|       | Reg. No. :        |           | 6126 |
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| ECE L | Question Paper Co | ode:50779 |      |

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017
Third Semester
Civil Engineering
MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Mechanical Engineering (Sandwich)/ Aeronautical Engineering/
Agriculture Engineering/ Automobile Engineering/ Biomedical Engineering/
Computer Science and Engineering/ Electrical and Electronics Engineering/
Electronics and Communication Engineering/ Electronics and Instrumentation
Engineering/ Geoinformatics Engineering/ Industrial Engineering/ Industrial
Engineering and Management/ Instrumentation and Control Engineering/
Manufacturing Engineering/ Marine Engineering/ Materials Science and
Engineering/Mechanical Engineering/Mechanical and Automation Engineering/
Mechatronics Engineering/ Medical Electronics/ Petrochemical Engineering/
Production Engineering/ Robotics and Automation Engineering/ Biotechnology,
Chemical Engineering/ Chemical and Electrochemical Engineering/
Food Technology/ Information Technology/ Petrochemical Technology/ Petroleum
Engineering/ Plastic Technology/Polymer Technology)
(Regulations 2013)

Time: Three Hours Maximum: 100 Marks

Answer ALL questions.

PART - A

(10×2=20 Marks)

- 1. Find the partial differential equation by eliminating the arbitrary function 'f' from the relation  $z = f(x^2 y^2)$ .
- 2. Find the complete integral of  $\sqrt{p} + \sqrt{q} = 1$ .
- 3. State Dirichlet's conditions for a given function f(x) to be expanded in Fourier series.
- 4. Write the complex form of Fourier series for a function f(x) defined in  $-l \le x \le l$ .

50779 5. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation? 6. State any two solutions of the Laplace equation  $u_{xx} + u_{yy} = 0$  involving exponential terms in x or y. 7. If F[f(x)] = F(s), then find F[f(ax)]. 8. State the convolution theorem for Fourier transforms. 9. Find the Z-transform of the function f(n) = 1/n. 10. Form the difference equation by eliminating arbitrary constant 'a' from  $y_n = a$ .  $2^n$ . PART - B (5×16=80 Marks) 11. a) i) Find the singular integral of  $z = px + qy + p^2 - q^2$ . ii) Find the general integral of (x-2z) p + (2z-y) q = y-x. (8)(OR) b) Solve the following equations. i)  $(D^2 + 2DD' + D'^2) z = e^{x - y} + xy$ ii)  $(D^2 - 5DD' + 6D'^2) z = y \sin x$ . 12. a) i) Find the Fourier series for a function  $f(x) = x + x^2 \text{ in } (-\pi, \pi)$  and hence deduce (8) ii) Find the Fourier series of y = f(x) up to first harmonic which is defined by the following data in  $(0, 2\pi)$ :  $\pi/3$  $2\pi/3$  $5\pi/3$  $4\pi/3$  $2\pi$ (8) f(x) 1.4 1.9 1.7 1.5

(OR)

b) i) Find the half-range cosine series for f(x) = x in  $(0, \pi)$ . Hence deduce the value

of 
$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$
 (8)

ii) Find the Fourier series for a function 
$$f(x) = \begin{cases} l - x, & 0 < x \le l \\ 0, & l < x \le 2l \end{cases}$$
 in (0, 2l). (8)

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- 13. a) A tightly stretched string of length l has its end fastened at x = 0, x = l. At t = 0, the string is in the form f(x) = kx(l x) and then released. Find the displacement at any point of the string at a distance x from one end and at any time t > 0. (16)
  - b) A rod of length l cm has its ends A and B kept at 0°C and 100°C respectively, until steady state conditions prevail. If the temperature at B is suddenly reduced to 0°C and maintained at 0°C, find the temperature distribution u(x, t) at a distance x from A at any time t.
    (16)
- 14. a) i) If  $F_S$  (s) and  $F_C$ (s) denote Fourier sine and cosine transform of a function f(x) respectively, then show that

$$F_S\{f(x) \sin ax\} = \frac{1}{2} \{F_C(s-a) - F_C(s+a)\}$$
 (4)

ii) Find the Fourier transform of a function  $f(x) = \begin{cases} 1 - |x| & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$  and hence

find the value of 
$$\int_{0}^{\infty} \frac{\sin^4 t}{t^4} dt$$
 by Parseval's identity. (12)

b) Find the Fourier sine and cosine transforms of a function  $f(x) = e^{-x}$ . Using Parseval's identity, evaluate:

(1) 
$$\int_{0}^{\infty} \frac{dx}{(x^2+1)^2}$$
 and (2)  $\int_{0}^{\infty} \frac{x^2 dx}{(x^2+1)^2}$  (16)

15. a) i) Find the Z-transform of  $\frac{2n+3}{(n+1)(n+2)}$ . (8)

ii) Find 
$$Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right]$$
 by using convolution theorem. (8)

(OR)

- b) i) Find the inverse Z-transform of  $\frac{z^3}{(z-1)^2(z-2)}$  by method of partial fraction. (6)
  - ii) Solve the difference equation  $y(n + 2) 7y(n + 1) + 12y(n) = 2^n$ , given that y(0) = 0 and y(1) = 0, by using Z-transform. (10)