UNIT II
OSCILLATORS

2.1 Introduction

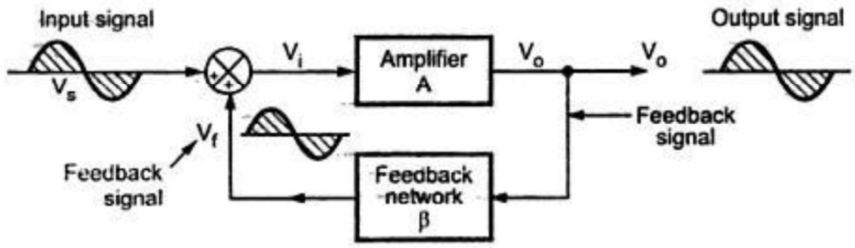
An oscillator is a circuit which basically acts as a generator, generating the output signal which oscillates with constant amplitude and constant desired frequency.

An oscillator does not require any input signal. An oscillator generates the output waveform of high frequency up to Giga hertz. It works as positive feedback.

Definition: Oscillator is an amplifier, which uses a positive feedback and without any external input signal, generates an output waveform at a desired frequency.

2.2 Basic Theory of Oscillators

As the phase of the feedback signal is same as that of the input applied, the feedback is called as '**positive feedback**'.



Here input V_i, output V_0 and gain A

$$A = \frac{V_0}{V_i} (called\ open\ loop\ gain)$$

For overall circuit supply voltage V_s and net output V_0

$$A_f = \frac{V_0}{V_s} (called\ closed\ loop\ gain)$$

Feedback is positive V_f is added to V_s generate input of amplifier V_i

$$V_i = V_f + V_s \ldots\ldots\ldots\ldots(1)$$

$$V_f = \beta V_0 \ldots\ldots\ldots\ldots(2)$$

Substituting (2) in (1)

$$V_i = \beta V_0 + V_s$$

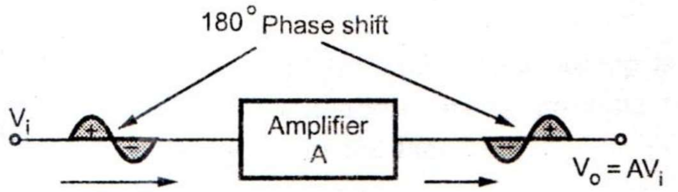
$$V_s = V_i - \beta V_0 \ldots\ldots\ldots(3)$$

$$A_f = \frac{V_0}{V_i - \beta V_0}$$

Dividing numerator and denominator by V_i

$$A_f = \frac{V_0/V_i}{1 - \beta V_0/V_i} = \frac{A}{1 - \beta A}$$

2.3 Barkhausen Criterion



Consider a basic inverting amplifier with an open loop gain A. the feedback network attenuation factor β is less than unity. As basic amplifier is inverting, it produces a phase shift of 180^0 between input and output.

$$V_0 = AV_i \dots\dots\dots (1)$$

$$V_f = -\beta V_0 \dots\dots\dots (2)$$

Substituting (2) in (1)

$$V_f = -\beta AV_i \dots\dots\dots. (3)$$

For oscillator V_f must act as V_i

$$V_i = -\beta AV_i$$

$$\boldsymbol{-\beta A = 1}$$

Barkhausen criterion states that,

1. The total phase shift around a loop, as the signal proceeds from input through amplifier, feedback network back to input again, completing a loop is exactly $0^0\ OR\ 360^0$, integral multiples of 2π radians.
2. The magnitude of the product of the open loop gain of the amplifier (A) and feedback factor (β) is unity.
   ie). $|A\beta| = 1$

2.4 Classification of Oscillators

1. **Based on output waveform**: Oscillators are classified as sinusoidal and non-sinusoidal oscillators.
2. **Based on circuit component**: Oscillator using R & C components called as RC oscillators. Oscillator using L & C components called as LC oscillators. Some oscillators' crystal used. So it is called as crystal oscillator.
3. **Based on range of operating frequency**: Oscillators are used to generate oscillation at audio frequency range which is 20Hz to 100-200KHz then oscillator are classified as LF and AF oscillators. While the oscillator used at the frequency range more than 200-300KHz to Giga Hertz classified as HF Oscillators.

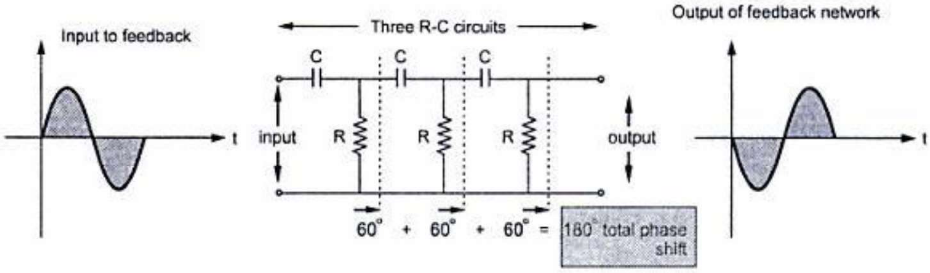
2.5 R-C Phase Shift Oscillator

RC phase shift oscillator basically consists of an amplifier and a feedback network consisting of resistors and capacitors arranged in ladder fashion. Hence such an oscillator is also called ladder type RC phase shift oscillator.

2.5.1 RC Feedback Network

RC network is used in feedback path. In oscillator, feedback network must introduce a phase shift of 180^0 to obtain total phase shift around a loop as 360^0.

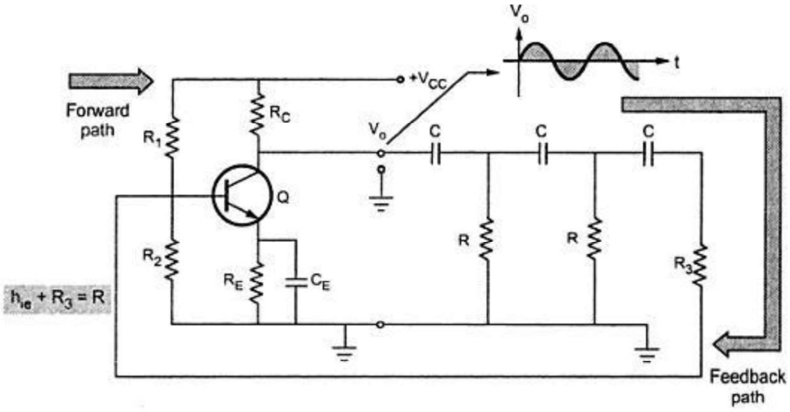
Thus if one RC network produces phase shift of $\phi = 60^0$ then to produce phase shift of 180^0 such three RC networks must be connected in cascade. Hence in RC phase shift oscillator , the feedback network consists of three RC sections each producing a phase shift of 60^0 , thus total phase shift due to feedback is 180^0 (3×60).



The network is also called **the ladder network**. All the resistance values and all the capacitance values are same, so that a particular frequency, each section of R and C produces a phase shift of 60^0

2.5.2 Transistorized RC Phase Shift Oscillator

In a practical, transistorized RC phase shift oscillator, a transistor is used as an active element of the amplifier stage.



The figure shows a practical transistorized RC phase shift oscillator which uses a common emitter single stage amplifier and a phase shifting network consisting of three identical RC sections.

The output of the feedback network gets loaded due to the low input impedance (h_{ie}) of a resistor. Hence an emitter follower input stage before the common emitter amplifier stage can be used, to avoid the problem of low input impedance. But if only single stage is to be used then the voltage shunt feedback, denoted by resistance R_3 in the figure is used, connected in series with the amplifier input resistance.

A phase shifting network is a feedback network, so output of the amplifier is given as an input to the feedback network. While the output of the feedback network is given as an input to the amplifier. Thus amplifier supplies its own input, through the feedback network.

Neglecting R_1 and R_2, as these are sufficiently large, we can write,

h_{ie} = Input impedance of the amplifier stage

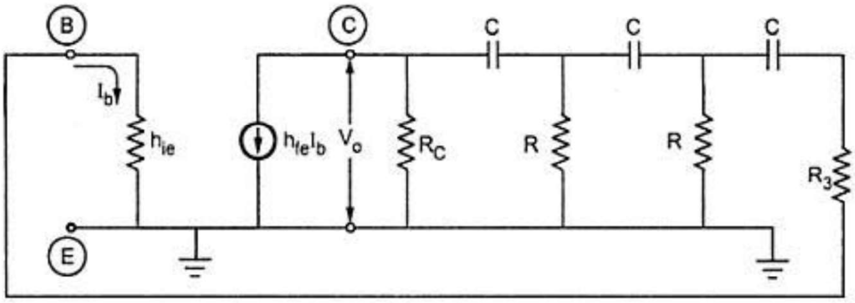
Now the resistance R_3 and h_{ie} are in series and the value of R_3 is also selected such that the resultant of the two resistance is R, which is the required value of the resistance, in last section of RC phase shifting network.

$$h_{ie} + R_3 = R$$

This ensures that all the three sections of the phase shifting network are identical.

2.5.3 Derivation of the Frequency of Oscillation

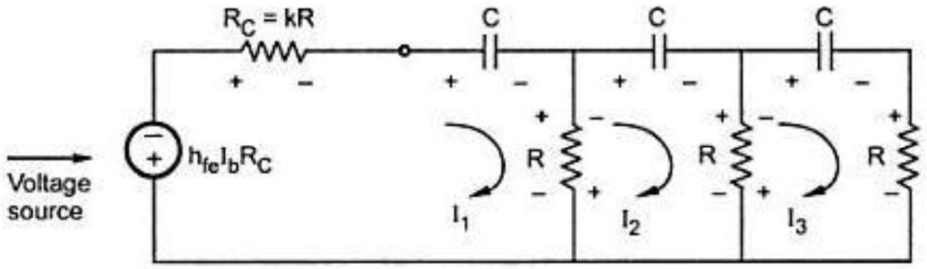
Replacing the transistor by its approximate h-parameter model, we get the equivalent oscillator circuit.



Now we can replace $h_{ie} + R_e$ as R from the equation. Similarly we can replace the current source $h_{fe}I_b$ by its equivalent voltage source. And assume the **ratio of the resistance R_C to R be** k.

$$\boldsymbol{k = \frac{Rc}{R}}$$

the modified equivalent circuit is shown below



Applying KVL for the various loops in the modified equivalent circuit we get, for Loop 1,

$$-I_1R_C - \frac{1}{j\omega C}I_1 - I_1R + I_2R - h_{fe}I_bR_c = 0$$

Replacing R_C by kR and $j\omega$ by s we get,

$$I_1\left[(k+1)R\frac{1}{sC}\right]-I_2R=-h_{fe}I_bkR$$

For Loop 2,

$$-\frac{1}{j\omega C}I_2-I_2R-I_2R+I_1R+I_3R=0$$

Replacing $j\omega$ by s we get,

$$-I_1R+I_2\left[2R+\frac{1}{sC}\right]-I_3R=0$$

For Loop 3,

$$-\frac{1}{j\omega C}I_3-I_3R-I_3R+I_2R=0$$

$$-I_2R+I_3\left[2R+\frac{1}{sC}\right]=0$$

Using Cramer's rule to solve I_3

$$D=\begin{vmatrix}(k+1)R+\frac{1}{sC} & -R & 0\\ -R & 2R+\frac{1}{sC} & -R\\ 0 & -R & 2R+\frac{1}{sC}\end{vmatrix}$$

$$=\left[(k+1)R+\frac{1}{sC}\right]\left\{\left[2R+\frac{1}{sC}\right]^2-R^2\right\}-R^2\left[2R+\frac{1}{sC}\right]$$

$$=\left[(k+1)R+\frac{1}{sC}\right]\left[2R+\frac{1}{sC}\right]^2-R^2\left[(k+1)R+\frac{1}{sC}\right]-R^2\left[2R+\frac{1}{sC}\right]$$

$$=\left[\frac{(k+1)sRC+1}{sC}\right]\left[\frac{2sRC+1}{sC}\right]^2-R^2\left[\frac{(k+1)sRC+1}{sC}\right]-R^2\left[\frac{2sRC+1}{sC}\right]$$

$$=\frac{[(k+1)sRC+1][2sRC+1]^2}{s^3R^3}-R^2\left[\frac{(k+1)sRC+1}{sC}\right]-R^2\left[\frac{2sRC+1}{sC}\right]$$

First term can be written as,

$$\frac{[(k+1)sRC+1][2sRC+1]^2}{s^3R^3}=(ksRC+sRC+1)(4s^2R^2C^2+4sRC+1)/_{s^3R^3}$$

$$=\frac{4ks^3R^3C^3+4s^3R^3C^3+4s^2R^2C^2+4ks^2R^2C^2+4s^2R^2C^2+4sRC+ksRC+sRC+1}{s^3R^3}$$

$$=\frac{s^3R^3C^3[4k+4]+s^2R^2C^2[4k+8]+sRC[5+k]+1}{s^3R^3}$$

Second and the third term can be combined to get,

$$=\frac{-R^2[ksRC+sRC+1]-R^2[2sRC+1]}{sC}$$

$$=\frac{-[2R^2+3sR^3C+ksR^3C]}{sC}$$

Combining the two terms and taking LCM as s^3C^3 we get,

$$D = \frac{s^3R^3C^3[4k+4] + s^2R^2C^2[4k+8] + sRC[5+k] + 1 - [2R^2 + 3sR^3C + ksR^3C]s^2C^2}{s^3C^3}$$

$$= \frac{s^3R^3C^3[3k+1] + s^2R^2C^2[4k+6] + sRC[5+k] + 1}{s^3C^3}$$

$$D_3 = \begin{vmatrix} (k+1)R + \frac{1}{sC} & -R & -h_{fe}I_bkR \\ -R & 2R + \frac{1}{sC} & 0 \\ 0 & -R & 0 \end{vmatrix}$$

$$= R^2\left(-h_{fe}I_bkR\right)$$

$$= -kR^3h_{fe}I_b$$

$$I_3 = \frac{D_3}{D}$$

$$I_3 = \frac{-kR^3h_{fe}I_bs^3C^3}{s^3R^3C^3[3k+1] + s^2R^2C^2[4k+6] + sRC[5+k] + 1}$$

Now I_3= output current of the feedback circuit

I_b = Input current of the amplifier

$I_C = h_{fe}I_b$= input current of the feedback circuit

$$\beta = \frac{output\ of\ feedback\ circuit}{input\ of\ feedback\ circuit} = \frac{I_3}{h_{fe}I_b}$$

And

$$A = \frac{output\ of\ amplifier\ circuit}{input\ of\ amplifier\ circuit} = \frac{I_3}{I_b} = h_{fe}$$

$$\boldsymbol{A\beta = \frac{I_3}{h_{fe}I_b} \times h_{fe} = \frac{I_3}{I_b}}$$

$$A\beta = \frac{\boldsymbol{-kR^3h_{fe}s^3C^3}}{\boldsymbol{s^3R^3C^3[3k+1] + s^2R^2C^2[4k+6] + sRC[5+k] + 1}}$$

Substituting $s = j\omega, s^2 = -j^2\omega^2 = -\omega^2, s^3 = j^3\omega^3 = -j\omega^3$

$$-A\beta = \frac{j\omega^3kR^3C^3h_{fe}}{-j\omega^3R^3C^3[3k+1] - \omega^2R^2C^2[4k+6] + j\omega RC[5+k] + 1}$$

Separating the real and imaginary parts in the denominator we get,

$$-A\beta = \frac{j\omega^3kR^3C^3h_{fe}}{[1 - 4k\omega^2R^2C^2 - 6\omega^2R^2C^2] - j[3k\omega^3R^3C^3 + \omega^3R^3C^3 - 5\omega RC - k\omega RC]}$$

Dividing numerator and denominator by $j\omega^3R^3C^3$

$$-A\beta = \frac{kh_{fe}}{\frac{[1-4k\omega^2R^2C^2-6\omega^2R^2C^2]}{j\omega^3R^3C^3} - j\frac{[3k\omega^3R^3C^3+\omega^3R^3C^3-5\omega RC-k\omega RC]}{j\omega^3R^3C^3}}$$

Replacing $-1/_j = j$,

$$-A\beta = \frac{kh_{fe}}{j\left\{\frac{1}{\omega^3R^3C^3} - \frac{4k}{\omega RC} - \frac{6}{\omega RC}\right\} + \left\{3k + 1 - \frac{5}{\omega^2R^2C^2} - \frac{k}{\omega^2R^2C^2}\right\}}$$

Replacing $\frac{1}{\omega \mathrm{CR}} = \alpha$ for simplicity

$$-A\beta = \frac{kh_{fe}}{\{3k + 1 - 5\alpha^2 - k\alpha^2\} + j\{\alpha^3 - 4k\alpha - 6\alpha\}}$$

To find frequency of oscillation, the imaginary part of the denominator term must be 0.

$$\therefore \alpha^3 - 4k\alpha - 6\alpha = 0$$

$$\alpha(\alpha^2 - 4k - 6) = 0$$

$\alpha^2 = 4k + 6$ Neglecting zero value

$$\alpha = \sqrt{4k + 6}$$

$$\frac{1}{\omega \mathrm{CR}} = \sqrt{4k + 6}$$

$$\omega = \frac{1}{\mathrm{RC}\sqrt{4k + 6}}$$

$$\boldsymbol{f = \frac{1}{2\pi RC\sqrt{4k + 6}}}$$

As per the barkhausen criterion$|\mathrm{A}\,\beta| = 1$

Substituting $\alpha = \sqrt{4k + 6}$ in the equation we get

$$-A\beta = \frac{kh_{fe}}{\{3k + 1 - (5 + k)(4k + 6)\}} = \frac{kh_{fe}}{\{3k + 1 - 20k - 30 - 4k^2 - 6k\}}$$

$$= \frac{kh_{fe}}{-4k^2 - 23k - 29}$$

Now $|\mathrm{A}\,\beta| = 1$

$$\left|\frac{kh_{fe}}{-4k^2 - 23k - 29}\right| = 1$$

$$kh_{fe} = 4k^2 + 23k + 29$$

$$h_{fe} = 4k + 23 + \frac{29}{k}$$

Minimum value of h_{fe} for the oscillations

To get minimum value of h_{fe}

$$\frac{\mathrm{d}h_{fe}}{\mathrm{dk}} = 0$$

$$\frac{\mathrm{d}}{\mathrm{dk}}\left[4k + 23 + \frac{29}{k}\right] = 0$$

$$4 - \frac{29}{k^2} = 0$$

$$k^2 = \frac{29}{4}$$

$$\boldsymbol{k^2 = 2.6925\ for\ minimum\ h_{fe}}$$

$$(h_{fe})_{min} = 4 \times 2.6925 + 23 + \frac{29}{2.6925}$$

$$(\boldsymbol{h_{fe}})_{\boldsymbol{min}} = \mathbf{44.54}$$

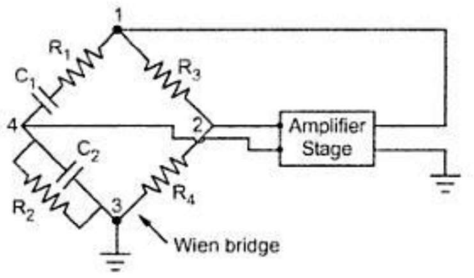
NOTE: Thus for the circuit to oscillate, we must select the transistor whose $(h_{fe})_{min}$ should be greater than 44.54.

2.6 Wien Bridge Oscillator

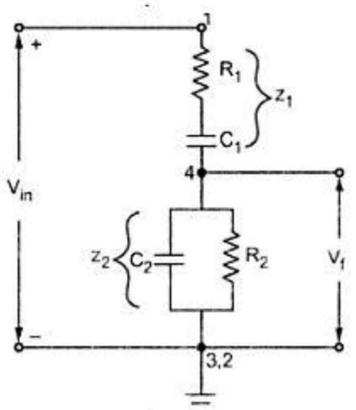
Generally in an oscillator, amplifier stage introduce 180^0 phase shift and feedback network introduces additional 180^0 phase shift, to obtain a phase shift of 360^0 (2π radians) around a loop. This is required condition for any oscillator. But **Wien Bridge oscillator uses a noninverting amplifier and hence does not provide any phase shift during amplifier stage**. As total phase shift required is 0^0 or $2n\pi$ radians, in wien bride type no phase shift is necessary through feedback.

Note: Thus the total phase shift around a loop is 0^0.

A basic wien bridge used in this oscillator and an amplifier stage is shown below.



Basic circuit of Wien bridge oscillator



Feedback network of Wien bridge oscillator

The output of the amplifier is applied between the terminals 1 and 3, which is the input to the feedback network. While the amplifier input is supplied from the diagonal terminals 2 and4, which is the output from the feedback network. Thus amplifier supplied its own input through the wien bridge as a feedback network.

The two arms of the bridge, namelyR_1, C_1 in series and R_2, C_2 in parallel are called **frequency sensitive arms.** This is becausethe components of these two arms decide the frequency of the oscillator. Let as find out the gain of the feedback network. As seen earlier, input V_{in} to the feedback network is between is 1 and 3 while output V_f of the feedback network is between 2 and 4. Such a feedback network is called **lead-lag network**. This is because at very low frequencies it acts like a lead while at very high frequencies it acts like lag network.

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

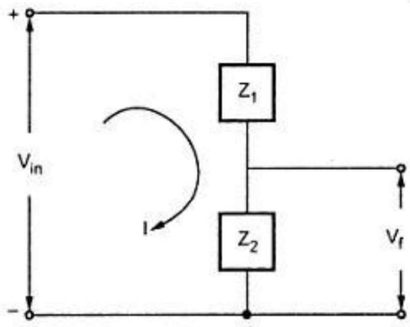
$$Z_2 = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2 \times \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}$$

$$Z_2 = \frac{R_2}{1 + j\omega R_2 C_2}$$

Replacing $j\omega = s$

$$Z_1 = \frac{1 + sR_1 C_1}{sC_1}$$

$$Z_2 = \frac{R_2}{1 + sR_2 C_2}$$



Simplified circuit

From the simplified circuit,

$$I = \frac{V_{in}}{Z_1 + Z_2}$$

And

$$V_f = IZ_2$$

$$V_f = \frac{V_{in}}{Z_1 + Z_2} Z_2$$

$$\beta = \frac{V_f}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

Substituting the values of $Z_1 \& Z_2$,

$$\beta = \frac{\left[\frac{R_2}{1 + sR_2 C_2}\right]}{\left[\frac{1 + sR_1 C_1}{sC_1}\right] + \left[\frac{R_2}{1 + sR_2 C_2}\right]}$$

$$\beta = \frac{sC_1 R_2}{(1 + sR_1 C_1)(1 + sR_2 C_2) + sC_1 R_2}$$

$$= \frac{sC_1 R_2}{1 + s(R_1 C_1 + R_2 C_2) + s^2 R_1 R_2 C_1 C_2 + sC_1 R_2}$$

$$= \frac{sC_1 R_2}{1 + s(R_1 C_1 + R_2 C_2 + C_1 R_2) + s^2 R_1 R_2 C_1 C_2}$$

Substituting $s = j\omega, s^2 = -j^2\omega^2 = -\omega^2$

$$\beta = \frac{j\omega C_1 R_2}{1 + j\omega(R_1 C_1 + R_2 C_2 + C_1 R_2) - \omega^2 R_1 R_2 C_1 C_2}$$

Separating real and imaginary values

$$\beta = \frac{j\omega C_1 R_2}{(1 - \omega^2 R_1 R_2 C_1 C_2) + j\omega(R_1 C_1 + R_2 C_2 + C_1 R_2)}$$

Rationalizing the above expression,

$$\beta = \frac{j\omega C_1R_2[(1-\omega^2R_1R_2C_1C_2) - j\omega(R_1C_1 + R_2C_2 + C_1R_2)]}{(1-\omega^2R_1R_2C_1C_2)^2 + \omega^2(R_1C_1 + R_2C_2 + C_1R_2)^2}$$

$$\beta = \frac{\omega^2C_1R_2(R_1C_1 + R_2C_2 + C_1R_2) + j\omega C_1R_2(1-\omega^2R_1R_2C_1C_2)}{(1-\omega^2R_1R_2C_1C_2)^2 + \omega^2(R_1C_1 + R_2C_2 + C_1R_2)^2}$$

To find frequency of oscillation imaginary part must be zero.

$$\omega C_1R_2(1-\omega^2R_1R_2C_1C_2) = 0$$

$$\omega^2R_1R_2C_1C_2 = 1$$

$$\omega^2 = \frac{1}{R_1R_2C_1C_2}$$

$$\omega = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$$

In practice, $R_1 = R_2 = R \; and \; C_1 = C_2 = C$ then

$$f = \frac{1}{2\pi\sqrt{R^2C^2}}$$

$$f = \frac{1}{2\pi RC}$$

At $R_1 = R_2 = R \; and \; C_1 = C_2 = C$ the gain of the feedback networks becomes,

$$\beta = \frac{\omega^2RC(3RC) + j\omega RC(1-\omega^2R^2C^2)}{(1-\omega^2R^2C^2)^2 + \omega^2(3RC)^2}$$

Substituting $f = \frac{1}{2\pi RC}$ i.e. $\omega = \frac{1}{RC}$

We get the magnitude of the feedback network at the resonating frequency of the oscillator as,

$$\beta = \frac{3}{0 + \frac{1}{R^2C^2} \times (3RC)^2} = \frac{3}{9}$$

$$\beta = \frac{1}{3}$$

The positive sign of β indicates that the phase shift by the feedback network is 0^0. To satisfy barkhausen criterion for sustained oscillations, we can write,

$$|A\beta| \geq 1$$

$$|A| \geq \frac{1}{|\beta|} \geq \frac{1}{\frac{1}{3}}$$

$$|A| \geq 3$$

This is required gain of the amplifier stage without any phase shift.

2.7 Twin-T Oscillator

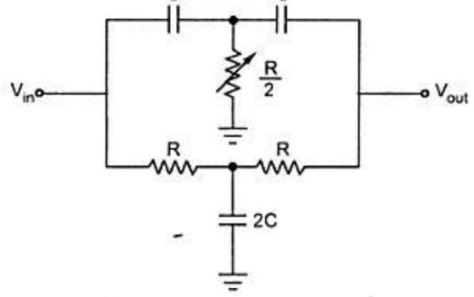
This is another type of RC oscillator, which **uses a typical circuit consisting of R and C called twin-T network or twin T filter.**

Note: Twin T filter is basically lead-lag circuit whose phase angle varies between +90^0 to -90^0 against the frequency.

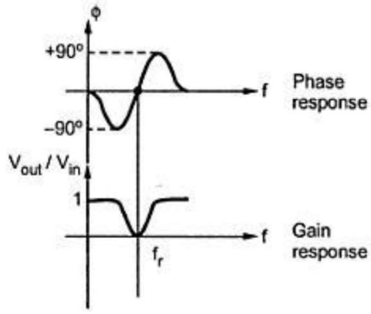
At $f=f_r$, its phase angle is **0^0** and it does not introduce any phase shift. Its gain is1 at low and high frequencies but at $f=f_r$, the gain reduces to zero. So it acts like a notch filter as it notches out frequencies near f_r. the equation for its resonating frequency is

$$f_r = \frac{1}{2\pi RC}$$

In fact a **twin-T filter is combination of high pass filter and low pass filter. The combined parallel combination of the two gives twin-T filter which is acts as notch filter**.



Twin-T circuit



Frequency response

The positive feedback is noninverting input is given though the potential divider of R_1 and R_2. The resistance of the potential divider is a lamp. The negative feedback to the inverting input is given through twin-T filter.

When power is given to the circuit the lamp resistance R_2 is low and positive feedback is maximum. This helps to build the oscillations. As oscillations grow, the lamp resistance R_2increases, decreasing the positive feedback. This controls the growing oscillations and makes them as the sustained oscillations. So lamp helps to stabilize the level of the output voltage.

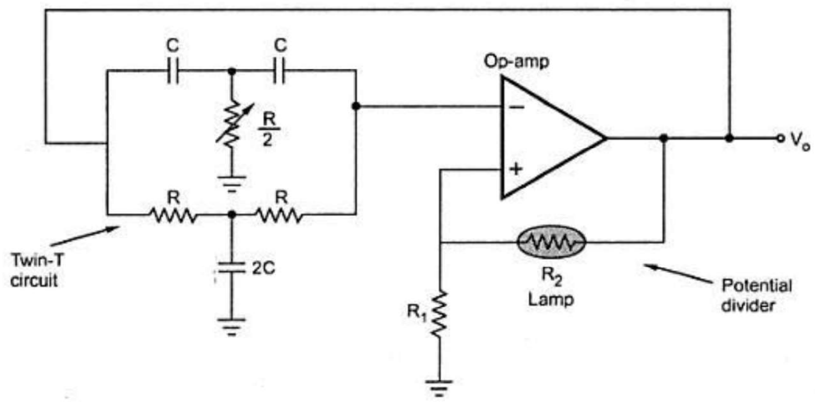


Fig :Twin-T Oscillator

2.8 LC Oscillators

The oscillators which use the elements L and C to produce the oscillations are called LC Oscillators. The circuit using elements L and C is called tank circuit or Oscillatory circuit, which is important part os LC oscillators.

2.9 Hartley Oscillator

A LC oscillator which uses **two inductive reactance and one capacitive reactance** in its feedback network is called **Hartley Oscillator**.

2.9.1 Transistorised Hartley Oscillator

The amplifier stage transistor in common emitter configuration.

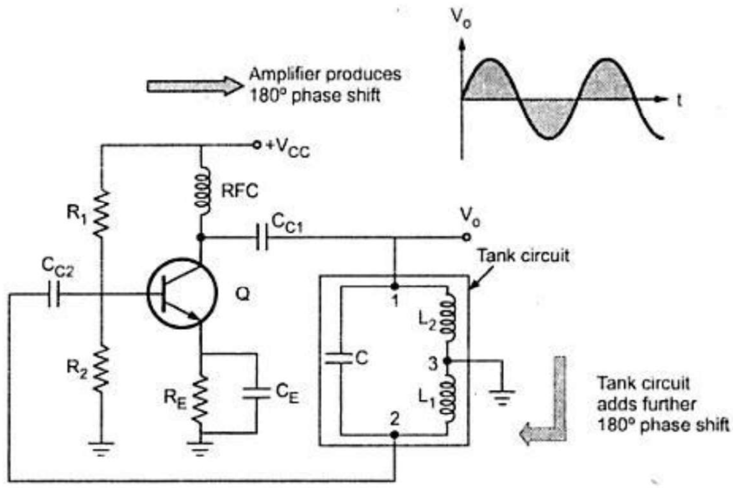


Fig :Transistorised Hartley Oscillator

The resistances R_1 and R_2 are the biasing resistances. The RFC is the radio frequency choke. Its reactance value is very high for high frequencies; hence it can be treated as open circuit. While for dc conditions, the reactance is zero hence causes no problem for dc capacitors.

The common emitter amplifier provides a phase shift of 180^0. As emitter is grounded, the base and the collector voltages are out of phase by 180^0. As the centre of L_1 and L_2 is grounded, when upper end becomes positive, the lower becomes negative and vice versa. So the LC feedback network gives as additional phase shift of 180^0, necessary to satisfy oscillation conditions.

2.9.2 Derivation of Frequency of Oscillation

The output which is the collector current is $h_{fe}I_B$ where I_B is the base current. Assuming coupling condensers are short, the capacitor C is between base and collector. The inductance L_1 is between base and emitter while the inductance L_2 is between collector and emitter. The equivalent circuit of the feedback is shown in the fig.

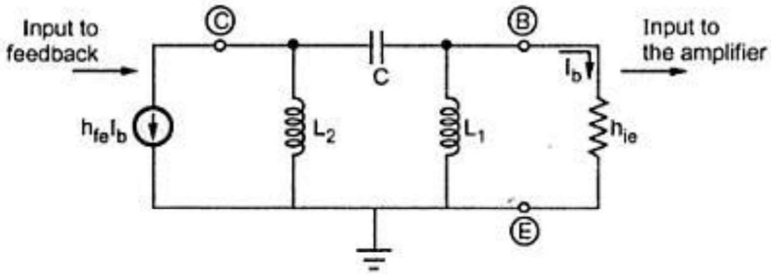


Fig: Equivalent circuit

As h_{ie} is the input impedance of the transistor. The output of the feedback is the current I_b, which is the input current of the transistor which is $I_C = h_{fe}I_b$, converting current source into voltage source we get,

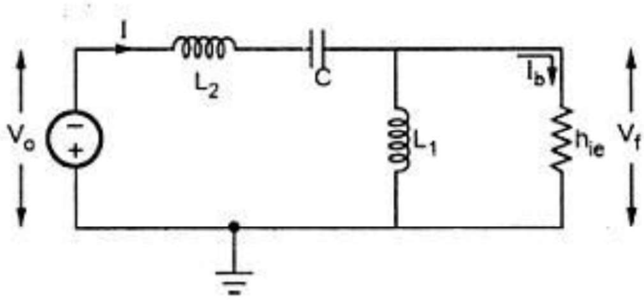


Fig: Simplified equivalent circuit

$$V_0 = h_{fe}I_bX_{L2} = h_{fe}I_bj\omega L_2 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now L_1 *and* h_{ie} are in parallel, so the total current I drawn from the supply is,

$$I = \frac{-V_0}{[X_{L2} + X_C] + [X_{L1} \parallel h_{ie}]} \ldots\ldots\ldots\ldots. (2)$$

Note: Negative sign, as current direction shown in opposite to the polarities of V_0

Now

$$X_{L2} + X_C = j\omega L_2 + \frac{1}{j\omega C} \qquad \text{and}$$

$$X_{L1} \parallel h_{ie} = \frac{j\omega L_1 h_{ie}}{j\omega L_1 + h_{ie}}$$

Substituting in the equation (2)

$$I = \frac{-h_{fe}I_bj\omega L_2}{\left[j\omega L_2 + \frac{1}{j\omega C}\right] + \left[\frac{j\omega L_1 h_{ie}}{j\omega L_1 + h_{ie}}\right]} \ldots\ldots\ldots\ldots. (3)$$

Replacing $j\omega = s$

$$I = \frac{-sh_{fe}I_bL_2}{\left[sL_2 + \frac{1}{sC}\right] + \left[\frac{sL_1 h_{ie}}{sL_1 + h_{ie}}\right]}$$

$$= \frac{-sh_{fe}I_bL_2}{\left[\frac{1+s^2L_2C}{sC}\right] + \frac{sL_1h_{ie}}{(sL_1+h_{ie})}}$$

$$= \frac{-sh_{fe}I_bL_2(sC)(sL_1+h_{ie})}{(1+s^2L_2C)(sL_1+h_{ie}) + (sC)(sL_1h_{ie})}$$

$$= \frac{-s^2h_{fe}I_bL_2C(sL_1+h_{ie})}{s^3L_1L_2C + sL_1 + h_{ie} + s^2CL_2h_{ie} + s^2CL_1h_{ie}}$$

$$I = \frac{-s^2h_{fe}I_bL_2C(sL_1+h_{ie})}{s^3L_1L_2C + s^2Ch_{ie}(L_1+L_2) + sL_1 + h_{ie}}$$

According to current division in parallel circuit,

$$I_b = I \times \frac{X_{L1}}{X_{L1} + h_{ie}}$$

$$= I \times \frac{j\omega L_1}{j\omega L_1 + h_{ie}}$$

$$I_b = I \times \left[\frac{sL_1}{sL_1 + h_{ie}}\right] \dots\dots\dots\dots\dots (4)$$

Substituting value of I from equation (3) in equation (4)

$$I_b = \left[\frac{-s^2h_{fe}I_bL_2C(sL_1+h_{ie})}{s^3L_1L_2C + s^2Ch_{ie}(L_1+L_2) + sL_1 + h_{ie}}\right] \times \left[\frac{sL_1}{sL_1 + h_{ie}}\right]$$

$$I_b = \frac{-s^3h_{fe}I_bL_1L_2C}{(s^3L_1L_2C + s^2Ch_{ie}(L_1+L_2) + sL_1 + h_{ie})}$$

$$1 = \frac{-s^3h_{fe}L_1L_2C}{(s^3L_1L_2C + s^2Ch_{ie}(L_1+L_2) + sL_1 + h_{ie})} \dots\dots\dots\dots (5)$$

Substituting $s = j\omega, s^2 = j^2\omega^2 = -\omega^2, s^3 = j^3\omega^3 = -j\omega^3$

$$1 = \frac{j\omega^3h_{fe}L_1L_2C}{-j\omega^3L_1L_2C - \omega^2Ch_{ie}(L_1+L_2) + j\omega L_1 + h_{ie}}$$

Separating real and imaginary values

$$1 = \frac{j\omega^3h_{fe}L_1L_2C}{[h_{ie} - \omega^2Ch_{ie}(L_1+L_2)] + j\omega L_1(1-\omega^2L_2C)} \dots\dots\dots\dots.. (6)$$

Rationalizing the R.H.S of the above equation,

$$1 = \frac{j\omega^3h_{fe}L_1L_2C\big[[h_{ie} - \omega^2Ch_{ie}(L_1+L_2)] - j\omega L_1(1-\omega^2L_2C)\big]}{[h_{ie} - \omega^2Ch_{ie}(L_1+L_2)] + j\omega L_1(1-\omega^2L_2C)}$$

$$1 = \frac{\omega^4h_{fe}L_1^2L_2C(1-\omega^2L_2C) + j\omega^3h_{fe}L_1L_2C[h_{ie} - \omega^2Ch_{ie}(L_1+L_2)]}{[h_{ie} - \omega^2Ch_{ie}(L_1+L_2)]^2 + \omega^2L_1^2(1-\omega^2L_2C)^2} \dots\dots\dots. (7)$$

To satisfy this equation, imaginary part of R.H.S must be zero.

$$\omega^3h_{fe}L_1L_2Ch_{ie}[1 - \omega^2C(L_1+L_2)] = 0$$

$$1 - \omega^2C(L_1+L_2) = 0$$

$$\omega^2C(L_1+L_2) = 1$$

$$\omega^2 = \frac{1}{C(L_1 + L_2)}$$

$$\omega = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

$$f = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

$$f = \frac{1}{2\pi\sqrt{CL_{eq}}}$$

`Where $L_{eq} = L_1 + L_2$

This is the frequency of the oscillations.To find condition for oscillation equating the real part (magnitudes of both sides) of the equation (7),

$$1 = \frac{\omega^4 h_{fe} L_1^2 L_2 C(1 - \omega^2 L_2 C)}{[h_{ie} - \omega^2 C h_{ie}(L_1 + L_2)]^2 + \omega^2 L_1^2 (1 - \omega^2 L_2 C)^2} \quad at\ \omega = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

$$1 = \frac{h_{fe} L_2}{(1 - \omega^2 L_2 C)} \quad at\ \omega = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

$$1 = \frac{h_{fe} L_2}{\left(1 - \frac{L_2 C}{C(L_1 + L_2)}\right)} = \frac{h_{fe} L_2}{L_1}$$

$$h_{fe} = \frac{L_1}{L_2}$$

This is the value of h_{fe} required to satisfy the oscillating condition.

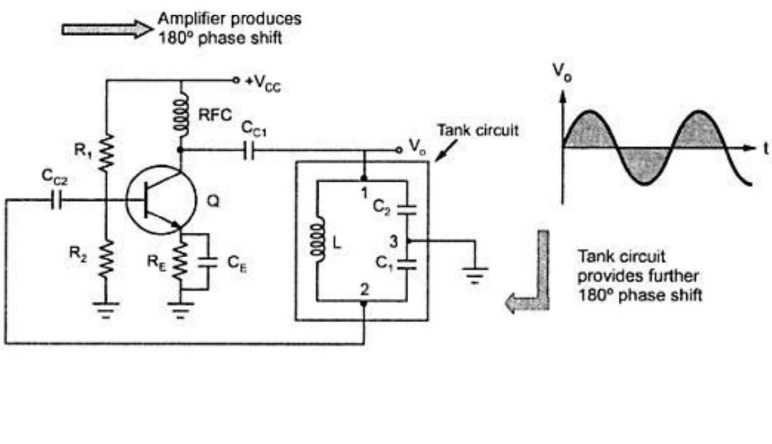
2.10 Colpitts Oscillators

A LC oscillator which uses **two capacitive reactances and oneinductivereactance** in its feedback network is called **ColpittsOscillators**.

2.10.1 TransistorisedColpitts Oscillators

The amplifier stage transistor in common emitter configuration.

The basic circuit is same as transistorized Hartley Oscillator, except the tank circuit. The common emitter amplifier causes a phase shift of 180^0, while the tank circuit adds further 180^0 phase shift, to satisfy the oscillating conditions.



2.10.2 Derivation of frequency of oscillation

The output current I_C which is $h_{fe}I_B$ acts as input to the feedback network. While the base current I_B acts as the output current of the tank circuit. The equivalent circuit of the feedback is shown in the fig.

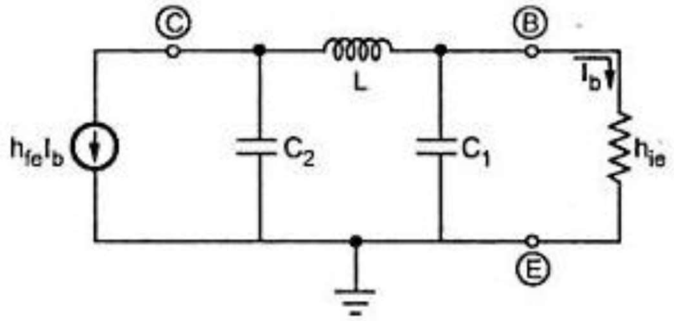


Fig : Equivalent circuit

Converting the current source into the voltage source, we get the equivalent circuit as shown in the fig.

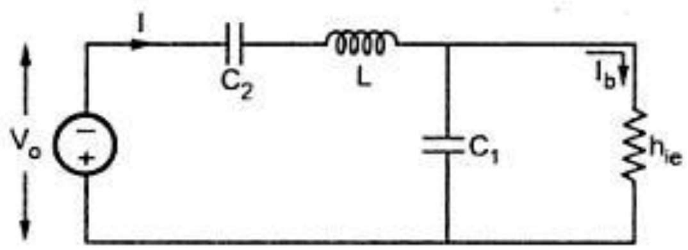


Fig : Simplified equivalent circuit

$$V_0 = h_{fe}I_bX_{C2} = h_{fe}I_b\frac{1}{j\omega C_2} \dots\dots\dots\dots\dots\dots (1)$$

Now C_1 and h_{ie} are in parallel, so the total current I drawn from the supply is,

$$I = \frac{-V_0}{[X_{C2} + X_L] + [X_{C1} \parallel h_{ie}]} \dots\dots\dots\dots (2)$$

Note: Negative sign, as current direction shown in opposite to the polarities of V_0

Now

$$X_{C2} + X_L = \frac{1}{j\omega C_2} + j\omega L \quad \text{and}$$

$$X_{C1} \parallel h_{ie} = \frac{\frac{1}{j\omega C_1} \times h_{ie}}{\left[\frac{1}{j\omega C_1} + h_{ie}\right]}$$

Substituting in the equation (2)

$$I = \frac{-h_{fe} I_b \frac{1}{j\omega C_2}}{\left[\frac{1}{j\omega C_2} + j\omega L\right] + \left[\frac{\frac{1}{j\omega C_1} \times h_{ie}}{\left[\frac{1}{j\omega C_1} + h_{ie}\right]}\right]} \ldots\ldots\ldots\ldots.(3)$$

Replacing $j\omega = s$

$$I = \frac{-h_{fe} I_b \frac{1}{sC_2}}{\left[\frac{1}{sC_2} + sL\right] + \left[\frac{\frac{1}{sC_1} \times h_{ie}}{\left[\frac{1}{sC_1} + h_{ie}\right]}\right]}$$

$$= \frac{-h_{fe} I_b \left(\frac{1}{sC_2}\right)}{\frac{(1+s^2LC_2)}{sC_2} + \left[\frac{h_{ie}}{1+sC_1 h_{ie}}\right]}$$

$$= \frac{-h_{fe} I_b \left(\frac{1}{sC_2}\right)(sC_2)(1 + sC_1 h_{ie})}{(1 + s^2LC_2)(1 + sC_1 h_{ie}) + h_{ie} sC_2}$$

$$= \frac{-h_{fe} I_b (1 + sC_1 h_{ie})}{s^3LC_2C_1h_{ie} + s^2LC_2 + sC_1h_{ie} + 1 + sC_2h_{ie}}$$

$$= \frac{-h_{fe} I_b (1 + sC_1 h_{ie})}{s^3LC_1C_2h_{ie} + s^2LC_2 + sh_{ie}(C_1 + C_2) + 1} \ldots\ldots\ldots\ldots.(4)$$

According to current division in parallel circuit,

$$I_b = I \times \frac{X_{C1}}{X_{C1} + h_{ie}}$$

$$= I \times \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + h_{ie}}$$

$$I_b = \frac{I}{(1 + SC_1 h_{ie})} \ldots\ldots\ldots\ldots\ldots(5)$$

Substituting value of I from equation (4) in equation (5)

$$I_b = \left[\frac{-h_{fe} I_b (1 + sC_1 h_{ie})}{s^3LC_1C_2h_{ie} + s^2LC_2 + sh_{ie}(C_1 + C_2) + 1}\right] \times \left[\frac{1}{(1 + SC_1 h_{ie})}\right]$$

$$I_b = \frac{-h_{fe} I_b}{s^3LC_1C_2h_{ie} + s^2LC_2 + sh_{ie}(C_1 + C_2) + 1}$$

$$1 = \frac{-h_{fe}}{s^3LC_1C_2h_{ie} + s^2LC_2 + sh_{ie}(C_1 + C_2) + 1} \ldots\ldots\ldots\ldots(6)$$

Substituting $s = j\omega, s^2 = j^2\omega^2 = -\omega^2, s^3 = j^3\omega^3 = -j\omega^3$

$$1 = \frac{-h_{fe}}{-j\omega^3LC_1C_2h_{ie} - \omega^2LC_2 + j\omega h_{ie}(C_1 + C_2) + 1}$$

Separating real and imaginary values

$$= \frac{-h_{fe}}{(1-\omega^2 LC_2) + j\omega h_{ie}(C_1 + C_2 - \omega^2 LC_1C_2)} \ldots\ldots\ldots\ldots\ldots(7)$$

To satisfy this equation, imaginary part of denominator R.H.S must be zero.

$$\omega h_{ie}(C_1 + C_2 - \omega^2 LC_1C_2) = 0$$

$$C_1 + C_2 - \omega^2 LC_1C_2 = 0$$

$$C_1 + C_2 = \omega^2 LC_1C_2$$

$$\omega^2 = \frac{C_1 + C_2}{LC_1C_2} = \frac{1}{L\left[\frac{C_1C_2}{C_1+C_2}\right]}$$

$$\omega = \frac{1}{\sqrt{L\left[\frac{C_1C_2}{C_1+C_2}\right]}}$$

Now $\frac{C_1C_2}{C_1+C_2}$ is nothing but the equivalent ot two capacitors C_1 and C_2 in series

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$\omega = \frac{1}{\sqrt{LC_{eq}}}$$

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

This is the frequency of the oscillations. To find condition for oscillation equating the real part (magnitudes of both sides) of the equation (7),

$$1 = \frac{-h_{fe}}{(1-\omega^2 LC_2)} \; at \; \omega^2 = \frac{C_1 + C_2}{LC_1C_2}$$

$$1 = \frac{-h_{fe}}{\left(1 - \left(\frac{C_1+C_2}{LC_1C_2} LC_2\right)\right)}$$

$$1 = \frac{-h_{fe}}{\left(1 - \left(\frac{C_1+C_2}{C_1}\right)\right)}$$

$$1 - \left(\frac{C_1 + C_2}{C_1}\right) = -h_{fe}$$

$$1 - 1 - \frac{C_2}{C_1} = -h_{fe}$$

$$h_{fe} = \frac{C_2}{C_1}$$

This is the value of h_{fe} required to satisfy the oscillating condition.

2.11 Clapp Oscillators(Modified form of colpitts oscillator of variable colpitts oscillator)

To achieve the frequency stability, colpitts oscillator circuit is slightly modified in practice, called **Clapp Oscillators.** Thebasic tank circuit with two **capacitive reactance's and**

oneinductivereactance remains same. But the modification in the tank circuit is that **one more capacitor C_3 is introduced in series with the inductance** as shown in fig

2.11.1 TransistorisedClapp Oscillators

The amplifier stage transistor in common emitter configuration.

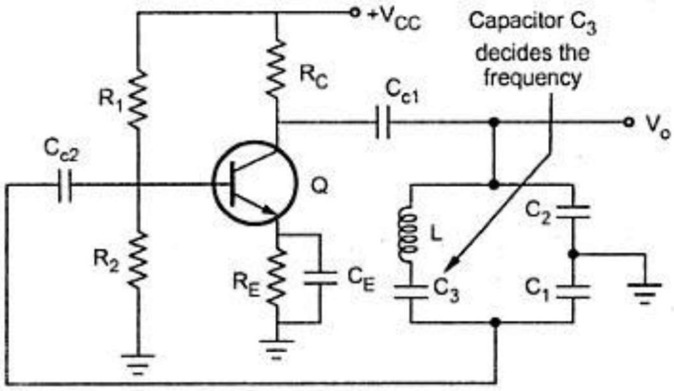


Fig :TransistorisedClapp Oscillators

2.11.2 Derivation of frequency of oscillation

The equivalent circuit of the feedback is shown in the fig.

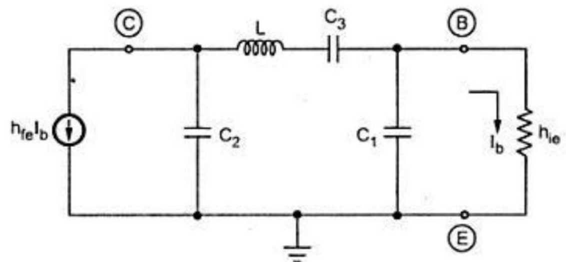
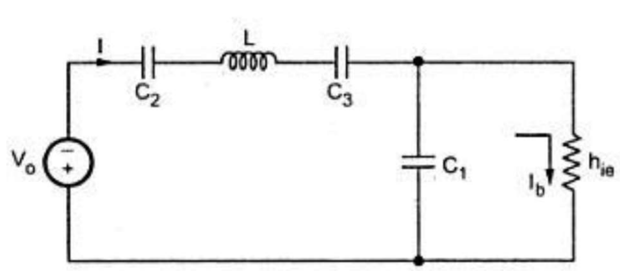


Fig : Equivalent circuit

Converting the current source into the voltage source, we get the equivalent circuit as shown in the fig.



$$V_0 = h_{fe} I_b X_{C2} = h_{fe} I_b \frac{1}{j\omega C_2} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now C_1 *and* h_{ie} are in parallel, so the total current I drawn from the supply is,

$$I = \frac{-V_0}{[X_{C2} + X_{C3} + X_L] + [X_{C1} \parallel h_{ie}]} \dots\dots\dots\dots(2)$$

Note: Negative sign, as current direction shown in opposite to the polarities of V_0

Now

$$X_{C2} + X_{C3} + X_L = \frac{1}{j\omega C_2} + \frac{1}{j\omega C_3} + j\omega L \qquad \text{and}$$

$$X_{C1} \parallel h_{ie} = \frac{\frac{1}{j\omega C_1} \times h_{ie}}{\left[\frac{1}{j\omega C_1} + h_{ie}\right]}$$

Substituting in the equation (2)

$$I = \frac{-h_{fe} I_b \frac{1}{j\omega C_2}}{\left[\frac{1}{j\omega C_2} + \frac{1}{j\omega C_3} + j\omega L\right] + \left[\frac{\frac{1}{j\omega C_1} \times h_{ie}}{\left[\frac{1}{j\omega C_1} + h_{ie}\right]}\right]} \dots\dots\dots\dots(3)$$

Replacing $j\omega = s$

$$I = \frac{-h_{fe} I_b \frac{1}{sC_2}}{\left[\frac{1}{sC_2} + \frac{1}{sC_3} + sL\right] + \left[\frac{\frac{1}{sC_1} \times h_{ie}}{\left[\frac{1}{sC_1} + h_{ie}\right]}\right]}$$

Multiplying by $sC_2$to denominator

$$= \frac{-h_{fe} I_b}{1 + \frac{C_2}{C_3} + s^2 L C_2 + \left[\frac{sC_2 h_{ie}}{1 + sC_1 h_{ie}}\right]}$$

$$= \frac{-h_{fe} I_b C_3}{C_3 + C_2 + s^2 L C_2 C_3 + \left[\frac{sC_2 C_3 h_{ie}}{1 + sC_1 h_{ie}}\right]}$$

$$= \frac{-h_{fe} I_b C_3 (1 + sC_1 h_{ie})}{C_3 + sC_1 C_3 h_{ie} + C_2 + sC_1 C_2 h_{ie} + s^2 L C_2 C_3 + s^3 L C_1 C_2 C_3 h_{ie} + sC_2 C_3 h_{ie}}$$

$$= \frac{-h_{fe} I_b C_3 (1 + sC_1 h_{ie})}{s^3 L C_1 C_2 C_3 h_{ie} + s^2 L C_2 C_3 + s h_{ie}[C_2 C_3 + C_1 C_2 + C_1 C_3] + C_2 + C_3} \dots\dots\dots(4)$$

According to current division in parallel circuit,

$$I_b = I \times \frac{X_{C1}}{X_{C1} + h_{ie}}$$

$$= I \times \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + h_{ie}}$$

$$I_b = \frac{I}{(1 + SC_1 h_{ie})} \dots\dots\dots\dots\dots(5)$$

Substituting value of I from equation (4) in equation (5)

$$I_b = \left[\frac{-h_{fe} I_b C_3 (1 + sC_1 h_{ie})}{s^3 L C_1 C_2 C_3 h_{ie} + s^2 L C_2 C_3 + s h_{ie}[C_2 C_3 + C_1 C_2 + C_1 C_3] + C_2 + C_3}\right] \times \left[\frac{1}{(1 + SC_1 h_{ie})}\right]$$

$$I_b = \frac{-h_{fe}I_bC_3}{s^3LC_1C_2C_3h_{ie} + s^2LC_2C_3 + sh_{ie}[C_2C_3 + C_1C_2 + C_1C_3] + C_2 + C_3}$$

$$1 = \frac{-h_{fe}C_3}{s^3LC_1C_2C_3h_{ie} + s^2LC_2C_3 + sh_{ie}[C_2C_3 + C_1C_2 + C_1C_3] + C_2 + C_3}$$

Substituting $s = j\omega, s^2 = j^2\omega^2 = -\omega^2, s^3 = j^3\omega^3 = -j\omega^3$

$$1 = \frac{-h_{fe}C_3}{-j\omega^3LC_1C_2C_3h_{ie} - \omega^2LC_2C_3 + j\omega h_{ie}[C_2C_3 + C_1C_2 + C_1C_3] + C_2 + C_3}$$

Separating real and imaginary values

$$1 = \frac{-h_{fe}C_3}{C_2 + C_3 - \omega^2LC_2C_3 + j\omega h_{ie}\{[C_2C_3 + C_1C_2 + C_1C_3] - \omega^2LC_1C_2C_3\}} \dots\dots\dots..(6)$$

To satisfy this equation, imaginary part of denominator R.H.S must be zero.

$$\omega h_{ie}\{[C_2C_3 + C_1C_2 + C_1C_3] - \omega^2LC_1C_2C_3\} = 0$$

$$C_2C_3 + C_1C_2 + C_1C_3 - \omega^2LC_1C_2C_3 = 0$$

$$\omega^2 = \frac{C_1C_2 + C_2C_3 + C_1C_3}{LC_1C_2C_3} = \frac{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}{L} = \frac{\frac{1}{C_{eq}}}{L}$$

$$\omega^2 == \frac{1}{LC_{eq}}$$

$$\omega = \frac{1}{\sqrt{LC_{eq}}}$$

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

Where $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

This is the frequency of the oscillations. To find condition for oscillation equating the real part (magnitudes of both sides) of the equation (6),

$$1 = \frac{-h_{fe}C_3}{C_2 + C_3 - \omega^2LC_2C_3} \; at \; \omega^2 = \frac{C_1C_2 + C_2C_3 + C_1C_3}{LC_1C_2C_3}$$

$$C_2 + C_3 - \omega^2LC_2C_3 = -h_{fe}C_3 \; at \; \omega^2 = \frac{C_1C_2 + C_2C_3 + C_1C_3}{LC_1C_2C_3}$$

$$C_2 + C_3 - \left(\frac{C_1C_2 + C_2C_3 + C_1C_3}{LC_1C_2C_3}\right)LC_2C_3 = -h_{fe}C_3$$

$$C_2 + C_3 - \left(\frac{C_1C_2 + C_2C_3 + C_1C_3}{C_1}\right) = -h_{fe}C_3$$

$$\frac{C_1C_2 + C_1C_3 - C_1C_2 - C_2C_3 - C_1C_3}{C_1} = -h_{fe}C_3$$

$$-\frac{C_2C_3}{C_1} = -h_{fe}C_3$$

$$h_{fe} = \frac{C_2}{C_1}$$

This is the value of h_{fe} required to satisfy the oscillating condition.

2.12 Armstrong Oscillator

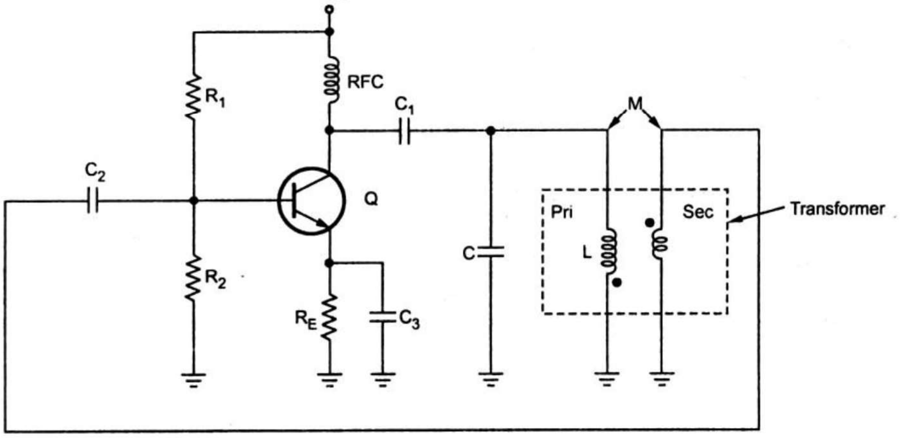


Fig :Armstrong Oscillator

In this circuit collector drives an LC resonant circuit. The primary winding of transformer L and capacitor C_3 forms a resonant circuit. The feedback signal is taken from a small secondary winding and feedback to the base. There is a phase shift of 180^0 in the transformer and another 180^0 phase shift is produced by amplifier as a result total phase shift is 0^0 or 180^0 which satisfies the Barkhauen Criterion, thus the circuit produce sustain oscillation.

2.13 Frankling Oscillator

In this circuit tank circuit connected in the collector circuit acts as load impedance it determines the frequency of oscillation.

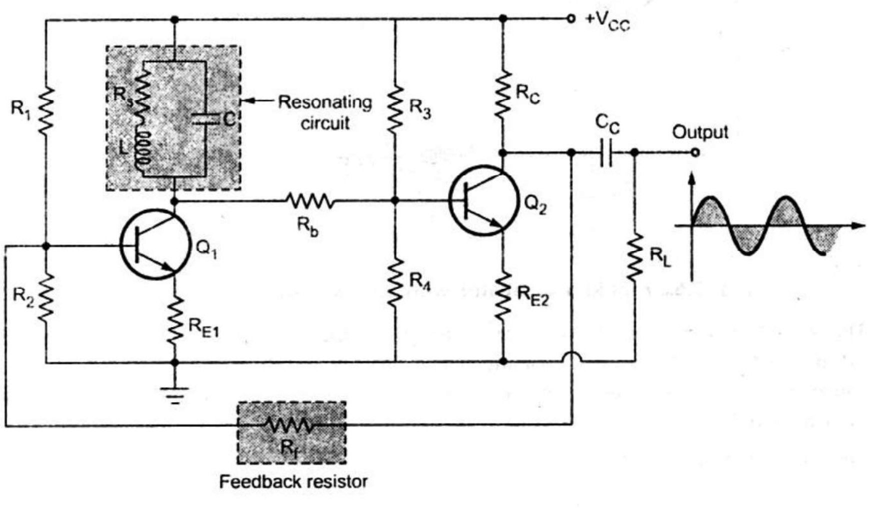
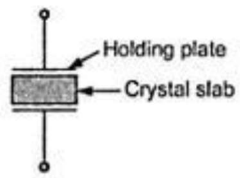


Fig :Frankling Oscillator

2.14 Crystal Oscillator

The crystals are either naturally accruing or synthetically manufactured, exhibiting the piezo electric effect. This means under the influence of the mechanical pressure, the voltage gets generated across the opposite faces of the crystal. If the mechanical force is applied in such a way to force the crystal to vibrate, the a.c voltage gets generated across it. The influence of mechanical vibration, the crystal generates an electrical signal to very constant frequency.

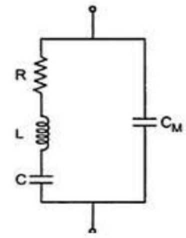
2.14.1 Construction Details



The natural shape of a quartz crystal is a hexagonal prism. But for its practical use, it is cut to the rectangular slab. This slab is than mounted between the two metal plates (Holding Plates).

2.14.2 A.C. Equivalent Circuit

When the crystal is not vibrating, it is a equivalent to a capacitance due to the mechanical mounting of a crystal. Such a capacitance existing due to the two metal plates separated by a dielectric like crystal slab is called 'Mounting Capacitance' C_M.



A.C. equivalent circuit of a crystal

When it is vibrating, there are internal frictional losses which are denoted by a resistance 'R' inductance 'L', capacitance 'C'. The mounting capacitance is shunt capacitance.

RLC forms a resonating circuit. The expression for the resonating frequency f_r is,

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{1+Q^2}}$$

Where,

Q- Quality factor of crystal

$$Q = \frac{\omega L}{R}$$

The Q factor of the crystal is very large, typically 20,000. Value of Q upto 10^6 also can be achieved. Hence $\sqrt{\frac{Q^2}{1+Q^2}}$ factor approaches to unity.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

The crystal frequency is in fact inversely proportional to the thickness of the crystal.

$$f = \frac{1}{t}$$

Where t- Thickness.

2.14.3 Series and parallel Resonance

The series resonance frequency is same as the resonating frequency given by

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

Under parallel resonance the equivalent capacitance is,

$$C_{eq} = \frac{C_M C}{C_M + C}$$

The parallel resonance frequency is given by,

$$f_p = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

2.14.4 Pierce Crystal Oscillator

The colpitts oscillator can be modified by using crystal to have as an inductor. The circuit is called pierce crystal oscillator.

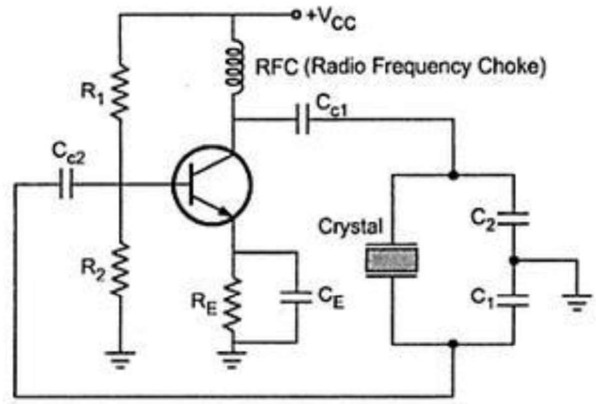


Fig :Pierce Crystal Oscillator

2.14.5 Miller Crystal Oscillator

The Hartley oscillator can be modified to get miller crystal oscillator. In Hartley oscillator uses two inductorsand one capacitor is required in the tank circuit. One inductor is replaced by crystal which acts as an inductor for frequency slightly greater than the series resonant frequency.

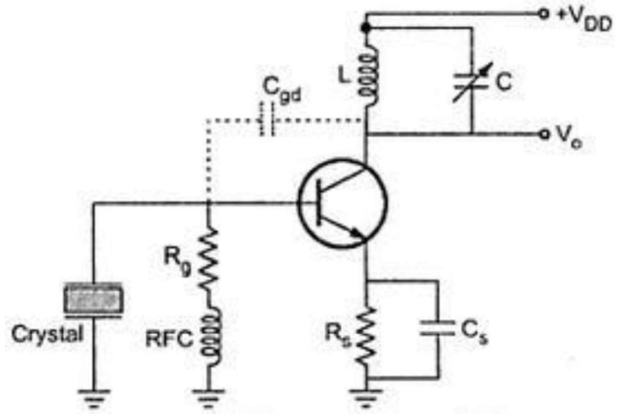


Fig :Miller Crystal Oscillator

2.15 Example with Solutions

Example 1: The frequency sensitive arms of the wien bridge oscillator uses $C_1 = C_2 = 0.001\mu F$ and $R_1 = 10K\Omega$ while R_2 is kept constant. The frequency is to be varied from 10kHz to 50kHz, by varying R_2. Find the minimum and maximum values of R_2.

Solution: The frequency of the oscillator is given by,

$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$f = 10kHz$$

$$10 \times 10^3 = \frac{1}{2\pi\sqrt{10 \times 10^3 \times R_2 \times (0.001 \times 10^{-6})^2}}$$

$$R_2 = 25.33k\Omega$$

Example 2: In a transistorized Hartley oscillator the inductance are 2mH and 20µH while the frequency is to be changed from 950kHz to 2050kHz. Calculate the range over which the capacitor is to be varied.

Solution: The frequency of the oscillator is given by,

$$f = \frac{1}{2\pi\sqrt{CL_{eq}}}$$

Where $L_{eq} = L_1 + L_2 = 2 \times 10^{-3} + 20 \times 10^{-6}$

$$= 0.00202kHz$$

$$f = f_{max} = 2050kHz$$

$$2050 \times 10^3 = \frac{1}{2\pi\sqrt{C \times 0.00202}}$$

$$C = 2.98pF$$

$$f = f_{min} = 950kHz$$

$$950 \times 10^3 = \frac{1}{2\pi\sqrt{C \times 0.00202}}$$

$$C = 13.89pF$$

Hence C must be varied from 2.98pF to 13.89pF, to get the required frequency variation.

Example 3: find the frequency of the oscillations of a transistorized colpitts oscillator having $C_1 = 150pF, C_2 = 1.5nF\ and\ L = 50\mu H$.

Solution: The frequency of the oscillator is given by,

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

Where,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{150 \times 10^{-12} \times 1.5 \times 10^{-9}}{(150 \times 10^{-12} + 1.5 \times 10^{-9})} = 136.363pF$$

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}} = \frac{1}{2\pi\sqrt{50 \times 10^{-6} \times 136.363 \times 10^{-12}}} = 1.927MHz$$

Example 3: A crystal $L = 0.4H, C = 0.085pF\ and\ C_M = 1pF\ with\ R = 5K\Omega$. Find

i. Series resonant frequency
ii. Parallel resonant frequency
iii. By what percent does the parallel resonant frequency exceed the series resonant frequency?
iv. Find the Q factor of the crystal.

Solution:

Series resonant frequency

$$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.4 \times 0.085 \times 10^{-12}}} = 0.856MHz$$

Parallel resonant frequency

$$C_{eq} = \frac{CC_M}{C + C_M} = \frac{0.085 \times 1}{0.085 \times 1} = 0.078pF$$

$$f_s = \frac{1}{2\pi\sqrt{LC_{eq}}} = \frac{1}{2\pi\sqrt{0.4 \times 0.078 \times 10^{-12}}} = 0.899MHz$$

% increase

$$\%\,increase = \frac{0.899 - 0.856}{0.856} \times 100 = 5.023\%$$

Q factor of the crystal

$$Q = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi \times 0.856 \times 10^6 \times 0.4}{5 \times 10^3} = 430.272$$