\right]\left[\frac{Z_R}{Z_0 + Z_R}\right] e^{-\sqrt{ZY}S}\right]$$

$$I = \frac{I_R}{2}\left[\frac{Z_0 + Z_R}{Z_0}\right]\left[e^{\sqrt{ZY}S} - \frac{I_R}{2}\left[\frac{Z_R - Z_0}{Z_R + Z_0}\right] e^{-\sqrt{ZY}S}\right] \text{...(19)}$$

Equal (17) and (19) are the first form of voltage and current.

Equal (16) may be rearranged as

$$E = \frac{E_R}{2}\left[\left[1 + \frac{Z_o}{Z_R}\right] e^{\sqrt{ZY}S} + \left[1 - \frac{Z_o}{Z_R}\right] e^{-\sqrt{ZY}S}\right]$$

$$E = \frac{E_R}{2}\left[e^{\sqrt{ZY}S} + \frac{Z_o}{Z_R} e^{\sqrt{ZY}S} + e^{-\sqrt{ZY}S} - \frac{Z_o}{Z_R} e^{-\sqrt{ZY}S}\right]$$

$$E = \frac{E_R}{2}\left[e^{\sqrt{ZY}S} + e^{-\sqrt{ZY}S} + \frac{Z_o}{Z_R}\left[e^{\sqrt{ZY}S} - e^{-\sqrt{ZY}S}\right]\right]$$

For example,

$$\text{Cash}\,\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

Sin h $\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

Sub in above equal,

$E = \frac{E_R}{2}[2\cosh\sqrt{ZYS} + 2 \cdot \frac{Z_o}{Z_R} \sin\sqrt{ZYS}]$

$E = E_R \text{cash}\sqrt{ZYS} + \frac{E_R Z_o}{Z_R} \sin\sqrt{ZYS}$

$Z_R = \frac{ER}{I_R}$

Sub the Z_R value in above equal ,

$E = E_R \cosh\sqrt{ZYS} + \frac{E_R Z_o}{[\frac{E_R}{I_R}]} \sinh\sqrt{ZYS}$

$E = E_R \cosh\sqrt{ZYS} + \frac{I_R E_R Z_o}{E_R} \sinh\sqrt{ZYS}$

$E = E_R \cosh\sqrt{ZYS} + I_R Z_o \sinh\sqrt{ZYS}$ ....(21)

The same procedure will be followed for the current equ,

equ (18) will be,

$I = I_R \cosh\sqrt{ZYS} + \frac{E_R}{Z_0} \sinh\sqrt{ZYS}$ ....(22)

equ (21) and (22) are the second form of voltage and current at any point on a transmission line.

PHYSICAL SIGNIFICANCE OF TRANSMISSION LINE (or) INFINITE LINE (or) THE TWO STANDARD FORM FOR INPUT IMPEDANCE OF THE TRANSMISSION LINE TERMINATED BY AN IMPEDANCE Z_R From the Fig 1.1.2 the equation for the current and voltage may be written for the sending end current 'I_S' of a line of length 'l' is,

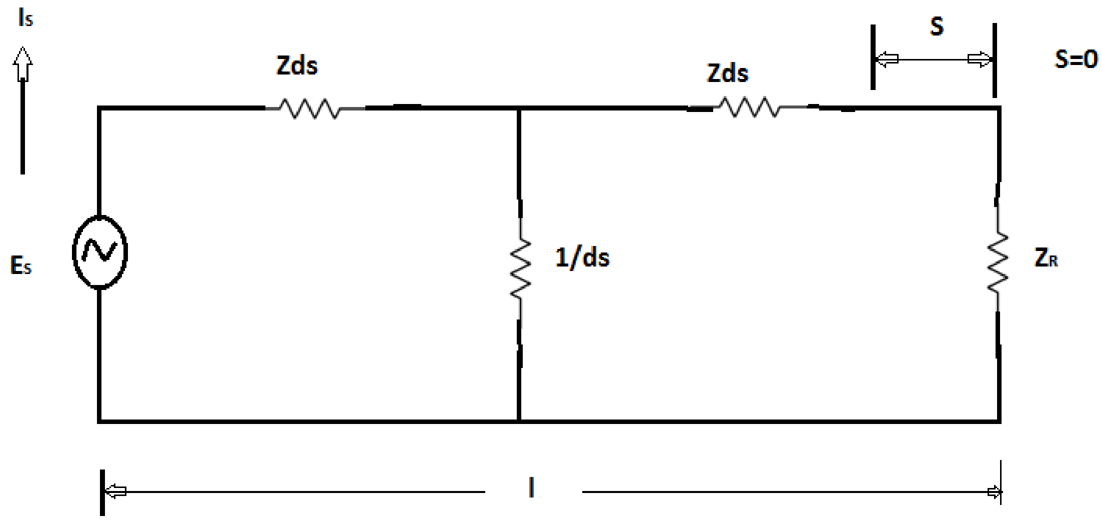

Fig: 1.1.2 A length l taken from an infinite line

The sending current equation is given by,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{E_R}{Z_O}\sinh\sqrt{ZY}.l$ $\quad [E_R = I_R Z_R]$

Sub E_R value in above equ,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{I_R Z_R}{Z_O}\sinh\sqrt{ZY}.l$

$I_S = I_R\,[\cosh\sqrt{ZY}.l + \frac{Z_R}{Z_O}\sinh\sqrt{ZY}.l]$ ....(1)

The sending voltage equation is given by,

$E_S = E_R \cosh\sqrt{ZY}.l + I_R Z_O \sinh\sqrt{ZY}.l$

$[E_R = I_R Z_R]$

$[I_R = \frac{E_R}{Z_R}]$

Sub I_R value in above equ,

$E_S = E_R \cosh\sqrt{ZY}.l + \frac{E_R Z_O}{Z_R}\sinh\sqrt{ZY}.l$

$E_S = E_R\,[\cosh\sqrt{ZY}.l + \frac{\sinh\sqrt{ZY}.l}{Z_R}]$ .... (2)

Since, We know that,

Propagation Constant $\gamma = \sqrt{ZY}$

Characteristic Impedance $Z_O = \sqrt{\frac{Z}{Y}}$

Sub γ value in equ (1) and (2),

From equ (1),

$I_S = I_R\,[\cosh \gamma l + \frac{Z_R}{Z_O} \sinh \gamma l]$ .... (3)

From equ (2),

$E_S = E_R\,[\cosh \gamma l + \frac{Z_O}{Z_R} \sinh \gamma l]$ .... (4)

Input Impedance $Z_S = \frac{E_S}{I_S}$ [E = IZ]

$$Z_S = \frac{E_R\,[\cosh \gamma l + \frac{Z_O}{Z_R}\sinh \gamma l]}{I_R\,[\cosh \gamma l + \frac{Z_R}{Z_O}\sinh \gamma l]}$$

$$Z_S = Z_R\left[\frac{\frac{Z_R \cosh \gamma l + Z_O \sinh \gamma l}{Z_R}}{\frac{Z_O \cosh \gamma l + Z_R \sinh \gamma l}{Z_O}}\right]$$

$E_R = I_R Z_R$

$Z_R = \frac{E_R}{I_R}$

$$Z_S = Z_R\left[\frac{Z_R \cosh \gamma l + Z_O \sinh \gamma}{Z_R} \; X \; \frac{Z_O}{Z_O \cosh \gamma l + Z \quad \sinh \gamma l}\right]$$

$$Z_S = Z_O\left[\frac{Z_R \cosh \gamma l + Z_O \sinh \gamma}{Z_O \cosh \gamma l + Z_R \sinh \gamma l}\right] \qquad \text{.......(5)}$$

This is the first standard form of input impedance of the transmission line.

$\text{Cosh}\theta = \frac{e^{\theta} + e^{-\theta}}{2}$

$\text{Sinh}\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

Sub the above formula in equ (5),

$$Z_S = Z_O \left[\frac{Z_R\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right) + Z_O\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)}{Z_O\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right) + Z_R\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)} \right]$$

$$Z_S = \frac{2Z_O}{2}\left[\frac{Z_R(e^{\gamma l}+e^{-\gamma l}) + Z_O(e^{\gamma l}-e^{-\gamma l})}{(e^{\gamma l}+e^{-\gamma l}) + Z_R(e^{\gamma l}-e^{-\gamma l})}\right]$$

$$Z_S = Z_O \left[\frac{Z_R e^{\gamma l} + Z_R e^{-\gamma l} + Z_O e^{\gamma l} - Z_O e^{-\gamma l}}{Z_O e^{\gamma l} + Z_O e^{-\gamma l} + Z_R e^{\gamma l} - Z_R e^{-\gamma l}}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l}[Z_R + Z_O] + e^{-\gamma l}[Z_R - Z_O]}{e^{\gamma l}[Z_R + Z_O] - e^{-\gamma l}[Z_R - Z_O]}\right]$$

$$Z_S = Z_O \frac{[Z_R + Z_O]}{[Z_R + Z_O]}\left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}\right] \quad \text{.........(6)}$$

This is the second standard form of input impedance of the transmission line.

WAVELENGTH AND VELOCITY OF PROPAGATION

WAVELENGTH:

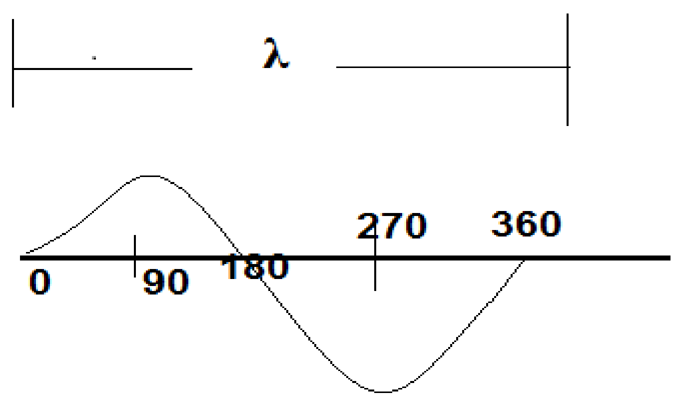

Fig: 1.1.3 Propagation of current from sending and towards receiving end

The distance with which the wave changes its phase by 2π radians is known as wavelength.

In Fig 1.1.3 the distance from the sending end to point 8 is thus one wavelength.

The distance the wave travels along the line where the phase angle is changing to 2π radians is known as wavelength.

It is denoted by λ,

$\lambda = \frac{2\pi}{\beta}$ …… (1)

And also we know,

$\lambda = \frac{v}{f}$

v = λ.f

v- Velocity

f- Frequency

$v = \frac{2\pi}{\beta}.F$

………(2)

$v = \frac{\omega}{\beta}$

$[\lambda = \frac{2\pi}{\beta}]$

[ω = 2πf]

VELOCITY OF PROPAGATION:

The velocity of propagation along the line depends on the change in the phase along the line. Therefore, this velocity is called phase velocity or wave velocity.

$\gamma = \text{in } (\frac{V_1}{V_2}) = \text{in } (\frac{I_1}{I_2})$

In general,

γ = α+jβ

$\gamma = \sqrt{ZY}$ ....(1)

Where,

Z = R+jωL

Y = G+jωC ………

(2) Sub equal (2) in equal (1)

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\gamma = \sqrt{RG + j\omega RC + j\omega LG - \omega^2 LC}$

$\alpha + j\beta = \sqrt{RG - \omega^2 LC + j(\omega RC + \omega LG)}$

Squaring on both sides,

$(\alpha + j\beta)^2 = RG - \omega^2 LC + j(\omega RC + \omega LG)$

$\alpha^2 + \beta^2 - 2j\alpha\beta = RG - 2LC + (\omega RC + \omega LG)$

Equating real and imaginary parts,

$\alpha^2 + \beta^2 = RC - \omega^2 LG$ ………

(3) $2\alpha\beta = \omega RC + \omega LG$

$2\alpha\beta = (RC + LG)$ ……………………………………(4)

From equal (3),

$\alpha^2 = RG - \omega^2 LC + \beta^2$ ……………………………………(5)

Squaring equal (4),

$4\alpha^2\beta^2 = \omega^2 (RC + LG)^2$ ……………………………… (6)

Sub equal (5) in equal (6)

$4(RG - \omega^2 LC + \beta^2)\, \beta 2 = \omega^2 (RC + LG)^2$

$4(RG\beta^2 - \omega^2 LC\, \beta^2 + \beta^4) = \omega^2 (RC + LG)^2$

$RG\beta^2 - \omega^2 LC\, \beta^2 + \beta^4 = \frac{\omega^2}{4} (RC + LG)^2$

$RG\beta^2 - \omega^2 LC\, \beta^2 + \beta^4 - \frac{\omega^2}{4} (RC + LG)^2 = 0$

$\beta^4 + \beta^2 (RG - \omega^2 LC) - \frac{\omega^2}{4} (RC + LG)^2 = 0$ …….(7)

The above equation so of the form of $ax^4+bx^2+c = 0$

$$x^2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

a=1, b= $RG - \omega^2 L$, c= - $\frac{\omega^2}{4}(RC + LG)^2$

$$\beta^2 = \frac{-(RG-\omega^2LC) \pm \sqrt{(RG-\omega^2LC)^2 - 4\left(-\frac{\omega^2}{4}(RC+LG)^2\right)}}{2}$$

Neglect the negative value,

$$\beta^2 = \frac{(\omega^2LC-RG)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}$$

$$\beta = \sqrt{\frac{(\omega^2LC-RG)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}} \quad \text{........(8)}$$

Sub β^2 value in equal (5),

$$\alpha^2 = RG - \omega^2LC + \left(\frac{(\omega^2LC-RG)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}\right)$$

$$\alpha^2 = \frac{2(RG-\omega^2LC)+(\omega^2LC-RG)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}$$

$$\alpha^2 = \frac{(RG-\omega^2LC)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}$$

$$\alpha = \sqrt{\frac{(RG-\omega^2LC)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}} \quad \text{...... (9)}$$

In a perfectly matched line R=0 and G=0,

Sub the above condition in β.

From equal (8),

$$\beta = \sqrt{\frac{(\omega^2LC)+\sqrt{(-\omega^2LC)^2}}{2}}$$

$$\beta = \omega\sqrt{LC}$$

The velocity of propagation of an ideal line is,

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\omega \sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$

The velocity of propagation is constant for a given L and C.

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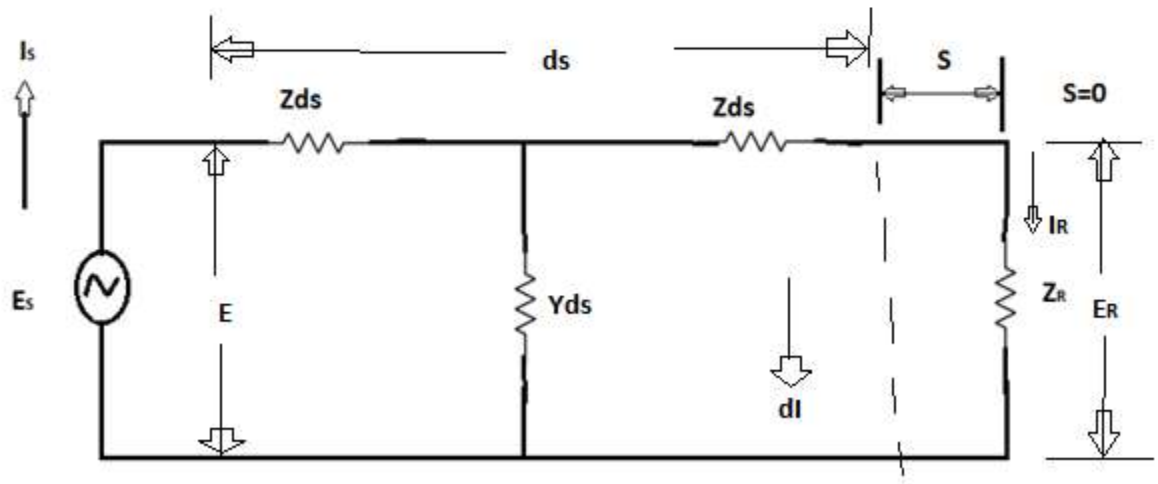

Fig: 1.1.1 A long line, with the elements of one of the infinitesimal sections shown.

From the Fig 1.1.1 consider an infinite length transmission line also an small element ds which located at a distance of s from the receiving end ( ). The sending end voltage and current are E_S and I_S. The receiving end voltage and current are E_R and I_R. The current and voltage at any point is E and I

The voltage in the length ds is dE.

dE = I Zds

$\frac{dE}{ds}$= IZ ………(1)

The current in the length ds is

dI = E Y ds

$\frac{dI}{ds}$= EY………………………………………………………………(2)

Diff equ (1) and (2)

From equ(1)…… $\frac{d^2E}{ds^2} = Z \frac{dI}{ds}$

From equ(2)…… $\frac{d^2I}{ds^2} = Y \frac{dE}{ds}$

Sub the values of $\frac{dI}{ds}$ and $\frac{dE}{ds}$ in above equ

$\frac{d^2E}{ds^2} = Z\ EY$

$\frac{d^2}{ds^2} = Y\ IZ$

$\frac{d^2E}{ds^2} - Z\ EY = 0$ (3)

$\frac{d^2}{ds^2} - Y\ IZ = 0$ (4)

These are the differential equations of transmission line.

From equ(3),

$(\frac{2}{ds^2} - Z\ Y)\ E = 0$

$(m^2 - Z\ Y)\ E = 0$

$(m^2 - Z\ Y) = 0$

$m^2 = Z\ Y$

$m = \pm\sqrt{ZY}$

The general solution can be written as $E = A\ e^{\sqrt{ZY}s} + B\ e^{-\sqrt{ZY}s}$ ………(5)

From equ(4),

$(\frac{2}{ds^2} - Z\ Y)\ I = 0$

$(m^2 - Z\ Y)\ I = 0$

$(m^2 - Z\ Y) = 0$

$m^2 = Z\ Y$

$m = \pm\sqrt{ZY}$

The general solution can be written as

$I = C e^{\sqrt{ZY}s} + e^{-\sqrt{ZY}s}$ ………(6)

where A, B, C, D are arbitrary constants

We need to find the values of A, B, C, D

By using the condition **s=0**

$I = I_R$

$E = E_R$

Use this condition in equ (5) and (6) From equ (5),

$E_R = A\, e^{\sqrt{ZY}(0)} + B\, e^{-\sqrt{ZY}(0)}$

$E_R = A + B$

From equ (6), IR = C + D …….(7)

Diff equ (5) and (6) with.r.to 's'

From equ(5)

$\frac{dE}{ds} = A\,\sqrt{ZY} e^{\sqrt{ZY}s} + B\,(-\sqrt{ZY})\, e^{-\sqrt{ZY}s}$

[using this formula, $e^{ax} = a\, e^{ax}$]

$\frac{dE}{ds} = A\,\sqrt{ZY} e^{\sqrt{ZY}s} - B\,\sqrt{ZY} e^{-\sqrt{ZY}s}$

$\frac{dE}{ds} = IZ$, sub in above equ

$IZ = A\,\sqrt{ZY} e^{\sqrt{ZY}s} - B\,\sqrt{ZY} e^{-\sqrt{ZY}s}$

$I = \frac{A\,\sqrt{ZY}\, e^{\sqrt{ZY}s} - B\,\sqrt{ZY}\, e^{-\sqrt{ZY}s}}{Z}$

$I = A\,\sqrt{\frac{Y}{Z}}\, e^{\sqrt{ZY}s} - B\,\sqrt{\frac{Y}{Z}}\, e^{-\sqrt{ZY}s}$ …………………………(8)

$(\frac{\sqrt{ZY}}{Z} = \sqrt{\frac{Y}{Z}})$

From equ (6),

$\frac{dI}{ds} = C\sqrt{ZY} e^{\sqrt{ZY}s} + D\,\sqrt{ZY} e^{-\sqrt{ZY}s}$

$E\,Y = C\sqrt{ZY} e^{\sqrt{ZY}s} + D\,\sqrt{ZY} e^{-\sqrt{ZY}s}$

$(\frac{dI}{ds} = E\,Y)$

$$E = \frac{C\sqrt{ZY}\, e^{\sqrt{ZY}s} - D\sqrt{ZY}\, e^{-\sqrt{ZY}s}}{Y}$$

$$E = C\sqrt{\frac{Z}{Y}}\, e^{\sqrt{ZY}s} - D\sqrt{\frac{Z}{Y}}\, e^{-\sqrt{ZY}s} \text{ ....(9)}$$

$$\left(\frac{\sqrt{ZY}}{Y} = \sqrt{\frac{Z}{Y}}\right)$$

At, s=0 , E = E_R and I = I_R

sub this values in (8) and (9)

from equ (8),

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \text{ ....(10)}$$

from equ (9),

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \text{ ....(11)}$$

from equ (7),

$$B = E_R - A$$

To find the value of A sub B value in equ (10)

$$I_R = A\sqrt{\frac{Y}{Z}} - (E_R - A)\sqrt{\frac{Y}{Z}}$$

$$I_R = A\sqrt{\frac{Y}{Z}} - E_R\sqrt{\frac{Y}{Z}} + A\sqrt{\frac{Y}{Z}}$$

$$I_R + E_R\sqrt{\frac{Y}{Z}} = 2A\sqrt{\frac{Y}{Z}}$$

$$2A\sqrt{\frac{Y}{Z}} = I_R + E_R\sqrt{\frac{Y}{Z}}$$

$$A = \frac{I_R + E_R\sqrt{\frac{Y}{Z}}}{2\sqrt{\frac{Y}{Z}}}$$

$$A = \frac{I_R}{2\sqrt{\frac{Y}{Z}}} + \frac{E_R\sqrt{\frac{Y}{Z}}}{2\sqrt{\frac{Y}{Z}}}$$

$$A = \frac{I_R\sqrt{\frac{Z}{Y}}}{2} + \frac{E_R}{2}$$

$$(\sqrt{\frac{Z}{Y}} = Z_o)$$

$$A = \frac{I_R Z_o}{2} + \frac{E_R}{2}$$

$$A = \frac{I_R Z_o}{2} + \frac{E_R}{2}$$

By using this formula, V = IR

E = IZ

$$I = \frac{E}{Z}$$

$$I_R = \frac{E_R}{Z_R}$$

sub I_R value in A,

$$A = \frac{\quad}{2} + \frac{E_R}{2}$$

$$A = \frac{E_R}{2}[1 + \frac{Z_o}{Z_R}] \qquad ....(12)$$

sub equ (12) in (7),

$$E_R = \frac{E_R}{2}[1 + \frac{Z_o}{Z_R}] + B$$

$$B = E_R - \frac{E_R}{2}[1 + \frac{\quad}{Z_R}]$$

$$B = E_R - \frac{E_R}{2} - \frac{Z_o}{Z_R} \cdot \frac{E_R}{2}$$

$$B = \frac{E_R}{2} - \frac{Z_o}{Z_R} \cdot \frac{E_R}{2}$$

$$B = \frac{E_R}{2}[1 - \frac{Z_o}{Z_R}] \qquad ....(13)$$

from equ (7),

$I_R = C + D$

$D = I_R - C$

sub the above value in (11),

$E_R = C\sqrt{\frac{Z}{Y}} - (I_R - C)\sqrt{\frac{Z}{Y}}$

$E_R = C\sqrt{\frac{Z}{Y}} - I_R\sqrt{\frac{Z}{Y}} + C\sqrt{\frac{Z}{Y}}$

$E_R = 2C\sqrt{\frac{Z}{Y}} - I_R\sqrt{\frac{Z}{Y}}$

$2C\sqrt{\frac{Z}{Y}} = E_R + I_R\sqrt{\frac{Z}{Y}}$

$C = \frac{E_R + I_R\sqrt{\frac{Z}{Y}}}{2\sqrt{\frac{Z}{Y}}}$

$C = \frac{E_R}{2\sqrt{\frac{Z}{Y}}} + \frac{I_R\sqrt{\frac{Z}{Y}}}{2\sqrt{\frac{Z}{Y}}}$

$C = \frac{E_R}{2\sqrt{\frac{Z}{Y}}} + \frac{I_R}{2}$

$C = \frac{E_R}{2Z_o} + \frac{I_R}{2}$

$(E_R = I_R Z_R)$

sub E_R value in C,

$C = \frac{I_R Z_R}{2Z_o} + \frac{I_R}{2}$

$C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]$ ....(14)

$D = I_R - C$

$D = I_R - \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right]$

$D = I_R - \frac{I_R}{2} - \frac{I_R}{2}\frac{Z_R}{Z_0}$

$$D = \frac{I_R}{2} - \frac{I_R}{2}\frac{Z_R}{Z_0}$$

$$D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right] \qquad \text{............(15)}$$

sub the A, B, C,D values in (5) and (6),

from equ (5), $E = A\, e^{\sqrt{ZY}s} + B\, e^{-\sqrt{ZY}s}$

$$E = \frac{E_R}{2}\left[1 + \frac{Z_o}{Z_R}\right] e^{\sqrt{ZY}s} + \frac{E_R}{2}\left[1 - \frac{Z_o}{Z_R}\right] e^{-\sqrt{ZY}s} \text{......................} (16)$$

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right] e^{\sqrt{ZY}s} + \frac{E_R}{2}\left[\frac{Z_R - Z_o}{Z_R}\right] e^{-\sqrt{ZY}s}$$

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right]\left[e^{\sqrt{ZY}S} + \left[\frac{Z_R - Z_o}{Z_R}\right]\left[\frac{Z_R}{Z_R + Z_o}\right] e^{-\sqrt{ZY}S}\right]$$

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right]\left[e^{\sqrt{ZY}S} + \left[\frac{Z_R - Z_o}{Z_R + Z_o}\right] e^{-\sqrt{ZY}S}\right] \qquad \text{.......(17)}$$

from equ (6), $I = C\, e^{\sqrt{ZY}s} + D e^{-\sqrt{ZY}s}$

$$I = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_0}\right] e^{\sqrt{ZY}s} + \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_0}\right] e^{-\sqrt{ZY}s} \text{..} (18)$$

$$I = \frac{I_R}{2}\left[\frac{Z_0 + Z_R}{Z_0}\right] e^{\sqrt{ZY}s} + \frac{I_R}{2}\left[\frac{Z_0 - Z_R}{Z_0}\right] e^{-\sqrt{ZY}s}$$

$$I = \frac{I_R}{2}\left[\frac{Z_0 + Z_R}{Z_0}\right]\left[e^{\sqrt{ZY}S} + \frac{I_R}{2}\left[\frac{Z_0 - Z_R}{Z_0}\right]\left[\frac{Z_R}{Z_0 + Z_R}\right] e^{-\sqrt{ZY}S}\right]$$

$$I = \frac{I_R}{2}\left[\frac{Z_0 + Z_R}{Z_0}\right]\left[e^{\sqrt{ZY}S} - \frac{I_R}{2}\left[\frac{Z_R - Z_0}{Z_R + Z_0}\right] e^{-\sqrt{ZY}S}\right] \text{..} (19)$$

equ (17) and (19) are the first form of voltage and current.

equ (16) may be rearranged as

$$E = \frac{E_R}{2}\left[\left[1 + \frac{Z_o}{Z_R}\right] e^{\sqrt{ZY}S} + \left[1 - \frac{Z_o}{Z_R}\right] e^{-\sqrt{ZY}S}\right]$$

$$E = \frac{E_R}{2}\left[e^{\sqrt{ZY}S} + \frac{Z_o}{Z_R} e^{\sqrt{ZY}S} + e^{-\sqrt{ZY}S} - \frac{Z_o}{Z_R} e^{-\sqrt{ZY}S}\right]$$

$$E = \frac{E_R}{2}\left[e^{\sqrt{ZY}S} + e^{-\sqrt{ZY}S} + \frac{Z_o}{Z_R}\left[e^{\sqrt{ZY}S} - e^{-\sqrt{ZY}S}\right]\right]$$

for example,

$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

sub in above equ,

$E = \frac{E_R}{2}[\,2\cosh\sqrt{ZYS} + \frac{2.\,Z_o}{Z_R}\sinh\sqrt{ZYS}]$

$E = E_R\cosh\sqrt{ZYS} + \frac{E_R\,Z_o}{Z_R}\sinh\sqrt{ZYS}$

$Z_R = \frac{ER}{I_R}$

sub the Z_R value in above equ,

$E = E_R\cosh\sqrt{ZYS} + \frac{E_R\,Z_o}{[\frac{E_R}{I_R}]}\sinh\sqrt{ZYS}$

$E = E_R\cosh\sqrt{ZYS} + \frac{I_R\,E_R\,Z_o}{E_R}\sinh\sqrt{ZYS}$

$E = E_R\cosh\sqrt{ZYS} + I_R\,Z_o\sinh\sqrt{ZYS}$ ....(21)

The same procedure will be followed for the current equ,

equ (18) will be,

$I = I_R\cosh\sqrt{ZYS} + \frac{E_R}{Z_0}\sinh\sqrt{ZYS}$ .... (22)

equ (21) and (22) are the second form of voltage and current at any point on a transmission line.

PHYSICAL SIGNIFICANCE OF TRANSMISSION LINE (or) INFINITE LINE (or) THE TWO STANDARD FORM FOR INPUT IMPEDANCE OF THE TRANSMISSION LINE TERMINATED BY AN IMPEDANCE Z

From the Fig 1.1.2 the equation for the current and voltage may be written for the sending end current 'I_S' of a line of length 'l' is,

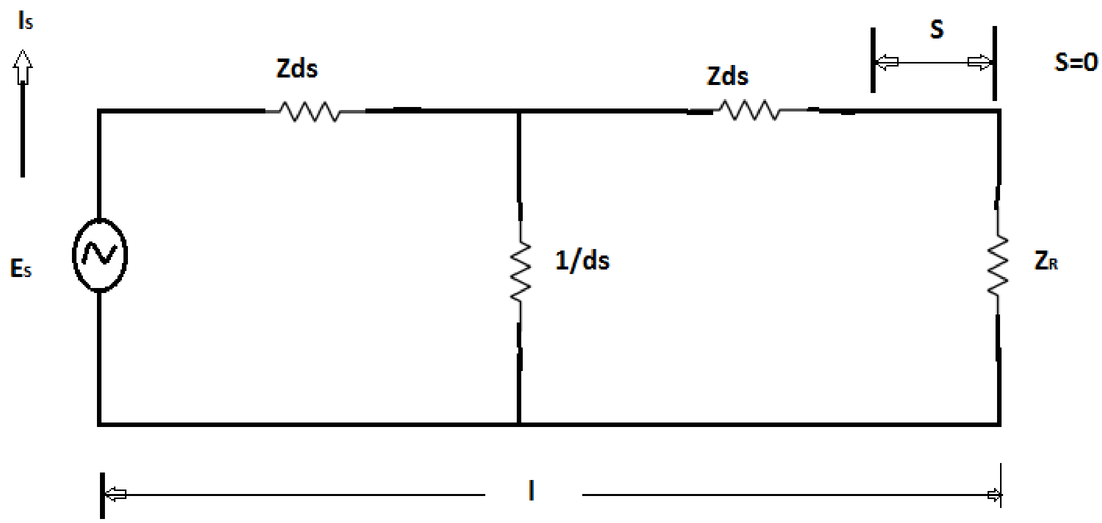

Fig: 1.1.2 A length l taken from an infinite line

The sending current equation is given by,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{E_R}{Z_0} \sinh\sqrt{ZY}.l$ $[E_R = I_R Z_R]$

Sub E_R value in above equ,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{I_R Z_R}{Z_0} \sinh\sqrt{ZY}.l$

$I_S = I_R [\cosh\sqrt{ZY}.l + \frac{Z_R}{Z_0} \sinh\sqrt{ZY}.l]$ ....(1)

The sending voltage equation is given by,

$E_S = E_R \cosh\sqrt{ZY}.l + I_R Z_0 \sinh\sqrt{ZY}.l$

$[E_R = I_R Z_R]$

$[I_R = \frac{E_R}{Z_R}]$

Sub I_R value in above equ,

$E_S = E_R \cosh\sqrt{ZY}.l + \frac{E_R Z_0}{Z_R} \sinh\sqrt{ZY}.l$

$E_S = E_R [\cosh\sqrt{ZY}.l + \frac{\sinh\sqrt{ZY}.l}{Z_R}]$ .... (2)

Since, We know that,

Propagation Constant $\gamma = \sqrt{ZY}$

Characteristic Impedance $Z_O = \sqrt{\frac{Z}{Y}}$

Sub γ value in equ (1) and (2),

From equ (1),

$I_S = I_R\,[\cosh \gamma l + \frac{Z_R}{Z_O}\sinh \gamma l]$ .... (3)

From equ (2),

$E_S = E_R\,[\cosh \gamma l + \frac{Z_O}{Z_R}\sinh \gamma l]$ .... (4)

Input Impedance $Z_S = \frac{E_S}{I_S}$ [E = IZ]

$$Z_S = \frac{E_R\,[\cosh \gamma l + \frac{Z_O}{Z_R}\sinh \gamma l]}{I_R\,[\cosh \gamma l + \frac{Z_R}{Z_O}\sinh \gamma l]}$$

$$Z_S = Z_R\left[\frac{\frac{Z_R\cosh \gamma l + Z_O\sinh \gamma l}{Z_R}}{\frac{Z_O\cosh \gamma l + Z_R\sinh \gamma l}{Z_O}}\right]$$

$E_R = I_R Z_R$

$Z_R = \frac{E_R}{I_R}$

$$Z_S = Z_R\left[\frac{Z_R\cosh \gamma l + Z_O\sinh \gamma}{Z_R} \times \frac{Z_O}{Z_O\cosh \gamma l + Z\ \sinh \gamma l}\right]$$

$$Z_S = Z_O\left[\frac{Z_R\cosh \gamma l + Z_O\sinh \gamma}{Z_O\cosh \gamma l + Z_R\sinh \gamma l}\right] \quad ....(5)$$

This is the first standard form of input impedance of the transmission line.

$\mathrm{Cosh}\theta = \frac{e^{\theta} + e^{-\theta}}{2}$

$\mathrm{Sinh}\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

Sub the above formula in equ (5),

$$Z_S = Z_O \left[\frac{Z_R\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right) + Z_O\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)}{Z_O\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right) + Z_R\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)} \right]$$

$$Z_S = \frac{2Z_O}{2}\left[\frac{Z_R(e^{\gamma l}+e^{-\gamma l}) + Z_O(e^{\gamma l}-e^{-\gamma l})}{(e^{\gamma l}+e^{-\gamma l}) + (e^{\gamma l}-e^{-\gamma l})}\right]$$

$$Z_S = Z_O \left[\frac{Z_R e^{\gamma l} + Z_R e^{-\gamma l} + Z_O e^{\gamma l} - Z_O e^{-\gamma l}}{Z_O e^{\gamma l} + Z_O e^{-\gamma l} + Z_R e^{\gamma l} - Z_R e^{-\gamma l}}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l}[Z_R + Z_O] + e^{-\gamma l}[Z_R - Z_O]}{e^{\gamma l}[Z_R + Z_O] - e^{-\gamma l}[Z_R - Z_O]}\right]$$

$$Z_S = Z_O \frac{[Z_R + Z_O]}{[Z_R + Z_O]}\left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}\right] \qquad \ldots\ldots\ldots(6)$$

This is the second standard form of input impedance of the transmission line.

WAVELENGTH AND VELOCITY OF PROPAGATION

WAVELENGTH:

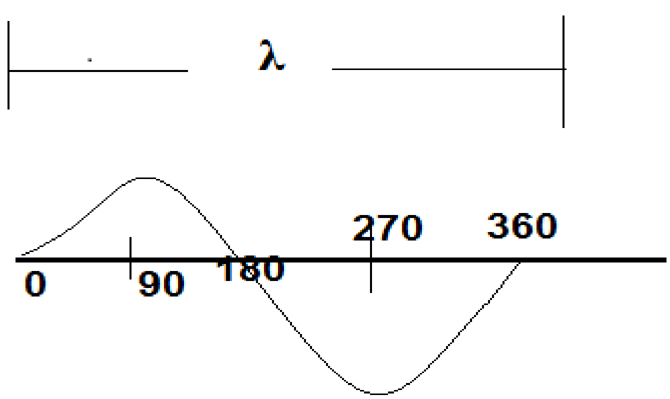

Fig: 1.1.3 Propagation of current from sending and towards receiving end

The distance with which the wave changes its phase by 2π radians is known as wavelength.

In Fig 1.1.3 the distance from the sending end to point 8 is thus one wavelength. The distance the wave travels along the line where the phase angle is changing to 2π radians is known as wavelength.

It is denoted by λ,

$$\lambda = \frac{2\pi}{\beta} \quad \ldots\ldots(1)$$

and also we know,

$$\lambda = \frac{v}{f}$$

v = λ.f

v- velocity

f- frequency

$$v = \frac{2\pi}{\beta}.f$$

$$v = \frac{\omega}{\beta} \quad \ldots\ldots\ldots(2)$$

$$[\lambda = \frac{2\pi}{\beta}]$$

[ω = 2πf]

VELOCITY OF PROPAGATION:

The velocity of propagation along the line depends on the change in the phase along the line. Therefore, this velocity is called phase velocity or wave velocity.

$$\gamma = \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{I_1}{I_2}\right)$$

In general,

γ = α+jβ

$$\gamma = \sqrt{ZY} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where,

Z = R+jωL

Y = G+jωC ………(2)

Sub equ (2) in equ (1)

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\gamma = \sqrt{RG + j\omega RC + j\omega LG - \omega^2 LC}$

$\alpha + j\beta = \sqrt{RG - \omega^2 LC + j(\omega RC + \omega LG)}$

Squaring on both sides,

$(\alpha + j\beta)^2 = RG - \omega^2 LC + j(\omega RC + \omega LG)$

$\alpha^2 + \beta^2 - 2j\alpha\beta = RG - {}^2 LC + j(\omega RC + \omega LG)$

Equating real and imaginary parts,

$\alpha^2 + \beta^2 = RC - \omega^2 LG$ ………(3)

$2\alpha\beta = \omega RC + \omega LG$

$2\alpha\beta = (RC + LG)$ ....(4)

From equ (3),

$\alpha^2 = RG - \omega^2 LC + \beta^2$ ....(5)

Squaring equ (4),

$4\alpha^2\beta^2 = \omega^2 (RC + LG)^2$ .... (6)

Sub equ (5) in equ (6)

$4(RG - \omega^2 LC + \beta^2)\beta^2 = \omega^2 (RC + LG)^2$

$4(RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4) = \omega^2 (RC + LG)^2$

$RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4 = \frac{\omega^2}{4} (RC + LG)^2$

$RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4 - \frac{\omega^2}{4} (RC + LG)^2 = 0$

$\beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4} (RC + LG)^2 = 0$ …….(7)

The above equation os of the form of ax^4+bx^2+c = 0

$$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a=1, b= $RG - \omega^2 L$, c= - $\frac{\omega^2}{4}(RC + LG)^2$

$$\beta^2 = \frac{-(RG-\omega^2 LC) \pm \sqrt{(RG-\omega^2 LC)^2 - 4\left(-\frac{\omega^2}{4}(RC+LG)^2\right)}}{2}$$

Neglect the negative value,

$$\beta^2 = \frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}} \quad \ldots\ldots..(8)$$

Sub β^2 value in equ (5),

$$\alpha^2 = RG - \omega^2 LC + \left(\frac{(\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}\right)$$

$$\alpha^2 = \frac{2\,(RG-\omega^2 LC) + (\omega^2 LC - RG) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\alpha^2 = \frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\alpha = \sqrt{\frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}} \quad \ldots\ldots(9)$$

In a perfectly matched line R=0 and G=0,

Sub the above condition in β.

From equ (8),

$$\beta = \sqrt{\frac{(\omega^2 LC) + \sqrt{(-\omega^2 LC)^2}}{2}}$$

$$\beta = \omega\sqrt{LC}$$

The velocity of propagation of a ideal line is,

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\omega \sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$

The velocity of propagation is constant for a given L and C.

1.4 INPUT AND TRANSFER IMPEDANCE:

INPUT IMPEDANCE:

The input impedance of a transmission line is given by,

$$Z_{in} = Z_S = Z_O \left[\frac{Z_R \cosh \gamma l + Z_O \sinh \gamma l}{Z_O \cosh \gamma l + Z_R \sinh \gamma l}\right]$$

$$Z_S = Z_O \frac{Z_R \cosh \gamma l}{Z_R \cosh \gamma l} \left[\frac{1 + \frac{Z_O \sinh \gamma l}{Z_R \cosh \gamma l}}{\frac{Z_O}{Z_R} + \frac{\sinh \gamma l}{\cosh \gamma l}}\right]$$

$$Z_S = Z_O \left[\frac{1 + \frac{Z_O \sinh \gamma l}{Z_R \cosh \gamma l}}{\frac{Z_O}{Z_R} + \tanh \gamma l}\right] \quad \ldots\ldots.(1)$$

TRANSFER IMPEDANCE:

Let,

$$Z_T = \frac{E_S}{I_R} = \frac{Voltage\ at\ the\ receiving\ end}{Current\ at\ the\ sending\ end}$$

Transfer impedance of the transmission line, Now, the terminating current (I_R) can be written as,

$$I_R = I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$$

Divide the above equ by I_R,

$$1 = \frac{I_S}{I_R} \cosh\gamma l - \frac{E_S}{Z_O I_R} \sinh\gamma l$$

$$1 = \frac{I_S}{I_R} \cosh\gamma l - \frac{Z_T}{Z_O} \sinh\gamma l \quad \ldots\ldots.(2)$$

We know that,

$$E_R = E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$$

$$I_S Z_O \sinh\gamma l = E_S \cosh\gamma l - E_R$$

$$I_S = \frac{E_S \cosh\gamma l - E_R}{Z_O \sinh\gamma l}$$

Sub equ (3) in equ

$$1 = \frac{\left(\frac{E_S \cosh\gamma l - E_R}{Z_O}\right)}{\sinh\gamma l \, Z_O} \sinh\gamma l$$

$$1 = \left(\frac{E_S \cosh\gamma l - E_R}{I_R Z_O}\right) \frac{\cosh\gamma l}{\sinh\gamma l} - \frac{Z_T}{Z_O} \sinh\gamma l$$

$$1 = \left(\frac{E_S \cosh\gamma}{I_R Z_O} - \frac{E}{I_R Z_O}\right) \frac{\cosh\gamma l}{\sinh\gamma l} - \frac{Z_T}{Z_O} \sinh\gamma l$$

$$1 = \left(\frac{Z_T \cosh\gamma l}{Z_O} - \frac{Z_R}{Z_O}\right) \frac{\cosh\gamma l}{\sinh\gamma l} - \frac{Z_T}{Z_O} \sinh\gamma l$$

$$1 = \frac{Z_T \cosh^2\gamma l}{Z_O \sinh\gamma l} - \frac{Z_R}{Z_O} \frac{\cosh\gamma}{\sinh\gamma l} - \frac{Z_T}{Z_O} \sinh\gamma l$$

$$1 + \frac{Z_R}{Z_O} \frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z_T \cosh^2\gamma l}{Z_O \sinh\gamma l} - \frac{Z_T}{Z_O} \sinh\gamma l$$

$$1 + \frac{Z_R}{Z_O} \frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z_T}{Z_O} \left(\frac{\cosh^2\gamma l}{\sinh\gamma l} - \sinh\gamma\right)$$

$$1 + \frac{Z_R}{Z_O} \frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z_T}{Z_O} \left(\frac{\cosh^2\gamma l - \sinh^2\gamma l}{\sinh\gamma l}\right)$$

$$\cosh^2\theta - \sinh^2\theta = 1$$

$$1 + \frac{Z_R}{Z_O} \frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z}{Z_O} \left(\frac{1}{\sinh\gamma l}\right)$$

$$Z_T = Z_O \sinh\gamma l \left[1 + \frac{Z_R}{Z_O} \frac{\cosh\gamma l}{\sinh\gamma l}\right]$$

$$Z_T = Z_O \sinh\gamma l \left[\frac{Z_O \sinh\gamma l + Z_R \cosh\gamma l}{Z_O \sinh\gamma l}\right]$$

$$Z_T = Z_O \sinh\gamma l + Z_R \cosh\gamma l$$

OPEN AND SHORT CIRCUIT IMPEDANCE:

FINITE LINE TERMINATED IN Z_O:

In Fig 1.4.1 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

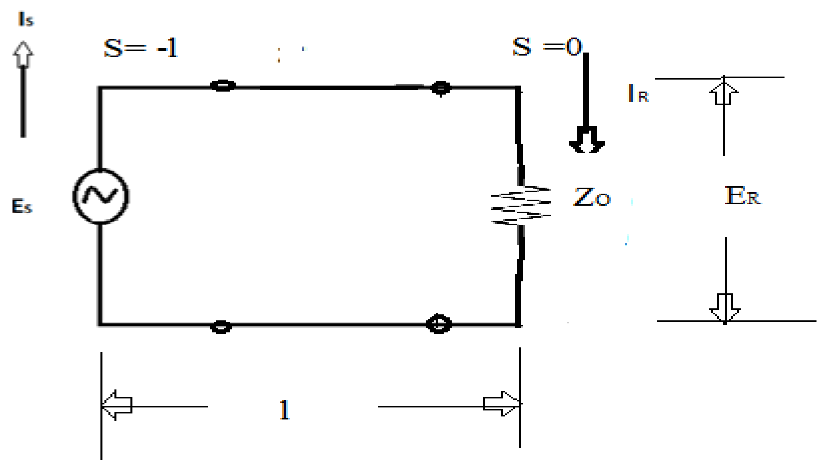

Fig: 1.4.1 Finite line terminated in Z_O

At distance S= l , E = E_R and I = I_R

$E_R = E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$

$I_R = I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$

$Z_R = \frac{E_R}{I_R}$

$Z_R = \frac{E_S \cosh\gamma l - I_S Z_O \sinh\gamma l}{I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l}$

FINITE LINE OPEN CIRCUITED AT DISTANCE END:

In Fig 1.4.2 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

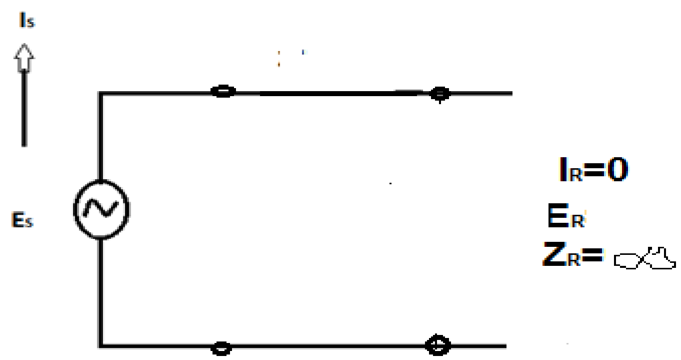

Fig: 1.4.2 Finite line open circuited at distance end

$E_R = E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$

$I_R = I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$

$I_R = 0$

$0 = I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$

$I_S \cosh\gamma l = \frac{E_S}{Z_O} \sinh\gamma l$

$\frac{E_S}{I_S} = Z_O \frac{\cosh\gamma l}{\sinh\gamma l}$

$Z_{OC} = Z_O \frac{\cosh\gamma l}{\sinh\gamma l}$

$Z_{OC} = Z_O \coth\gamma l$

If $l = \infty$

$Z_{OC} = Z_O \coth\gamma(\infty)$

$Z_{OC} = Z_O$

FINITE LINE SHORT CIRCUITED AT DISTANCE END:

In Fig 1.4.3 shows that the wave is progressing from the receiving end toward the load, the initial value equal to the reflected voltage at the load for open circuit. This is incident wave.

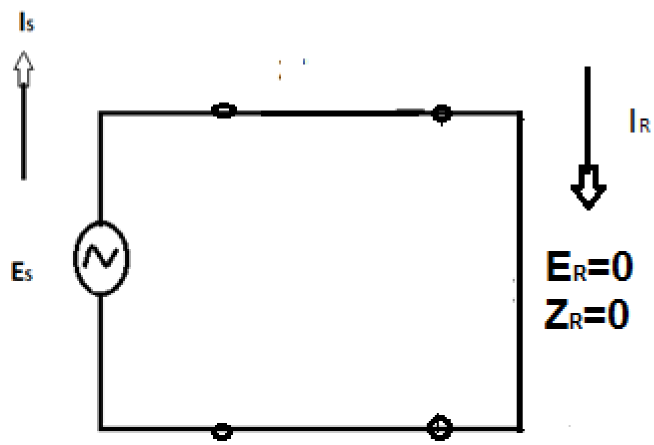

Fig: 1.4.3 Finite line short circuited at distance end

$E_R = E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$

$I_R = I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$

$E_R = 0$

$0 = E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$

$E_S \cosh\gamma l = I_S Z_O \sinh\gamma l$

$\frac{E_S}{I_S} = Z_O \frac{\sinh\gamma l}{\cosh\gamma l}$

$Z_{SC} = Z_O \tanh\gamma l$

If $l = \infty$

$Z_{SC} = Z_O \tanh\gamma(\infty)$

$Z_{SC} = Z_O$

Multiply the Z_{OC} & Z_{SC} expression.

$Z_{OC}\ Z_S = Z_O \coth\gamma l Z_O \tanh\gamma l$

$Z_{OC}\ Z_{SC} = Z_O^2 \frac{1}{\coth\gamma l} \tanh\gamma l$

$Z_{OC} Z_{SC} = Z_O^2$

$Z_O = \sqrt{Z_{OC} Z_{SC}}$

INPUT IMPEDANCE INTERMS OF Z_O AND REFLECTION COFFIECIENT:

We know that,

Input impedance of the transmission line is,

$$Z_S = Z_O \left[\frac{Z_R \cosh \gamma l + Z_O \sinh \gamma l}{Z_O \cosh \gamma l + Z_R \sinh \gamma l}\right]$$

$$Z_S = Z_O \left[\frac{Z_R\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right)+Z_O\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)}{Z_O\left(\frac{e^{\gamma l}+e^{-\gamma l}}{2}\right)+Z_R\left(\frac{e^{\gamma l}-e^{-\gamma l}}{2}\right)}\right]$$

$$Z_S = \frac{2Z_O}{2}\left[\frac{Z(e^{\gamma l}+e^{-\gamma l})+Z_O(e^{\gamma l}-e^{-\gamma l})}{Z(e^{\gamma l}+e^{-\gamma l})+Z_R(e^{\gamma l}-e^{-\gamma l})}\right]$$

$$Z_S = Z_O \left[\frac{Z_R e^{\gamma l}+Z_R e^{-\gamma l}+Z_O e^{\gamma l}-Z_O e^{-\gamma l}}{Z_O e^{\gamma l}+Z_O e^{-\gamma l}+Z_R e^{\gamma l}-Z_R e^{-\gamma l}}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l}[Z_R+Z_O]+e^{-\gamma l}[Z_R-Z_O]}{e^{\gamma l}[Z_R+Z_O]-e^{-\gamma l}[Z_R-Z_O]}\right]$$

$$Z_S = Z_O \frac{[Z_R+Z_O]}{[Z_R+Z_O]}\left[\frac{e^{\gamma l}+e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}{e^{\gamma l}-e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l}+e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}{e^{\gamma l}-e^{-\gamma l}\left[\frac{Z_R-Z_O}{Z_R+Z_O}\right]}\right]$$

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$Z_S = Z_O \left[\frac{e^{\gamma l}+e^{-\gamma l}K}{e^{\gamma l}-e^{-\gamma l}K}\right]$$

The above equation is interms of Z_O and reflection coefficient.

1.5 REFLECTION LOSS:

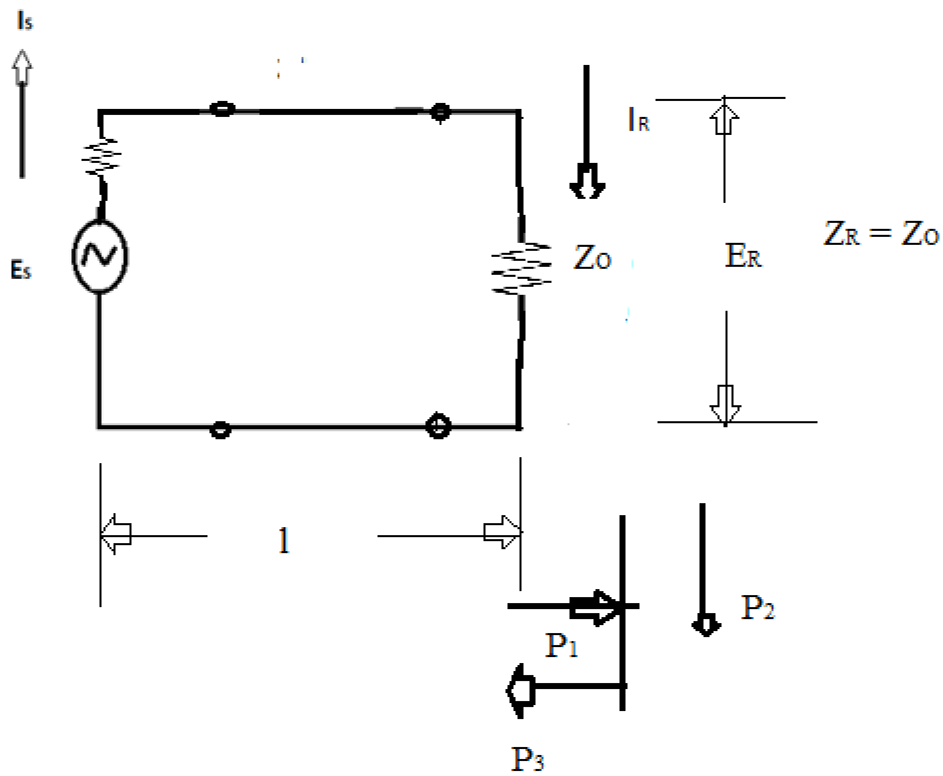

Fig: 1.5.1 Generator of impedance Z_1 connected to the load Z_2

If Z_2 is not equal to Z_1, impedance mismatch causes the chance in the ratio of voltage to current or of energy transmitted by the electric field to that transmitted by the magnetic field and thus a portion of the energy reflected by the load that will be shown in Fig 1.5.1.

$P_1 = P_2 + P_3$

UNDER MISMATCHED COMDITION:

A part of the energy is rejected and reflected by the load the energy delivered to the load is always less than the actual energy to be delivered. This loss is called reflection loss.

The reflection loss is defined as the number of Nipper's and Decibel's by which the current in the load under image matched condition would exceed the current actually flowing in the load.

Reflection loss = $[\frac{|I_2'|}{|I_2|}]$ Nipper

I_2' - actual current

I_2 – observed current

Reflection loss = $20\,log[\frac{|I_2'|}{|I_2|}]$ Decibel

Where,

I_2' is load current under image matched condition.

I_2 is actual load current under mismatched condition.

P_1 is power at receiving end due to incident wave.

P_2 is power observed by the load

P_3 is power rejected back to the line.

$P_1 = P_2 + P_3$

Wkt,

P = I^2

I α $\sqrt{P}$

I = $P^{\frac{1}{2}}$

Reflection loss = $20\,log[\frac{(P_1)^{\frac{1}{2}}}{(P_2)^{\frac{1}{2}}}]$

Reflection loss = $20\,log[\frac{P_1}{P_2}]^{\frac{1}{2}}$

Reflection loss = $10\,log[\frac{P_1}{P_2}]$

And also we know,

K = $\frac{P_2}{P_1}$

Reflection loss = $10 \log [\frac{1}{K}]$

K = $\frac{Z_R - Z_0}{Z_R + Z_0}$

Reflection loss = $10 \log [\frac{Z_R + Z_0}{Z_R - Z_0}]$

If E_R and I_R are the values of voltage and current at the receiving end due to incident wave, then the values of voltage and current at the receiving end to the reflected wave is KE_R and KI_R

$P_1 = E_R I_R$

$P_3 = KE_R . KI_R$

$P_3 = K^2 E_R I_R$

$P_3 = K^2 P_1$

$P_2 = P_1 - P_3$

$P_2 = P_1 - K^2 P_1$

$P_2 = P_1(1 - K^2)$

Wkt,

Reflection loss = $10 \log [\frac{P_1}{P_1(1-K^2)}]$

Reflection loss = $10 \log [\frac{1}{(1-K^2)}]$

K = $\frac{Z_R - Z_0}{Z_R + Z_0}$

Reflection loss = $10 \log [\frac{1}{1-(\frac{Z_R - Z_0}{Z_R + Z_0})^2}]$

Reflection loss = $10 \log [\frac{1}{\frac{(Z_R + Z_0)^2 - (Z_R - Z_0)^2}{(Z_R + Z_0)^2}}]$

Dodddwnload

Reflection loss = $10 \log \left[\frac{+ Z_0}{2\sqrt{Z_R Z_0}}\right]^2$

Reflection loss = $20 \log \left[\frac{+ Z_0}{2\sqrt{Z_R Z_0}}\right]$

Reflection loss = $20 \log \left[\frac{1}{|K|}\right]$

$|K|$ is reflection factor

The ratio in which indicate the change in current with the load due to reflection at the mismatched junction is called reflection factor.

It is given by,

$|K| = \frac{2\sqrt{Z_R Z_0}}{Z_R + Z_0}$

RETURN LOSS:

It is defined as the ratio of actual power to the reflected power.

Return loss = $10 \log \left[\frac{P_1}{P_3}\right]$

Return loss = $10 \log \left[\frac{1}{K^2 P_1}\right]$

Return loss = $10 \log \left[\frac{1}{K^2}\right]$

Return loss = $10 \log \left[\frac{1}{K}\right]^2$

Return loss = $20 \log \left[\frac{1}{K}\right]$

Wkt,

$K = \frac{Z_R - Z_0}{Z_R + Z_0}$

Return loss = $20 \log \left[\frac{Z_R + Z_0}{Z_R - Z_0}\right]$

1.2 WAVEFORM DISTORTION

In an ideal transmission line of a signal through the transmission line when the input signal or the sending end signal is not the same at the receiving end.

Then the signal is said to be distorted.

The distortion is to be classified into,

i. Frequency Distortion

ii. Phase Distortion

i. FREQUENCY DISTORTION:

The attenuation constant (α) is a function of frequency.

The signal transmitted along the line will be attenuated to the different extent.

For example, a voice signal consists of many frequencies will be transmitted along the transmission line and all the frequencies will not be attenuated equally along the transmission line.

Hence the received signal is not the exact replica of the input signal at the sending end. Such a distortion is called as frequency distortion.

$$\alpha = \sqrt{\frac{(RG-\omega^2LC)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}}$$

ii. PHASE DISTORTION

$$\beta = \sqrt{\frac{(\omega^2LC-RG)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}}$$

We know that,

$$v = \frac{\omega}{\beta}$$

The phase distortion is also depends on frequency.

Thus the velocity of propagation v also varies with frequency. So some of the signal reach the receiving end very fast while some waves will be delayed then the others will not have same transmission time.

Thus the receiving end signal is not the exact replica of the sending end signal. This kind of distortion is called as phase distortion or delayed distortion.

THE DISTORTION LESS LINE:

The distortion less line does not distort the signal phase, but does introduce a signal loss line they are not super conductors. This is known as Heaviside distortion.

Already we know that,

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) \pm \sqrt{(RG - \omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}}$$

The value must be made independent of frequency by making LG + RC = 0

Derive the condition for a distortion less line,

$Z = R + j\omega L$

$Y = G + j\omega C$

$\gamma = \alpha + j\beta$

$\gamma = \sqrt{ZY}$

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\gamma = \sqrt{RG + j\omega RC + j\omega RC - \omega^2 LG}$

$\gamma = \sqrt{RG - \omega^2 LC + (RC + LG)}$ ………(1)

To make a frequency independent; in the imaginary part LG + RC = 0

Sub this condition in equ (1),

$\gamma = \sqrt{RG - \omega^2 LC + (0)}$

$\gamma = \sqrt{RG - \omega^2 LC}$

$(-1 = j^2)$

Sub in above value,

$\gamma = \sqrt{RG + j^2\omega^2 LC}$

$\gamma = \sqrt{\ } + \sqrt{j^2\omega^2 LC}$

$\gamma = \alpha + j\beta$

$\alpha + j\beta = \sqrt{RG} + j\omega\sqrt{LC}$

$\alpha = \sqrt{RG}$

$\beta = \omega\sqrt{LC}$

Where the phase velocity,

$v_p = \frac{\omega}{\beta}$

$v_p = \frac{\omega}{\omega\sqrt{LC}}$

$\boldsymbol{v_p = \frac{1}{\sqrt{LC}}}$

<u>CHARACTERISTIC IMPEDANCE OF DISTORTION LESS LINE:</u>

$Z_0 = R_0 + jX_0 = \sqrt{\frac{Z}{Y}}$

$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$Z_0 = \sqrt{\frac{R(1+j\omega\frac{L}{R})}{G(1+j\omega\frac{C}{G})}}$

$\frac{L}{R} = \frac{C}{G}$

LG = RC

$\frac{L}{C} = \frac{R}{G}$

$Z_0 = \sqrt{\frac{R}{G}\frac{(1+j\omega\frac{C}{G})}{(1+j\omega\frac{C}{G})}}$

$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$

Therefore the distortion less line must satisfy the conditions

i. LG = RC

ii. $\alpha = \sqrt{RG}$

iii. $\beta = \omega\sqrt{LC}$

iv. $v = \frac{1}{\sqrt{LC}}$

v. $Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$

vi. $\alpha = \sqrt{G}$

TELEPHONE CABLE:

We know, propagation constant,

$\gamma = \sqrt{ZY}$

Z = R+jωL

Y = G+jωC

γ = α+jβ

$\alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\alpha + j\beta = \sqrt{L\left(\frac{R}{L} + j\omega\right) C\left(\frac{}{C} + j\omega\right)}$

$\alpha + j\beta = \sqrt{LC\left(\frac{R}{L} + j\omega\right)\left(\frac{}{C} + j\omega\right)}$

If there is no attenuation,

$\frac{R}{L} = \frac{G}{C}$

$\alpha + j\beta = \sqrt{LC\left(\frac{R}{L} + j\omega\right)\left(\frac{}{L} + j\omega\right)}$

$\alpha + j\beta = \sqrt{LC\left(\frac{R}{L} + j\omega\right)^2}$

$\alpha+j\beta= \sqrt{LC}\left(\frac{R}{L} \pm j\omega\right)$

$\alpha+j\beta = \frac{R}{L}\sqrt{LC} + j\omega\sqrt{LC}$

$\alpha+j\beta =\sqrt{LC}\left(\frac{R}{L} \pm j\omega\right)$

$\alpha+j\beta = \frac{G}{C}\sqrt{LC} + j\omega\sqrt{LC}$

(or)

Separate the real and imaginary terms,

$\alpha = \frac{R}{L}\sqrt{LC}$ (or) $\frac{G}{C}\sqrt{LC}$

$\beta = \omega\sqrt{LC}$

We know that,

$v_p = \frac{\omega}{\beta}$

$v_p = \frac{\omega}{\omega\sqrt{LC}}$

$v_p = \frac{1}{\sqrt{LC}}$

CHARACTERISTIC IMPEDANCE:

$Z_0 = \sqrt{\frac{Z}{Y}}$

$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$Z_0 = \sqrt{\frac{L(\frac{R}{L}+j\omega)}{C(\frac{G}{C}+j\omega)}}$

For distortionless line,

$\frac{R}{L} = \frac{G}{C}$

$Z_0 = \sqrt{\frac{L(\frac{R}{L}+j\omega)}{C(\frac{G}{C}+j\omega)}}$

$Z_0 = \sqrt{\frac{L}{C}}$

LOADING AND DIFFERENT METHODS OF LOADING:

A distortion less transmission line must satisfy the condition LG = RC therefore $\frac{L}{C} = \frac{R}{G}$

The attenuation of a transmission line is given by,

$$\alpha = \sqrt{\frac{(RG-\omega^2LC)+\sqrt{(RG-\omega^2LC)^2+(\omega^2(RC+LG)^2)}}{2}}$$

It is observed that α depends on four primary constants in addition to the frequency(L, C, R, G)

DIFFERENT LOADING METHODS

There are two loading methods

i. Inductance Loading
ii. Capacitance Loading

Capacitance loading techniques increases the impedance and attenuation

Inductance loading is mostly used in transmission lines.

The types of inductance loading methods are,

i. Lumped Loading
ii. Continuous Loading (or) Uniform Loading

i. LUMPED LOADING:

In this method, lumped inductors or loading coils are placed in series along the transmission lines at suitable intervals. Hence, it is called lumped loading. It will increase the total effective inductance.

The cut-off frequency is given by,

$$f_c = \frac{1}{\pi\sqrt{L_c C\ d}}$$

$$f_c \ \alpha \ \frac{1}{\sqrt{LC}}$$

L_c – inductance of the loading coil and cable per km.

C – Capacitance per km

d – Spacing between the coils

ii. CONTINUOUS LOADING:

In this method, wing the cable with a high permeability material. The inductors use perm-alloy or molybdenum. In Fig 1.2.1 the coil is wound of the largest gauge of content with small size and each winging is divided into two equal parts.

In a uniformly loaded cables, assume

i) G = 0

ii) Wavelength is large

Loading coils are placed into steel pots.

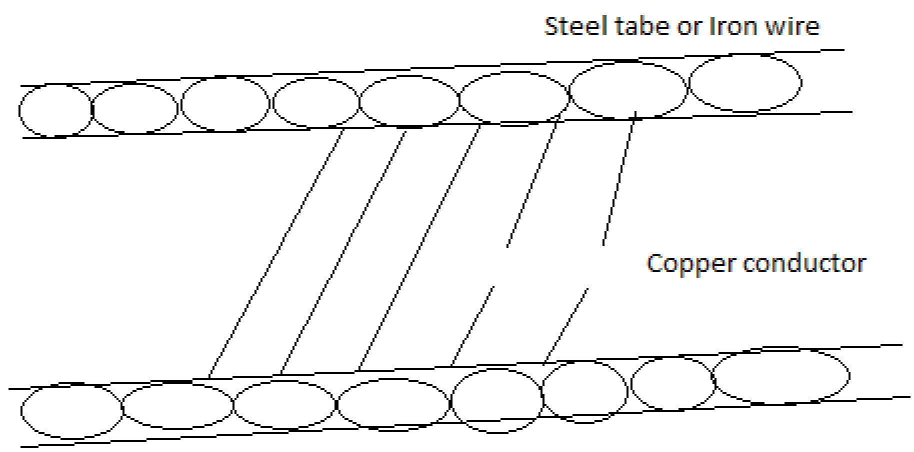

Fig: 1.2.1 Continuous loading coils