

1.7. Bending of beams

A beam is defined as a rod (or) bar of uniform cross section whose length is very much greater than its other dimensions, such as breadth and thickness.

It is commonly used in the construction of bridges to support roofs of the buildings etc. Since the length of the beam is much greater than its other dimensions the shearing stresses are very small.

Assumptions:

- The length of the beam should be large compared to other dimensions.
- The applied load should be large compared to the weight of the beam.
- The cross section of the beam remains constant and hence the geometrical moment of inertia also remains constant.
- The shearing stresses are negligible.

Consider a beam ABCD, which is made up of a large number of thin plane layers are placed one above the other. One end of the beam AD is fixed in the rigid support and a Load is applied in the other end BC.

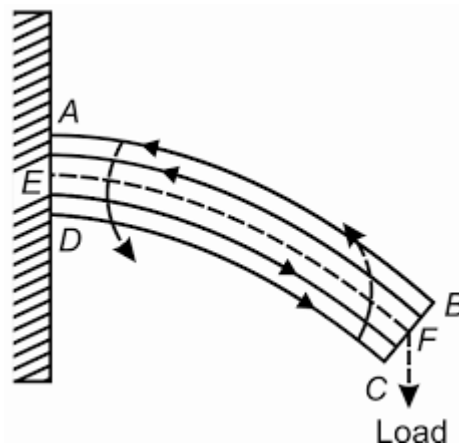


Figure.1.7.1.Bending of beam.

Taking the longitudinal section ABCD of the bent beam, the layers in the upper half are elongated while those in the lower half are compressed.

In the middle there is a layer (MN) which is not elongated or compressed due to bending of the beam. This layer is called the '*neutral surface*' and the line (MN) at which the neutral layer intersects the plane of bending is called the '*neutral axis*'.

The layers below MN are compressed and those above MN are elongated and there will be such pairs of layers one above MN and one below MN experiencing same forces of elongation and compression due to bending and each pair forms a couple.

*The resultant moments of all these internal couples are called the **internal bending moment** and in the equilibrium condition, this is equal to the external bending moment*

1.7.1.INTERNAL BENDING MOMENT OF THE BEAM

When a beam bent, the restoring couple arises. This couple balances the external couple due to external load is called **internal bending moment of the beam**.

At equilibrium,

Restoring couple = Bending couple

A beam may be assumed to consist of a number of parallel longitudinal metallic fibers placed one over the other and are called as filaments. Let the beam be subjected to deforming forces at its ends, due to which it bends. Let us consider a filament AB at the center of the beam. It is found that the filaments (layers) lying above AB gets elongated, while the filaments lying below AB gets compressed.

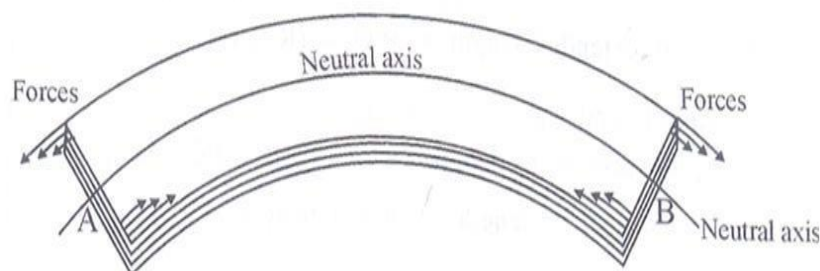


Fig 1.7.1.1 Bending of beam.

Therefore the filament i.e. layer AB which remains unaltered taken as the reference axis called as Neutral axis and the plane is called as neutral plane. Further, the deformation of any filament can be measured with reference to the neutral axis. The moment of couple due to elastic reactions (restoring couple) which balances the bending couple due to applied load is called the bending moment. Let us consider a beam under the action of deforming forces. The beam bends into a circular arc.

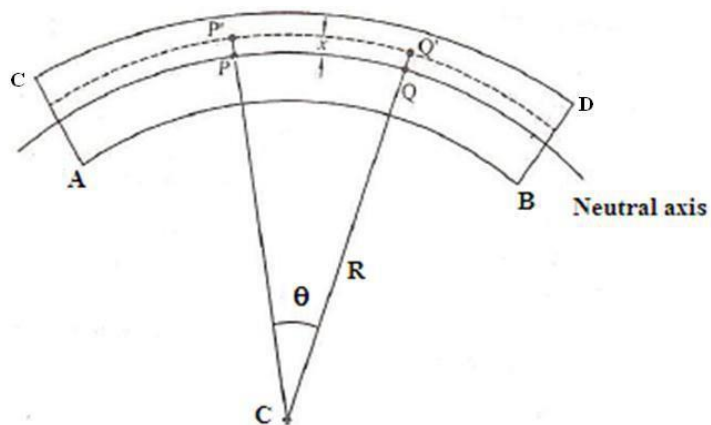


Fig 1.7.1.2 Internal bending moment of the beam.

Let PQ be the neutral axis of the beam and P' Q' be another filament at distance y from PQ. If R is the radius of curvature of the neutral axis and θ is the angle subtended by it at its centre of curvature 'C'.

Then we can write original length PQ = Radius x Angle

$$= R\theta \dots\dots\dots(1)$$

If $R + x$ is the radius of curvature of the filament P'Q'.

$$P'Q' = (R + x) \theta \dots\dots\dots(2)$$

Extension Produced in the filament P'Q' due to bending = P'Q' - PQ

$$= (R + x) \theta - R\theta$$

$$= x\theta \dots\dots\dots(3)$$

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{x\theta}{R\theta} = \frac{x}{R}$$

The Young's modulus of the filament P'Q'

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

Longitudinal stress on the filament P'Q' = Y × longitudinal strain

$$= Y \times \frac{x}{R}$$

If A is the area of cross-section of the filament, then the tensile force on the filament.

$$= \text{Longitudinal stress} \times \text{area}$$

$$= Y \frac{x}{R} \times A$$

We know, moment of longitudinal force about the neutral axis = tensile force × distance

$$= Y \times \frac{x}{R} \times A \times x = \frac{Y}{R} Ax^2$$

$$\text{Moment of all the forces about the neutral axis} = \frac{Y}{R} \sum Ax^2 = \frac{Y}{R} I$$

I_g is the geometrical moment of inertia and it is represented as AK^2

Where A is the area of the cross-section and K is radius of gyration.

In equilibrium,

$$\text{Bending moment of the beam} = \text{Moment of force}$$

$$\text{Internal bending moment of the beam} = \frac{Y}{R} I \dots \dots \dots (4)$$

SPECIAL CASES

Rectangular cross section

If b and d are the breadth and thickness of the beam, then $A = bd$ and $K^2 = \frac{d^2}{12}$

$$I_g = AK^2 = \frac{bd^3}{12}$$

Using the value of $I_g = \frac{bd^3}{12}$ in equation (4)

$$\square \text{Internal bending moment of the rectangular beam} = \frac{bd^3 Y}{12R} \dots \dots \dots (5)$$

Circular cross section

If r be the radius of the beam, then $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

$$I_g = AK^2 = \frac{\pi r^4}{4}$$

Using the value $I_g = \frac{\pi r^4}{4}$ in equation (4)

$$\text{Internal bending moment of the circular beam} = \frac{\pi r^4 Y}{4R} \dots\dots\dots (6)$$

1.7.2. DEPRESSION OF A CANTILEVER

DEFINITION

A light beam clamped horizontally at one end and loaded with a weight $W = Mg$ at the free end is called a **cantilever**.

In equilibrium,

$$\text{External bending moment} = \text{Internal bending moment}$$

THEORY

Let us consider a beam fixed at one end and loaded at its other free end as shown in fig1.7.2.2. Let AB is the neutral axis of a cantilever (a beam or rod) of length ' l ' is fixed at the end A and loaded at the free end B by a weight W . Due to load applied the cantilever is depressed to B' .

Let BB' represents the vertical depression at the free end.

Due to the load applied at the free end, a couple is created between the two forces. (i.e)

- (i) Force (load ' W ') applied at the free end towards downward direction and

(ii) Reaction (R) acting in the upward direction at the supporting end.

This external bending couple tends to bend the beam in the clockwise direction. But, since one end of the beam is fixed, the beam cannot rotate. Therefore the external bending couple must be balanced by another equal and opposite couple, created due to the elastic nature of the body called as internal bending moment.

Consider the section of the cantilever P at a distance 'x' from the fixed end A. Q is another point at a distance dx from P i.e., PQ = dx. It is at a distance (l-x) from the loaded end B'. Considering the equilibrium of the portion PB', there is a force of reaction W of P. Let O be the centre of curvature and R be the radius of curvature.

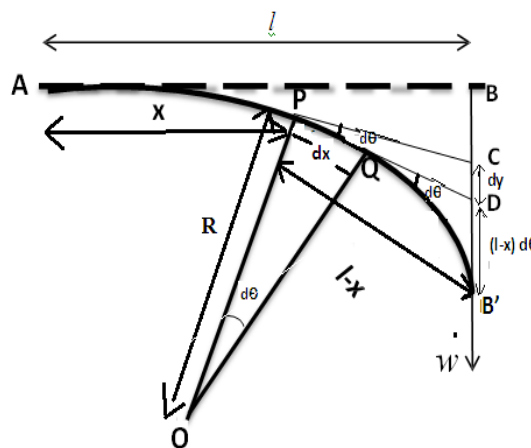


Fig 1.7.2.2. Depression of a cantilever.

•The external bending moment = $W \times PB' = W (l-x)$ (1)

•Internal bending moment of the cantilever = $\frac{Y}{R} I_g$ (2)

Where Y–Young’s modulus of the cantilever.

I_g - Geometrical moment of inertia of its cross-section.

In the equilibrium position,

External bending moment = Internal bending moment

$$W(l-x) = \frac{YI}{R^g}$$

$$R = \frac{YI_g}{W(l-x)} \dots\dots\dots (3)$$

Let Q be another point on the bent cantilever at small distance ‘ dx ’from P. Since P and Q are very near, we can assume that the radius of curvature R is practically the same.

The tangents are drawn at P and Q meeting the vertical line BB’ at C and D. Let dθ be the angle between the tangents at P and Q.

$$\therefore \sin d\theta \approx d\theta = \frac{dx}{R}$$

$$R = \frac{dx}{d\theta} \dots\dots\dots (4)$$

Substituting the value of R from (3) in (4), we have

$$\frac{dx}{d\theta} = \frac{YI_g}{W(l-x)}$$

$$d\theta = \frac{W(l-x)dx}{YI_g} \dots\dots\dots (5)$$

If ‘ dy ’ is the depression due to the curvature at PQ

$$dy = (l-x)d\theta \dots\dots\dots (6)$$

Substituting value of dθ

$$dy = (l-x) \frac{W(l-x)}{YI_g} dx$$

$$dy = \frac{W(l-x)^2}{YI_g} dx \dots\dots\dots (7)$$

To find the total depression at the free end of the cantilever equation (7) has to be integrated from 0 to l .

$$\int dy = \int_0^l W \frac{(l-x)^2}{YI_g} dx$$

$$y = \frac{W}{YI_g} \int_0^l (l-x)^2 dx = \frac{W}{YI_g} \int_0^l (l^2 + x^2 - 2lx) dx$$

$$y = \frac{W}{YI_g} \left[lx + \frac{x^3}{3} - 2l \frac{x^2}{2} \right]_0^l$$

$$y = \frac{W}{YI_g} \left[l^3 + \frac{l^3}{3} - l^3 \right]$$

$$y = \frac{Wl^3}{3YI_g} \dots\dots\dots (8)$$

Equation (8) gives the depression produced by the cantilever at the free end.

SPECIAL CASES

Rectangular cross section

If *b* and *d* are the breadth and thickness of the beam, then $A = bd$ and $K^2 = \frac{d^2}{12}$

Substituting the values of $I_g = AK^2 = I_g = \frac{bd^3}{12}$ and $W = Mg$ in equation (8)

Depression $y = \frac{4Mg l^3}{Ybd^3} \dots\dots\dots (9)$

Circular cross section

If *r* be the radius of the beam, then $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

Substituting the values of $I_g = AK^2 = \frac{\pi r^4}{4}$ and $W = Mg$ in equation (8)

Depression $y = \frac{4Mgl^3}{3Y\pi r^4} \dots\dots\dots (10)$

EXPERIMENTAL DETERMINATION OF YOUNG'S MODULUS BY CANTILEVER DEPRESSION METHOD

CONSTRUCTION

One end of the beam is rigidly clamped at one end to the edge of the table using G-clamp. A tall pin P is fixed vertically to the free end of the bar. A loop of cotton string or a hook is attached to this end of the bar and a weight hanger is

suspended from it. A travelling microscope is focused on the tip of the pin as shown in fig.

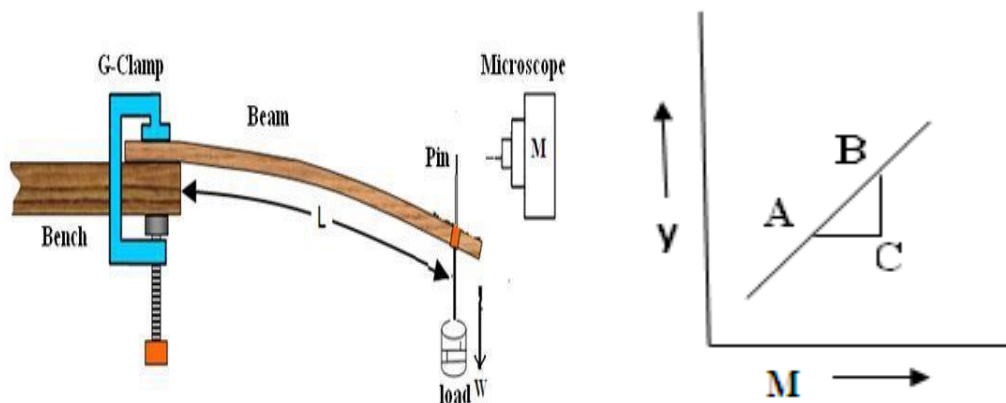


Fig 1.7.2.3 Experiential verification of a cantilever.

PROCEDURE

A dead load without any slotted weights is attached to the hook. The microscope is adjusted such that the horizontal cross wire coincides with the tip of the image of the pin and the reading on the vertical scale is taken. Loads are added to the hanger in steps of 50g and every time, the readings are noted on the vertical scale. A travelling microscope is focused on the tip of the pin as shown in fig. These observations are also repeated while unloading in the same. Steps and the readings are tabulated. The mean depression 'y' for a load 'M' kg is found from the tabulated readings.

The observations are tabulated as follows.

Graphical method

A graph is drawn between the load (M) along X-axis and elevation (y) along Y-axis. It is found to be a straight line as shown in fig. The slope of the straight line gives the value of (y/M).

Load 10^{-3} kg	Microscope readings						Mean 10^{-2} m	depression (y) for 100 g 10^{-2} m	Mean (M/y) Kgm^{-1}
	Loading			Unloading					
	MSRc m	VSR cm	TR cm	MSRc m	VSR Cm	TR Cm			
W									
W+50									
W+100									
W+150									
W+200									

Hence Young's modulus of the cantilever can be calculated as

$$Y = \frac{4gl^3}{bd^3} \left(\frac{M}{y} \right)$$

5.5 Hexagonal Closed Packed Structure (HCP)

The hexagonal closed packed (HCP) structure consists of three layers of atoms. The bottom layer has six corner atoms and one face centred atom. The middle layer has three full atoms. The upper layer has six corner atoms and one face centred atom.

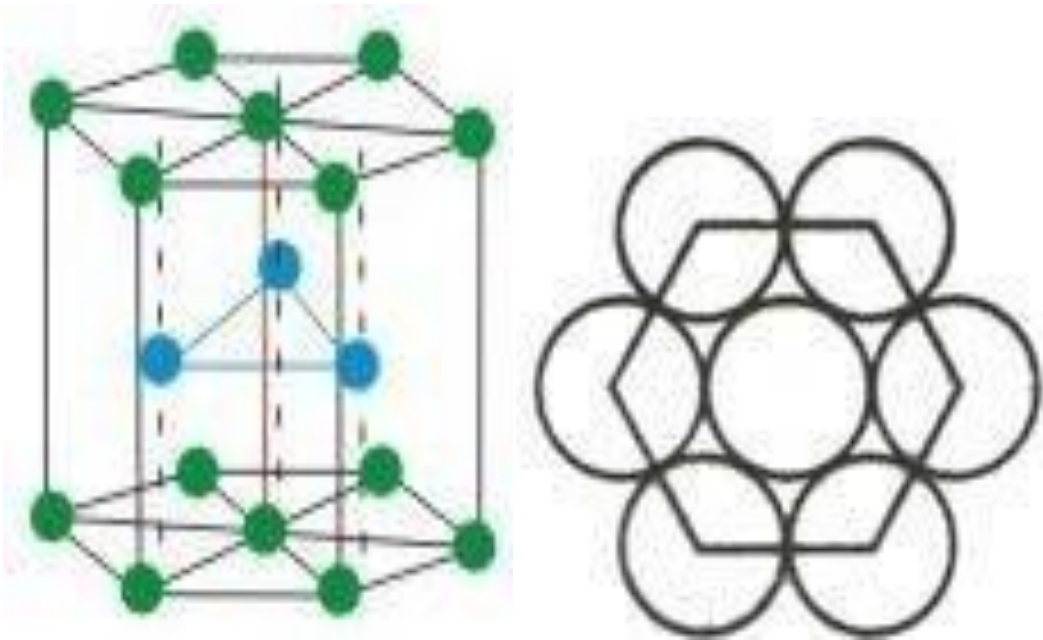


Fig:5.5.1- Hexagonal Closed Packed Structure

Number of atoms per unit cell

In order to calculate the total number of atoms in the hcp structure, let us consider the bottom layer of atoms. The bottom layer consists of six corner atoms and one face centred atom. Each and every corner atom contributes $\frac{1}{6}$ of its part to one unit cell. Thus,

Hence,

the total number of atoms contributed by the face centred atom is $= \frac{1}{6} \times 6 = 1$

The face centred atom contributes $\frac{1}{2}$ of its part to one unit cell. Therefore, the total number of atoms present in the case of the bottom layer is $= 1 + \frac{1}{2} = \frac{3}{2}$

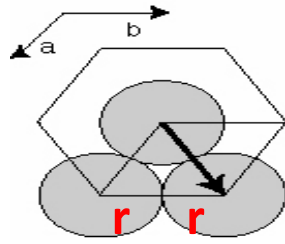
Similarly, the upper layer also has $\frac{3}{2}$ number of atoms. The middle layer has three full atoms. Therefore, the total number of atoms present in a unit cell is $= \frac{3}{2} + \frac{3}{2} + 3 = 6$

The total number of atoms present in the case of hcp crystal structure is six.

Atomic Radius

To find the atomic radius of HCP structure, consider any two corner atoms. It has to

be noted that, each and every corner atom touches



each other; therefore they are the nearest neighbours.

From figure,

We can write $a = 2r$

Atomic radius $r = \frac{a}{2}$

Co-Ordination Number

The HCP structure is considered to have three layers via. (i) Bottom layer (B_L), (ii) Top layer (T_L), (iii) Middle layer (M_L) as shown in figure.

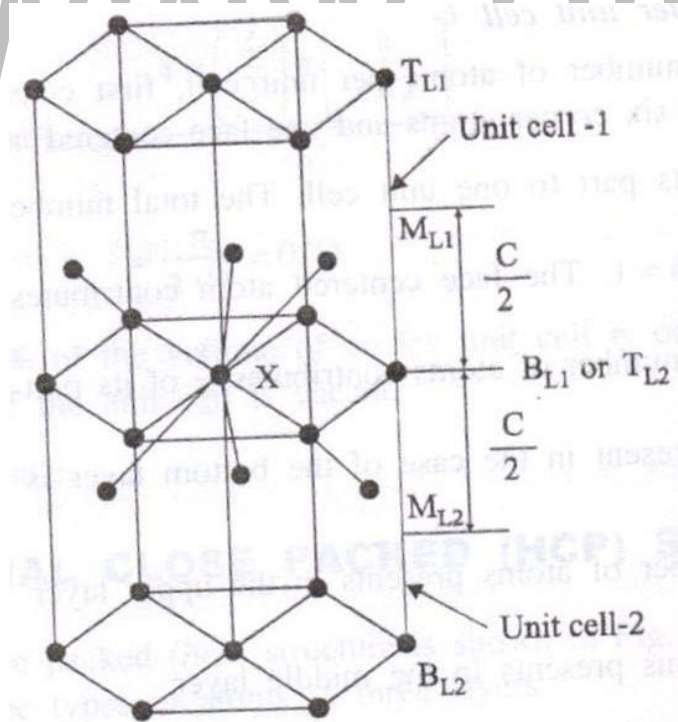


Fig:5.5.2- Hexagonal Closed Packed Structure

In the top and bottom layers, the base centred atom is surrounded by six corner atoms. In the middle layer we have 3 atoms stacked inside the unit cell as shown in figure.

Let us consider two unit cells as shown in figure. Let 'X' be the reference atom taken in the bottom layer (BL₁) of unit cell – I (or top layer [TL₂] of unit cell 2). This atom has 6 neighbouring atom in its own plane. Further at a distance of $c/2$ it has 3 atoms in the middle layer (ML₁) of unit cell-I and 3 more atoms in the middle layer (ML₂) of unit cell-2. Therefore, the total numbers of neighbouring atoms are $6+3+3=12$.

Relation between 'c' and 'a' [c/a ratio]

In hexagonal system, 'c' is the height of the unit cell of HCP structure and 'a' is the distance between two neighbouring atoms. Now

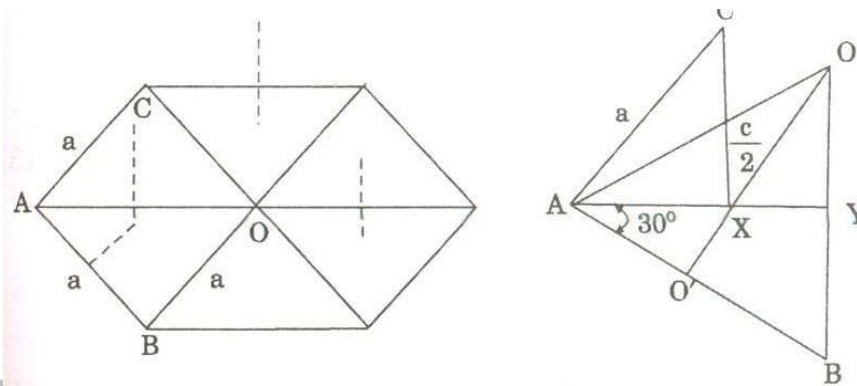


Fig.5.5.3- layer of HCP Structure

consider the triangle ABO in the bottom layer. Here A, B and O are the lattice points and exactly above these atoms at a perpendicular distance ' $c/2$ ' the next layer atom lies at C.

In Δ ABY,

$$\cos 30^\circ = \frac{AY}{AB}$$

(or) $AY = AB \cos 30^\circ$

Since, $AB = a$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$AY = a \frac{\sqrt{3}}{2} \text{----- (1)}$$

But from figure, $AX = \frac{2}{3} AY$
=

Substituting the AY value in the above equation, we get

$$AX = \frac{2}{3} a \frac{\sqrt{3}}{2}$$

$$AX = \frac{a}{\sqrt{3}} \quad (2)$$

In ΔAXC

$$AC^2 = AX^2 + CX^2$$

Substituting the given values in the above equation we get,

$$AC = a; AX = \frac{a}{\sqrt{3}} \text{ and } CX = \frac{c}{2}, \text{ we get}$$

$$a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left(\frac{c}{2}\right)^2$$

$$a^2 = \frac{a^2}{3} + \frac{c^2}{4}$$

$$\frac{c^2}{4} = a^2 - \frac{a^2}{3}$$

$$\frac{c^2}{4} = \frac{3a^2 - a^2}{3}$$

$$\frac{c^2}{4} = \frac{2a^2}{3}$$

$$\frac{c^2}{a^2} = \frac{8}{3} \quad (3)$$

Taking square root on both sides of equation (3) we get,

$$\sqrt{\frac{c^2}{a^2}} = \sqrt{\frac{8}{3}}$$

$$\frac{c}{a} = \sqrt{\frac{8}{3}} = 1.633 \quad (4)$$

Atomic Packing Factor (APF)

We know that,

$$APF = \frac{u}{v} \dots\dots\dots (5)$$

Where

u = Total number of atoms per unit cell X volume of one atom

v = Total volume of the unit cell

The number of atoms per unit cell in HCP structure = 6

Volume of one atom (spherical) is = $\frac{4}{3} \pi r^3$

We know that, the atomic radius of HCP is $r = \frac{a}{2}$(6)

Therefore,

Volume occupied by the total number of atoms per unit cell is, $U = 6 \times \frac{4}{3} \pi r^3 \dots\dots\dots (7)$

Substitute equation (6) in (7) we get

$$U = \frac{24}{3} \pi \left(\frac{a}{2}\right)^3$$

$$= \frac{24\pi a^3}{24}$$

$$= \pi a^3 \dots\dots\dots (8)$$

For HCP,

The volume of the unit cell is

$$= \text{Area of the base} \times \text{Height} \dots\dots\dots (9)$$

Area of the base = Area of six triangles

Area of the base = 6 × area of the one triangle (AOB)

From figure,

The area of the triangle AOB = $\frac{1}{2} (BO) \times (AY)$

Substituting the value of $BO = a$ and $AY = \frac{a\sqrt{3}}{2}$ we have

$$\text{Area of triangle AOB} = \frac{1}{2} (a) \frac{a\sqrt{3}}{2}$$

$$\frac{a^2\sqrt{3}}{4} \quad \text{Area of the}$$

$$\text{base} = 6 \times \frac{a^2\sqrt{3}}{4}$$

$$\text{Area of the base} = \frac{3a^2\sqrt{3}}{2} \quad \text{-----} \quad (10)$$

We know that,

$$\text{The height of the hcp structure} = c \quad \text{-----} \quad (11)$$

Substituting equations (10) and (11) in equation (9) we get

$$\text{Volume of the unit cell is } (v) = \frac{3a^2\sqrt{3}}{2} c$$

Substituting equations (8) and (12) in equation (5), we get

$$\text{Atomic packing factor} = \frac{\pi a^3}{\frac{3a^2\sqrt{3}c}{2}}$$

$$\frac{2\pi}{3\sqrt{3}} \left[\frac{a}{c} \right]^3$$

since $\frac{a}{c} = \left[\frac{\sqrt{2}}{3} \right]^2$ we get

$$\text{Atomic packing factor} = \frac{2\pi}{3\sqrt{3}} \left[\frac{\sqrt{2}}{3} \right]^3$$

$$\begin{aligned} \text{Atomic packing factor} &= \frac{2\pi \sqrt{3}}{3\sqrt{3} \cdot 2\sqrt{2}} \\ &= \frac{\pi}{3\sqrt{2}} \end{aligned}$$

$$\text{Atomic packing factor} = 0.74$$

Therefore we can say that 74% volume of the unit cell of HCP is occupied by atoms and remaining 26% volume is vacant. Thus, the packing density is 74%, which is the same as that of FCC structure. Hence, HCP structure is also termed as tightly or closely packed structure.

UNIT – 1

PROPERTIES OF MATTER (Elasticity)

Elasticity – Stress-strain diagram and its uses – factors affecting elastic modulus and tensile strength. Torsional stress and deformations – twisting couple – torsion pendulum: theory and experiment – bending of beams. Bending moment – cantilever: theory and experiment. Uniform and non-uniform bending: theory and experiment – I-shaped girders – stress due to bending in beams

Introduction of Elasticity

Elasticity is a branch of physics which deals with the elastic property of materials. When an external force is applied to a body, there will be some change in its length, shape and volume. When the external force is removed, if the body regains its original shape and size, then the body is said to be a perfectly elastic body.

If the body does not regain its original shape (or) size, after the removal of the applied force, then it is said to be a perfectly plastic body. In nature no body is perfectly elastic (or) perfectly plastic.

Therefore elasticity is the property of the body by virtue of which it tends to regain its original shape (or) size after the removal of deforming forces applied externally to it.

1.1. Fundamental definitions

1.1.1. Deforming force

The force which changes or tries to change the shape or size of a body without moving it as a whole is called deforming force.

1.1.2. Elasticity

Elasticity is the properties of solid material to regain their original shape or size after remove the deforming force acting on it.

Example: Rubber, a fiber of quartz crystal.

- Elasticity is the molecular property of matter.
- It is different in different materials.

1.1.3. Plasticity

If the body does not regain its original shape and size, when the applied force is removed, then it is a perfect plastic body. This property is called plasticity.

Example: Putty, Wood, Mud.

1.1.4. Stress

The restoring force acting per unit area inside the body is known as stress.

Stress = restoring force/area

$$= F/A \text{ (Nm}^{-2}\text{)}$$

This restoring force is equal and opposite of the applied force. Therefore, stress is also defined as;

The applied force per unit cross sectional area of the body is called stress.

Types of stress

- Linear (longitudinal)stress
- Shearing (tangential)stress
- Volume (bulk)stress

1.1.5. Strain

Relative change in dimension or shape of a body by the applied force is called strain. Strain is also measured by the ratio of change in dimension to original dimension.

Types of strain

- Linear (longitudinal)strain
- Shearing (tangential)strain
- Volume (bulk)strain

1.1.6. Hooke'sLaw

It states that “*Within the elastic limit, stress in a body is directly proportional to strain produced on it*”

1.2. Stress – Strain diagram

Consider a wire which is rigidly fixed at one end and gradually loaded at the other end. The corresponding strain produced for the different loads are noted until the wire breaks down. A graph is plotted between strains along X- axis and stress along Y- axis. This graph is known as **stress - strain diagram**.

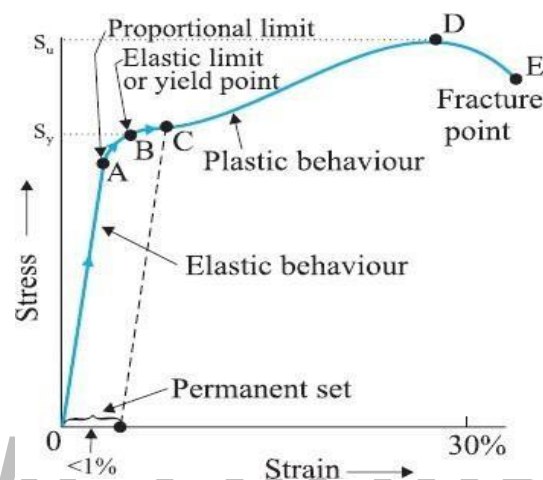


Fig.1.2.1 stress-strain diagram.

- In the region between O to A, the curve is linear. In this region, **Hooke's law** is obeyed and the solid behaves as an elastic body.
- In the region from A to B, stress and strain are not proportional. The point B in the curve is known as **yield point** (also known as **elastic limit**) and the corresponding stress is known as yield strength (S_y) of the material.
- If the load is increased further, strain increases rapidly even for a small change in the stress. When the load is removed at point C, the body does not regain its original shape. In this case, even when the stress is zero, the strain is not zero. The material is said to have a **permanent set**. The deformation is said to be **plastic deformation**.

- At the point D on the graph, the stress developed is maximum, which is called as **ultimate tensile strength (S_u)** of the material.
- Beyond this point, additional strain is produced even by a reduced applied force and **fracture occurs at point E**. If the ultimate strength and fracture points **are close, the material is said to be brittle** and if they are **far, the material is said to be ductile**.

Uses of stress – strain diagram

1. It is used to categorize the materials (ductile, brittle and plastic).
2. It is used to found the amount of deformation.
3. It is used to calculate the strength of the materials

1.3. Factors affecting Elasticity

It is found that some bodies lose their elastic property even within the elastic limit, due to elastic fatigue. Therefore, the manufacturer should choose the material in such a way that it should regain its elastic property even when it is subjected to large number of cycles of stresses. *The temporary loss of elastic property of a body by the continuous applied strain or stress is called elastic fatigue.*

Apart from elastic fatigue, some materials will have change in their elastic property because of the following factors.

- Effect of stress
 - Effect of change in temperature
 - Effect of impurities
 - Effect of hammering, rolling and annealing
 - Effect of crystalline nature
- **Effect of stress**

When a material is subjected to large number of cycles of stresses, it loses its elastic property even within the elastic limit. Therefore, the working stress on the material should be kept lower than the ultimate tensile strength.

- **Effect of change in temperature**

A change in temperature affects the elastic properties of a material. Normally the elasticity increases with the decrease in temperature and vice versa. For example, lead is not a good elastic material

- **Effect of impurities**

1. The elastic property of a material is either increased or decreased due to the addition of impurities. It depends upon the elastic properties of the impurity.

2. For example, an elastic property of gold is increases by the addition of potassium (or) copper in it.

- **Effect of hammering, rolling and annealing**

1. The crystal grains are break up into small unit by hammering and rolling, which turns increase the elastic properties of a body.

2. Annealing is a process to heat and cool the body gradually and it increase the crystalline size, which is ultimately reduces the elastic property of a body.

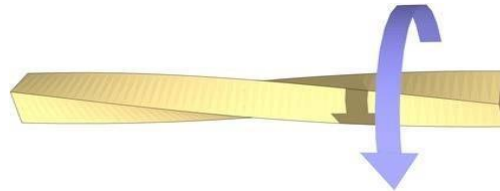
- **Effect of crystalline nature**

The modulus of elasticity of a body is more when it is in single crystal and in polycrystalline state it is comparatively small.

TORSIONAL STRESS AND DEFORMATION

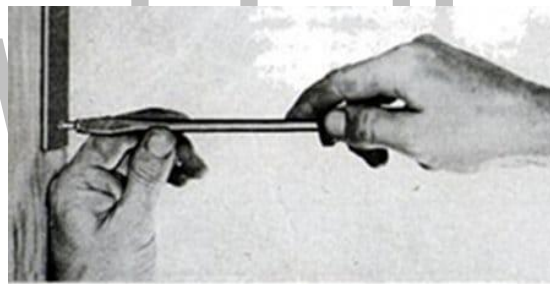
1.4. Torsional Stress and Deformations

Shear stress is produced about a longitudinal axis of a structural member by the application of twisting couple to the end of the structural member is known as



torsional stress. Torsion is the twisting of a straight bar when it is loaded by twisting couple or torque. It tends to produce rotation about the longitudinal axes of the bar.

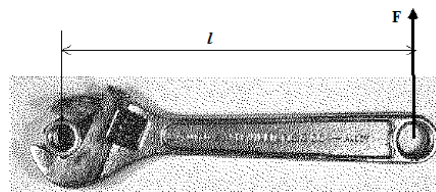
For instant, when we turn a screw driver to produce torsion our hand applies torque “ T ” to the handle and twist the shank of the screw driver.



1.5. Twisting Couple (Torque)

The twisting of a structural member about its longitudinal axis by two equals and opposite torques is expressed through a certain angle is called twisting couple.

The stress is produced in this process is not tensile or compressive, it is said to be



shearing or shear stress. The strain is measured by an angle in unit of radians.

The simple example is that of using a wrench to tighten a nut on a bolt as shown in given figure. If the bolt, wrench, and force are all perpendicular to one another.

1.6. TWISTING COUPLE ON A WIRE

If we have a wire or cylinder, clamped at one end, and twisted at the other through an angle θ , about its axis, it is said to be under tension, due to the elasticity of the material of the wire or the cylinder, a restoring couple is set up in it, equal and opposite to the twisting couple.

Consider a cylindrical wire of length l and radius a . The cylindrical wire is clamped to a fixed support. This wire is made up of a number of cylindrical tubes (co-axial) whose radii vary from zero to „ r “. Let us consider one such cylinder, as shown in fig (b) with radius „ x “ and thickness „ dx “.

Let AB be a line on the elementary tube which is parallel to the axis of the tube. Consider a couple applied at the bottom end of the wire, which results in twisting of the wire through an angle θ . In the twisted state, the position of AB will be taken as AB" as shown in fig 1.6.1

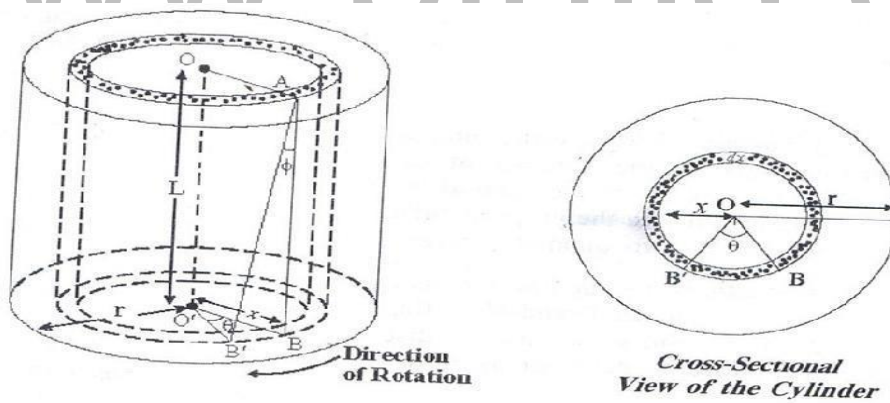


Fig (a)

Fig (b)

Fig 1.6.1 Twisting couple on a wire.

From fig (a)

The angle $ABB' = \phi$

$$BB' = l\phi \dots\dots\dots (1)$$

The displacement decreases, as the move from the rim of the cylinder to the Centre. At the Centre, it will become zero. This means that shearing strain is maximum at the rim and minimum at the Centre.

Consider a hollow cylinder along the plane AB and flattened out. Therefore, we get a rectangle OABO' before twisting and OABB' after twisting. The angle through which this the hollow cylinder is sheared.

From the cross sectional view of the cylinder

$$BB' = x\theta \quad \dots\dots\dots (2)$$

From equations (1) and (2), we get,

$$l\phi = x\theta$$

$$\text{Shearing strain } \phi = \frac{x\theta}{l} \quad (3)$$

We know that,

$$\text{Rigidity modulus (n)} = \frac{\text{shearing stress}}{\text{shearing strain}}$$

$$\text{Shearing stress} = n \times \text{Shearing strain}$$

$$= n \frac{x\theta}{l} \quad \dots\dots\dots (4)$$

Let the area of the elementary tube = $2\pi x dx$

The shearing force on this area = Shearing stress \times Area

$$= n \frac{x\theta}{l} \times 2\pi x dx$$

$$= \frac{2\pi n \theta x^2 dx}{l} \quad \dots\dots\dots (5)$$

The moment of force about the axis of the wire = $\frac{2\pi n \theta x^2 dx}{l} \times x$

$$= \frac{2\pi n \theta x^3 dx}{l} \quad \dots\dots\dots (6)$$

The twisting couple applied to the whole wire can be obtained by integrating equation 6 between the limit $x=0$ and $x=r$

$$= \int_0^r \frac{2\pi n \theta x^3 dx}{l}$$

$$= \frac{2\pi n\theta^r}{l} \int_0^r x^3 dx$$

$$= \frac{2\pi n \theta}{l} \frac{x^4}{4} \Big|_0^r$$

$$\boxed{C = \frac{1}{2} \frac{n\pi r^4 \theta}{l}} \dots\dots\dots (7)$$

If $\theta=1$ radian

The twisting couples per unit angular twist of the wire

$$\boxed{C = \frac{1}{2} \frac{n\pi r^4}{l}}$$

1.6.1. TORSIONAL PENDULUM

When a body is fixed at one end and twisted about its axis by means of a torque at the other end, then the body is said to be under torsion. The torsion involves shearing strain and hence modulus involved is the rigidity modulus.

Torsional pendulum consists of a suspension wire with one end is fixed and the other end is fixed to the center of the circular disc. Let l be the length of the suspension wire and r be the radius of the suspension wire. When a heavy circular disc is rotated in a horizontal plane, so that the wire is twisted through an angle θ . The various elements of the wire will undergo shearing strain and restoring couple is produced. Now if the disc is released, the disc will produce torsional oscillations. The couple acting on the disc produces an angular acceleration in it, which is proportional to the angular displacement and is always directed towards its mean position.

Total energy of the torsion pendulum = P.E +K.E

The potential energy confined to the wire is equal to the work done in twisting the disc, thereby creating a restoring couple (C).

The restoring couple through an angle $\theta = \int_0^\theta \text{moment of couple} \times d\theta$

$$= \int_0^{\theta} C\theta d\theta = \frac{C\theta^2}{2} \dots\dots\dots (8)$$

Let ω be the angular velocity with which the disc oscillates, due to the restoring couple, then

The kinetic energy of the rotating disc (deflecting couple) = $\frac{I\omega^2}{2}$ (9)

I be the moment of inertia of the circular disc

Total energy = $\frac{C\theta^2}{2} + \frac{I\omega^2}{2} = \text{constant}$ (10)

Differentiating equation (4) with respect to „t“ we get,

$$C\theta \frac{d\theta}{dt} + I\omega \frac{d\omega}{dt} = 0 \dots\dots\dots (11)$$

Since the angular velocity $\omega = \frac{d\theta}{dt}$ and the angular acceleration $\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

□Equation (11) becomes

$$C\theta \frac{d\theta}{dt} + I \frac{d\theta}{dt} \frac{d^2\theta}{dt^2} = 0$$

$$\frac{d\theta}{dt} \left[C\theta + I \frac{d^2\theta}{dt^2} \right] = 0$$

Here $\frac{d\theta}{dt} \neq 0 \quad \therefore \left[C\theta + I \frac{d^2\theta}{dt^2} \right] = 0$

\therefore Angular acceleration = $\frac{d^2\theta}{dt^2} = -\frac{C\theta}{I}$ (12)

The negative sign indicates that the couple tends to decrease the twist on the wire.

TIME PERIOD OF TORSIONAL OSCILLATION

The time period of torsional oscillation $T = 2\pi \frac{\text{angular displacement}}{\text{angular acceleration}}$

$$T = 2\pi \sqrt{\frac{\theta}{C\theta/I}} = 2\pi \sqrt{\frac{I}{C}} \dots\dots\dots (13)$$

Frequency of oscillation $f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{C}{I}}$
--

..... (14)

RIGIDITY MODULUS OF THE WIRE

Let l be the length of the suspension wire and r be the radius of the suspension wire. We know

The twisting couple per unit angular twist of the wire $C = \frac{1}{2} \frac{n\pi r^4}{l}$ (15)

Substituting equation (9) in equation (7) we get

$$T = 2\pi \sqrt{\frac{I}{n\pi r^4 / 2l}} = 2\pi \sqrt{\frac{2Il}{n\pi r^4}}$$

$$T^2 = 4\pi^2 \frac{2Il}{n\pi r^4} = \frac{8\pi Il}{nr^4} \dots\dots\dots (16)$$

Rigidity modulus of the wire $n = \frac{8\pi Il}{T^2 r^4}$
--

Thus torsional pendulum is used to find the rigidity modulus of the various materials.

Experimental verification of torsional pendulum

A torsion pendulum is constructed as shown in Figure.

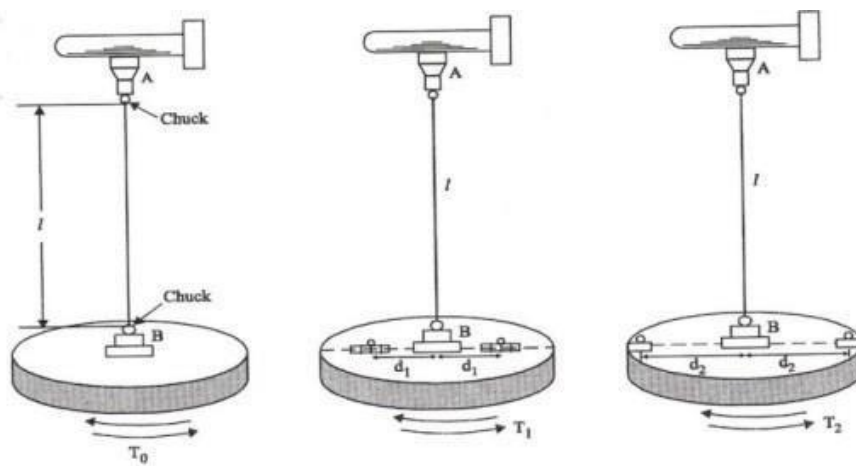


Fig.1.6.1.2 Torsion pendulum

Measure carefully the length of the suspension wire between the two chucks. Standing in front of the pendulum, gently set it in torsional oscillation without any lateral movement.

Note the time for 10 oscillations. T_0 , the period of oscillation of the pendulum without any masses in it calculated. Take two readings. Find the mean.

Two equal symmetrical masses (m) are placed on the disc on either side, close to the suspension wire. The closest distance ' d_1 ' from the center of the symmetrical mass and the center of the suspension wire is found. Set the pendulum to oscillate and note the time for 10 oscillations. From that the period of oscillation T_1 is calculated. Take two readings find the mean.

Two equal masses are now moved to the extreme ends so that the edges of masses coincide with the edge of the disc and the centers are equaled distant. The distance „ d_2 “ from the center of the symmetrical mass and the center of the suspension wire is noted. Set the pendulum to oscillate and note the time for 10 oscillations. Take two readings. Calculate the mean period of oscillation T_2 . All the measured parameters are tabulated in the given table,

Position of the Symmetrical masses	Time taken for 10 oscillations			Time period	Square of the time period
	Trail 1	Trail 2	Mean		
Unit	s	s	s	s	s
Without any Masses				$T_0 =$	$T_0^2 =$
With masses at closest distance $d_1 = \dots \times 10^{-2} \text{ m}$				$T_1 =$	$T_1^2 =$
With masses at farther distance $d_2 = \dots \times 10^{-2} \text{ m}$				$T_2 =$	$T_2^2 =$

Measure carefully, the diameter ($2r$) of the wire at various places, with a screw gauge. Find the mean of the diameter and calculate the radius. Note the mass (m) of the one symmetrical mass. The moment of inertia of the disc and rigidity modulus of the wire is calculated using the formula

$$n = \frac{\dots}{\dots}$$

1.8. UNIFORM BENDING

ELEVATION AT THE CENTRE OF THE BEAM LOADED AT BOTH ENDS

DEFINITION

The beam is loaded uniformly on its both ends; the bent beam forms an arc of a circle. The elevation is produced in the beam. This type of bending is known as uniform bending.

THEORY

Let us consider a beam AB of negligible mass, supported symmetrically on two knife edges C and D. It is loaded with equal weight W at each end. Let 'l' be the length between the two knife edges and 'a' be the length between the knife edge and the load. Due to the applied load the beam bends into an arc of circle and produces an elevation 'y'. Let P be any section of the beam.

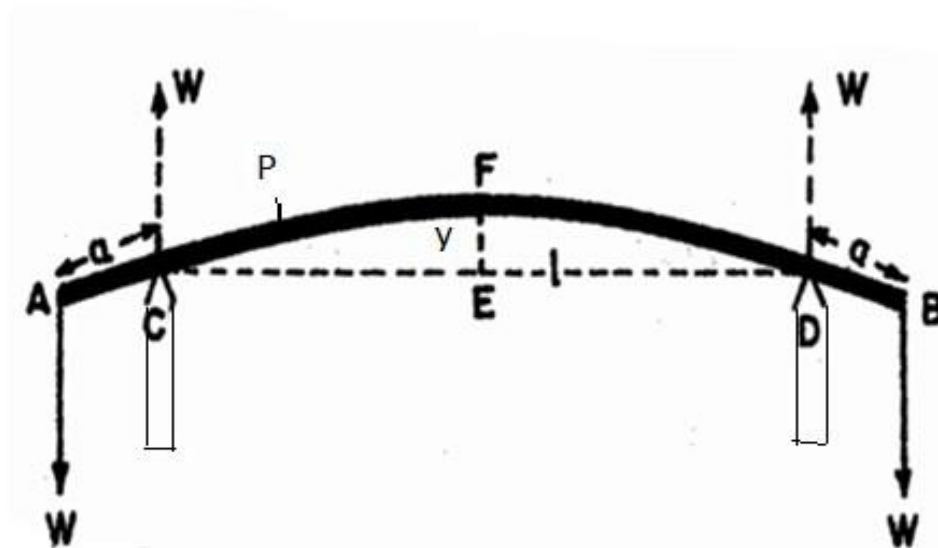


Fig 1.8.1 Young's modulus of the beam by uniform bending.

At the equilibrium position of the section PA of the beam two equal forces, the applied load W at A (downward) and the normal reaction W at C (upward) are acting in the opposite direction constitute a couple.

$$\begin{aligned} \text{The External bending moment} &= (W \times AE) - (W \times CE) \\ &= W(AE - CE) \\ &= W \times AC = Wa \quad \dots\dots\dots (1) \end{aligned}$$

Where, R is the radius of curvature.

$$\text{Internal bending moment} = \frac{Y}{R} I_g \quad \dots\dots\dots (2)$$

Where Y—Young’s modulus of the cantilever.

I_g - Geometrical moment of inertia of its cross-section.

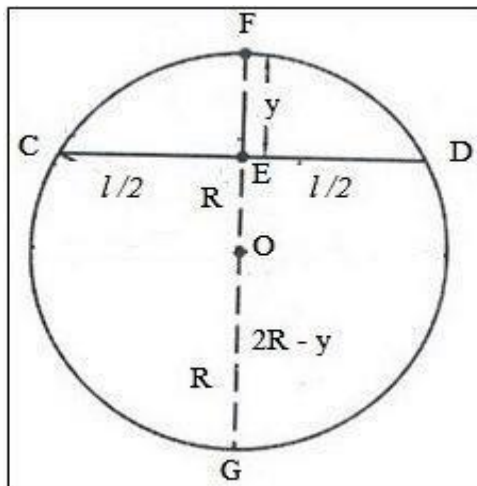
R - Radius of the curvature of the neutral axis at P.

At Equilibrium,

External bending moment = Internal bending moment

$$Wa = \frac{Y}{R} I_g \quad \dots\dots\dots (3)$$

Since, Wa is a constant, R is also constant. Therefore the beam bends into an arc of a circle of radius R. Hence the bending in this case is said to be uniform.



From the property of circle, $CE \times ED = EF \times EG$

$$\frac{l}{2} \times \frac{l}{2} = y(2R - y)$$

$$\frac{l^2}{4} = 2Ry - y^2 \quad \dots\dots\dots (4)$$

Here y is the elevation produced in the beam. Since the elevation is very small compared to the radius of curvature of the circle, y^2 can be neglected.

$$\frac{l^2}{4} = 2Ry$$

$$R = \frac{l^2}{8y} \quad \dots\dots\dots (5)$$

Substituting equation (5) in equation (3)

$$Wa = \left[\frac{YI_g}{8y} \right]$$

Young's modulus by uniform bending $Y = \frac{Wal^2}{8yI_g} \quad \dots\dots\dots (6)$

SPECIAL CASES

RECTANGULAR CROSS SECTION

If b and d are the breadth and thickness of the beam, then $A = bd$ and $K^2 = \frac{d^2}{12}$

Substituting the values of $I_g = AK^2 = I_g = \frac{bd^3}{12}$ and $W = Mg$ in equation (6)

$$Y = \frac{3Mgal^2}{2bd^3y} \quad \dots\dots\dots (7)$$

Young's modulus of the material of the beam by uniform bending can be calculated as using the relation (7).

CIRCULAR CROSS SECTION

If r be the radius of the beam, then $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

Substituting the values of $I_g = AK^2 = \frac{\pi r^4}{4}$ and $W = Mg$ in equation (6)

$$Y = \frac{Wal^2}{8yI_g} = \frac{Mgal^2}{2y\pi r^4} \quad \dots\dots\dots (8)$$

EXPERIMENTAL DETERMINATION

CONSTRUCTION

A rectangular beam of uniform –section is supported horizontally on two knife –edges A and B as shown in Figure. Two weight hangers of equal masses are suspended from the ends of the beam. A pin is arranged vertically at the mid-point of the beam. A microscope is focused on the tip of the pin.

PROCEDURE

Dead loads are attached to the hangers. The microscope is adjusted such that the horizontal cross-wire coincides with the tip of the image of the pin and the readings on the vertical scale are taken. Equal weights in steps of 50g are added to both hangers simultaneously and the reading of the microscope in the vertical scale is noted. The experiment is repeated for decreasing order of magnitude of the equal masses. The observations are then tabulated and the mean elevation(y) at the mid-point of the bar is determined. The length of the bar between the knife edges ' l ' is measured. The distance of one of the weight hangers from the nearest knife edge ' a ' is measured. The breadth (b) and thickness (d) of the bar are measured by using vernier calipers and screw gauge. The young's modulus of the material of the beam is determined by the relation.

$$Y = \frac{3M g a l^2}{2b d^3 y}$$

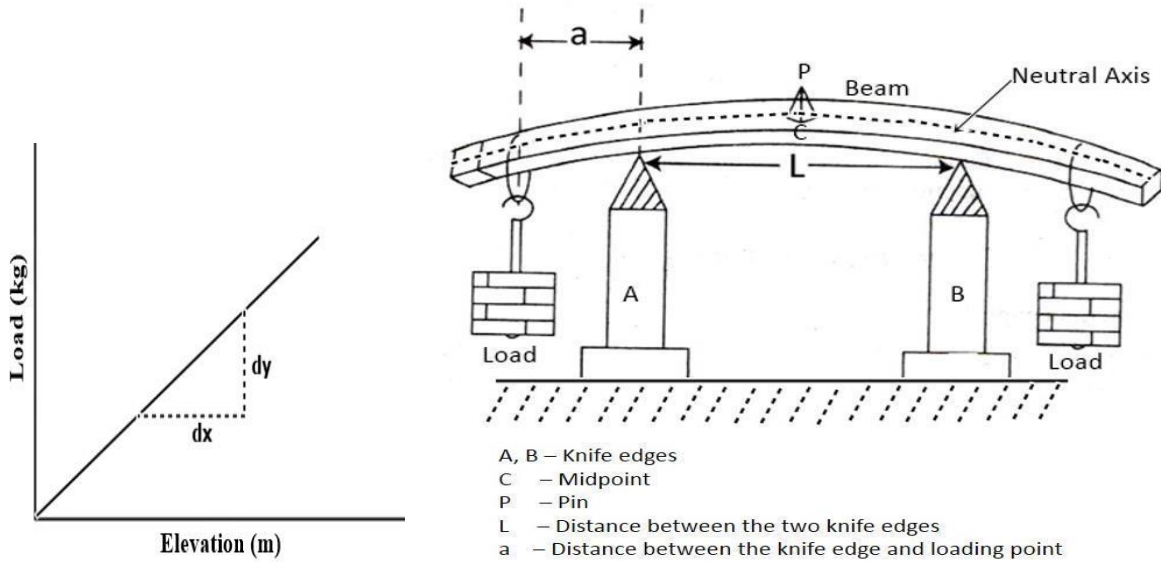


Fig 1.8.3(a) Graphical representation of Young's modulus of the beam by uniform bending.

Fig1.8.3 (b) Experimental arrangement.

Table: Experimental verification of Young's modulus by uniform bending

Load 10^{-3} kg	Microscope readings						Mean 10^{-2} m	elevation (y) for 100 g 10^{-2} m	Mean (M/y) Kgm^{-1}
	Loading			Unloading					
	MSR cm	VSR cm	TR cm	MSR cm	VSR cm	TR Cm			
W									
W+50									
W+100									
W+150									
W+200									

1.9. NON-UNIFORM BENDING

DEFINITION

If the beam is loaded at its midpoint, the depression is produced will not form an arc of a circle. This type of bending is called non uniform bending.

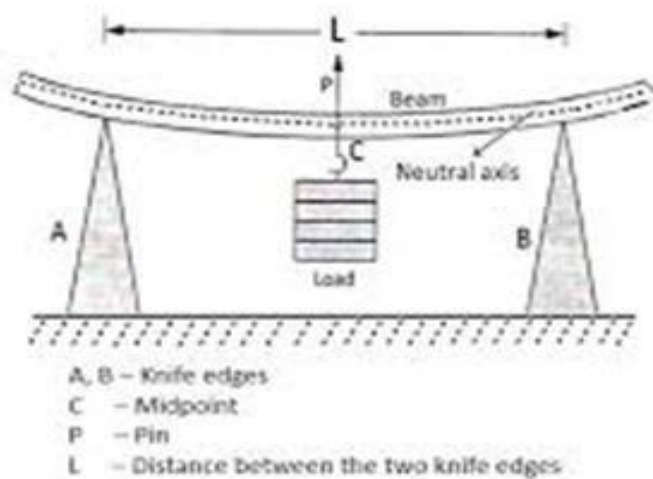


Fig1.9.1. Young's modulus of the beam by non-uniform bending.

THEORY

Consider a uniform beam AB of length L arranged horizontally on two knife edges at its ends A and B. Let it be loaded in the midpoint of the beam with a weight W. The reaction at each Knife edge is equal to $\left(\frac{W}{2}\right)$ in the upward direction. Let 'y' be the depression at the midpoint C.

The bent beam is considered as equivalent to two inverted cantilevers fixed at C each of length $\left(\frac{l}{2}\right)$ and each loaded at A and B with a weight $\left(\frac{W}{2}\right)$ acting vertically upwards. In the case of a

cantilever of length (l) and load (w), the depression = $\frac{Wl^3}{3YI_g}$

Hence for cantilever of length $\left(\frac{l}{2}\right)$ and load $\left(\frac{W}{2}\right)$ depression is $y = \frac{\left(\frac{W}{2}\right)\left(\frac{l}{2}\right)^3}{3YI_g}$

$$y = \frac{Wl^3}{48YI_g} \dots\dots\dots (1)$$

Young's modulus by non-uniform bending $Y = \frac{Wl^3}{48 y I_g} \dots\dots\dots (2)$

SPECIAL CASES

RECTANGULAR CROSS SECTION

If b and d are the breadth and thickness of the beam, then $A = bd$ and $K^2 = \frac{d^2}{12}$

Substituting the values of $I_g = AK^2 = I_g = \frac{bd^3}{12}$ and $W = Mg$ in equation (2)

$$\text{Equation (2)} \rightarrow Y = \frac{Wl^3}{48 y I_g} = \frac{Mgl^3}{48y \left(\frac{bd^3}{12} \right)}$$

$\therefore Y = \frac{Mgl^3}{4bd^3y} \text{ Nm}^{-2} \dots\dots\dots (3)$

Young's modulus of the material of the rectangular beam by non-uniform bending can be calculated as using the relation (3).

CIRCULAR CROSS SECTION

If r be the radius of the beam, then $A = \pi r^2$ and $K^2 = \frac{r^2}{4}$

Substituting the values of $I_g = AK^2 = \frac{\pi r^4}{4}$ and $W = Mg$ in equation (2)

$$Y = \frac{Wl^3}{48 y I_g} \Rightarrow Y = \frac{M g l^3}{12 y \pi r^4} \dots\dots\dots (4)$$

Young's modulus of the material of the circular beam by non-uniform bending can be calculated as using the above relation (4).

EXPERIMENT ARRANGEMENT

The given beam AB of rectangular cross section is arranged horizontally on two knife edges k_1 and k_2 near the ends A and B. A weight hanger is suspended and a pin is fixed vertically at midpoint O. A microscope is focused on the tip of the pin. The initial reading on the vertical scale of the microscope is taken. A suitable mass M is added to the hanger. The beam is depressed. The cross wire is adjusted to coincide with the tip of the pin. The reading of the microscope is noted. The depression corresponding to the mass M is found.

The experiment is repeated by increasing and decreasing the mass step by step. The corresponding readings are tabulated. The average value of depression y is found from the observation.

Table:1.6 Experimental verification of Young’s modulus by non-uniform bending

Load 10^{-3} kg	Microscope readings						depression (y) for 100 g 10^{-2} m	Mean (M/y) Kgm^{-1}	
	Loading			Unloading					Mean 10^{-2} m
	MSR cm	VSR cm	TR cm	MSR cm	VS R cm	TR cm			
W									
W+50									
W+100									
W+150									
W+200									

The breadth (b) and thickness (d) of the bar are measured by using Vernier calipers and screw gauge. The young’s modulus of the material of the beam is determined by the relation,

$$Y = \frac{Mgl^3}{4bd^3 y} \text{ Nm}^{-2}$$

1.10. I-SHAPE GIRDERS

The elastic behavior of materials plays an important role in everyday life. For example while designing a building, the structural design of the columns, beams and supports require knowledge of strength of materials used. Have you ever thought why the beams used in construction of bridges; supports etc. have a cross-section of the type I?

A girder is a support beam used in construction. Girder is the term used to denote the main horizontal support of a structure, which supports smaller beams. A girder is commonly used more in the building of bridges and planes.

The girders with upper and lower section broadened and the middle section tapered, so that it can withstand heavy loads over it. The cross section of the girder takes the shape of the capital letter I and hence it is called as I-shape girders.

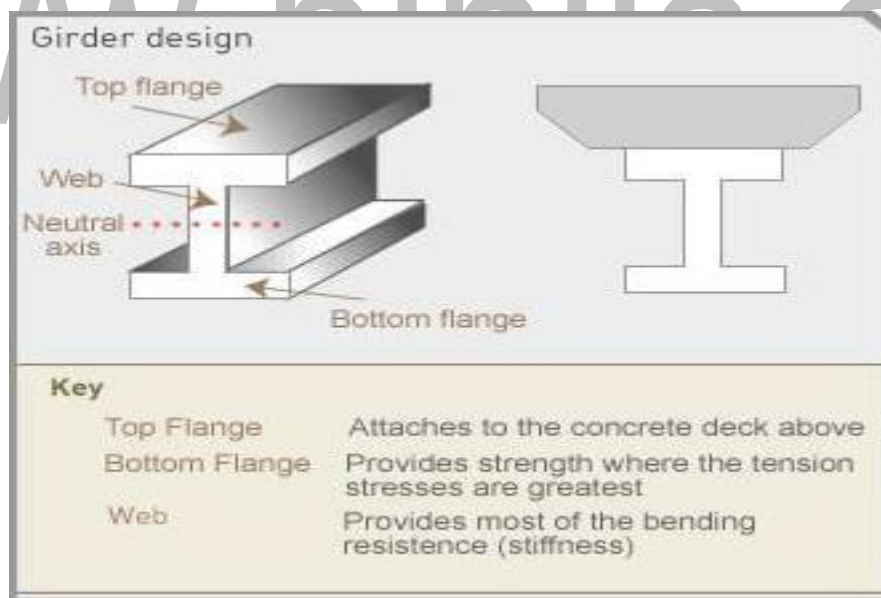


Figure. 1.10 I-shape girders.

The vertical plate in the middle is known as the web and the top and bottom plates are referred to as flanges, steel is one of the most common materials used to make I-beams, since it can withstand very heavy loads, although other materials, such as aluminum are sometimes used. When a beam is used as a girder for a given load, depression must be minimum.

- We know that the depression at the mid-point of a beam loaded at that point is given by

$$Y = \frac{Mgl^3}{4bd^3 y} \text{ Nm}^{-2}$$

Here, the depression can be minimized by,

- Decreasing the load (Mg)
- Decreasing the length (l)
- Increasing the Young's modulus (Y)
- Increasing the breadth (b)
- Increasing the thickness of the girder (d)

Since the length (l) and Young's modulus (Y) of the beam are the fixed quantity, it cannot be altered.

- Therefore the breadth and thickness are adjusted to minimize the depression.

• Thus the girders are made of I shape and are called I shape girders.

APPLICATIONS OF I-SHAPE GIRDERS

- Support beam for commercial and residential construction.
- Support frames and columns for trolley ways, lifts and hoists.
- Construction of platforms.
- Trailer and truck bed framing.
- Construction of bridges.
- Machine bases.
- Iron rails include in railway tracks.

5.9 Crystal Growth:

Crystal growth is a major stage of a crystallization process, and consists in the addition of few atoms, ions, or polymer strings into the characteristic arrangement of crystalline Bravais lattice. The growth typically follows an initial stage of either homogeneous or heterogeneous nucleation, unless a seed crystal purposely added to start the growth

Solidification (or) Crystallization

Solidification is the on transformation of material from liquid to solid state cooling.

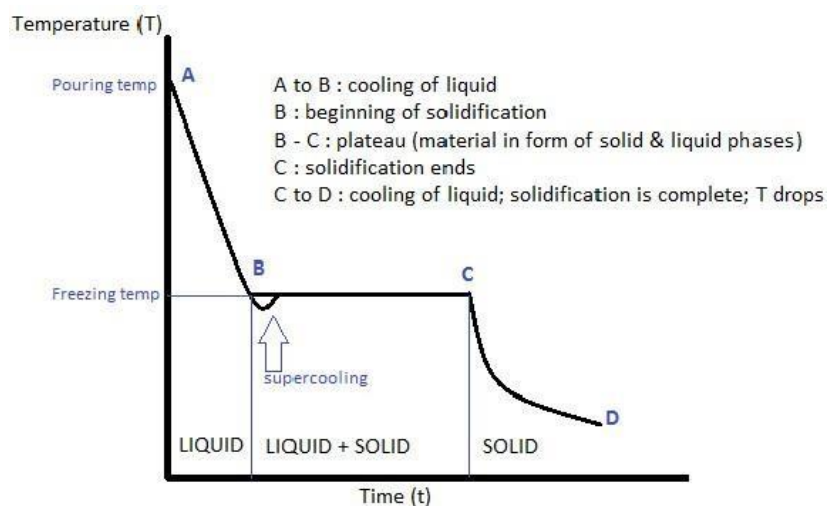


Fig 5.9.1-Transformation of material from liquid to solid state

When a liquid solidifies (solidification process), the energy of each atom is reduced. This energy is given out as latent heat. For a pure metal it occurs at a fixed temperature. During solidification, the disordered structure of the liquid transforms to the orderly arrangement depending upon the time of solidification. This solidification process greatly influences the crystal growth.

Crystal Growth Techniques

Crystal can be grown using the following basic growth techniques. They are,

- Low temperature solution growth
- High temperature solution growth (Flux growth)
- Melt growth
- Hydrothermal growth
- Gel growth
- Vapour phase growth

Melt Growth

Melt technique is the process of crystallization by fusion and re-solidification of the starting materials from the melt. The growth from melt is further sub-grouped into two techniques.

- ❖ Czochralski technique
- ❖ Bridgman technique (high temperature technique)

Czochralski Method or Crystal Pulling Method

Principle:

Crystals are grown by a gradual layer by layer condensation of melt. It is based on liquid-solid phase transition initiated by a seed crystal.

Description

1. The pure material which is to be grown in the form of single crystal is taken in the crucible.
2. The material is heated above the melting point using induction heater.
3. Thus melt is obtained in the crucible. A small defect free single crystal, called seed crystal is introduced into the melt.
4. The seed crystal is attached to a rod, which is rotated slowly.
5. The seed crystal is rotated and gradually pulled out of the melt.

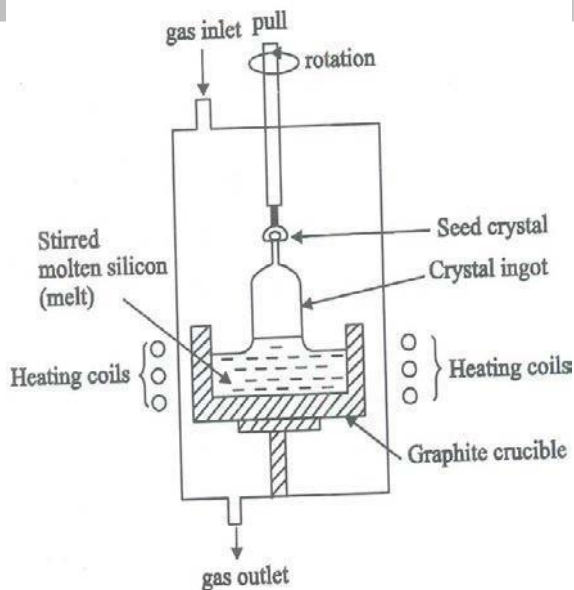


Fig 5.9.2-Czochralski Method

7. A single crystal grows on the seed crystal.
8. The seed crystal acts as i) a nucleation centre
ii) heat sink through which the latent heat of solidification escapes.

9. The temperature difference between the melt and the seed crystal establishes the desired growth condition.
10. The seed is slowly with-drawn from the melt by pulling and rotating such that the crystal diameter is gradually increased to the desired value.
11. The seed crystal is dipped into a melt held at a temperature slightly above the melting point.
12. The shape of the crystal is initially in the form of thin neck and then the diameter of the crystal is increased to the required size. This is known as necking procedure.
13. By using necking procedure and pulling mechanism, bulk single crystals can be obtained by Czochralski method.

Advantages

- It produces defect free crystal.
- It produces large single crystal.
- It allows convenient chemical composition of crystal.
- The dopant distribution in the crystal will be uniform.

Limitations

- High vapour pressure of the materials can be used.
- Possibility of liquid phase encapsulation occurs during solidification.
- It may produce contamination of melt by the crucible.

5.10 Bridgman Techniques

Principle

The selective cooling of the molten material is used to form single crystal by solidification along a particular direction. It is classified into two types. They are,

- Vertical Bridgman Technique
- Horizontal Bridgman Technique

In both techniques, the melt in a sealed crucible is progressively frozen from one end to other end.

Description

1. The material to be grown in the form of a single crystal which is taken in a cylindrical crucible.

2. Crucible is made of platinum and tapered conically with pointed tip at the bottom.
3. The crucible is suspended in the upper furnace until the material in the crucible is completely melted into molten liquid.

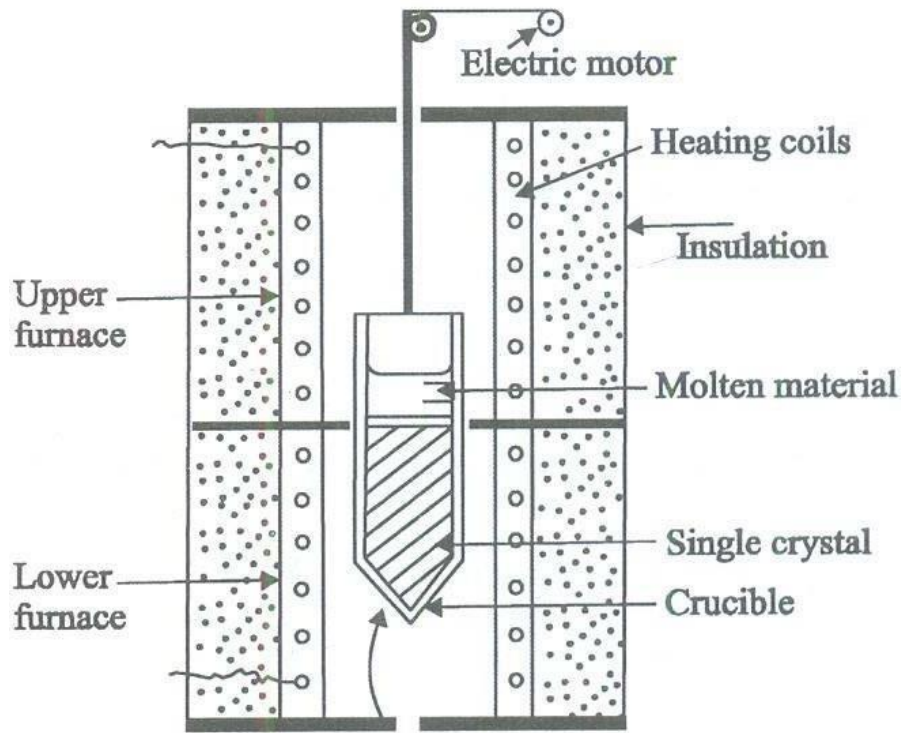


Fig 5.10.1 Bridgman Technique

4. Crucible is then slowly lowered from upper furnace into lower furnace with the help of an electric motor.
5. Temperature of the lower furnace is maintained below the melting point of the material inside the crucible.
6. Thus, a bulk single pure crystal can be grown in the crucible by lowering the crucible at steady rate (1 – 30 mm/hr.).

Advantages

- ❖ It is relatively cheaper than other pulling techniques.
- ❖ Simpler technology
- ❖ Melt composition can be controlled during the growth.
- ❖ It enables easy stabilization of temperature gradient

Disadvantages

- ❖ Growth rate is very low.

- ❖ Sometimes instead of single crystals, polycrystals may grow.
- ❖ Since the material is in contact with the walls of the container for long period, it leads to dislocations of the nucleus.
- ❖ This technique can't be used for materials which decompose before melting.

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