

Question Paper Code: 57502

## B.E./B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Third Semester

Civil Engineering

## MA 6351 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulations 2013)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions.

 $PART - A (10 \times 2 = 20 Marks)$ 

- 1. Form the partial differential equation by eliminating the arbitrary functions from  $f(x^2 + y^2, z xy) = 0.$
- 2. Find the complete solution of the partial differential equation  $p^3 q^3 = 0$ .
- 3. Find the value of the Fourier series of  $f(x) = \begin{cases} 0 \text{ in } (-c, 0) \\ 1 \text{ in } (0, c) \end{cases}$  at the point of discontinuity x = 0.
- 4. Find the value of  $b_n$  in the Fourier series expansion of  $f(x) = \begin{cases} x + \pi & \text{in } (-\pi, 0) \\ -x + \pi & \text{in } (0, \pi) \end{cases}$

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- 5. Classify the partial differential equation  $u_{xx} + u_{xy} = f(x, y)$ .
- 6. Write down all the possible solutions of one dimensional heat equation.
- 7. State Fourier integral theorem.
- 8. Find the Fourier transform of a derivative of the function f(x) if  $f(x) \to 0$  as  $x \to \pm \infty$ .
- 9. Find  $Z\left\{\frac{1}{n!}\right\}$
- 10. Find  $Z \{(\cos \theta + i \sin \theta)^n\}$ .

## $PART - B (5 \times 16 = 80 Marks)$

11. (a) (i) Solve the equation 
$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$
. (8)

(ii) Find the singular integral of the equation 
$$z = px + qy + \sqrt{1 + p^2 + q^2}$$
. (8)

OR

(b) (i) Solve: 
$$(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$$
. (8)

(ii) Solve: 
$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$$
 (8)

12. (a) (i) Find the Fourier series of 
$$f(x) = x$$
 in  $-\pi < x < \pi$ . (6)

(ii) Find the Fourier series expansion of 
$$f(x) = |\cos x| \text{ in } -\pi < x < \pi$$
. (10)

OR

(b) (i) Find the half range sine series of 
$$f(x) = x \cos \pi x$$
 in (0, 1). (8)

(ii) Find the Fourier cosine series up to third harmonic to represent the function given by the following data: (8)

x: 0 1 2 3 4 5

y: 4 8 15 7 6 2

13. (a) Find the displacement of a string stretched between two fixed points at a distance of 2*l* apart when the string is initially at rest in equilibrium position and points of  $\frac{x}{2} = \frac{1}{2} \sin(\theta_0 t)$ 

the string are given initial velocities v where  $v = \begin{cases} \frac{x}{l} & \text{in } (0, l) \\ \frac{2l - x}{l} & \text{in } (l, 2l) \end{cases}$ , x being the

distance measured from one end.

(16)

OR

- (b) A long rectangular plate with insulated surface is l cm wide. If the temperature along one short edge is  $u(x, 0) = k(lx x^2)$  for 0 < x < l, while the other two long edges x = 0 and x = 1 as well as the other short edge are kept at 0 °C, find the steady state temperature function u(x, y). (16)
- 14. (a) Find the Fourier cosine and sine transform of  $f(x) = e^{-ax}$  for  $x \ge 0$ , a > 0. Hence

deduce the integrals 
$$\int_{0}^{\infty} \frac{\cos sx}{a^2 + s^2} ds$$
 and  $\int_{0}^{\infty} \frac{s \sin sx}{a^2 + s^2} ds$ . (16)

OR

- (b) (i) Find the Fourier transform of  $f(x) = e^{\frac{x^2}{2}}$  in  $(-\infty, \infty)$ . (8)
  - (ii) Find the Fourier transform of f(x) = 1 |x| if |x| < 1 and hence find the

value of 
$$\int_{0}^{\infty} \frac{\sin^4 t}{t^4} dt$$
. (8)

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- 15. (a) (i) Find the Z-transforms of  $\cos \frac{n\pi}{2}$  and  $\frac{1}{n(n+1)}$ . (8)
  - (ii) Using convolution theorem, evaluation  $Z^{-1}\left\{\frac{z^2}{(z-a)^2}\right\}$ . (8)

OR

- (b) (i) Find the inverse Z-transform of  $\frac{z}{z^2 2z + 2}$  by residue method. (8)
  - (ii) Solve the difference equation  $y_{n+2} + y_n = 2$ , given that  $y_0 = 0$  and  $y_1 = 0$  by using Z-transforms. (8)