**UNIT- III**
**RANDOM PROCESS**

## 3.1 RANDOM VARIABLE

**Definition:** A function which takes on any value from the sample space and it's range is some set of real numbers is called a random variable.

**Example 1:**

Consider the example of tossing a fair coin twice. The sample space is S= {HH, HT, TH, TT} and all four outcomes are equally likely. Then we can define a random variable X as follows

| Sample Point Ω | Value of the random Variable X = x | P{X = x} |
|---|---|---|
| HH | 0 | ¼ |
| HT | 1 | ¼ |
| TH | 2 | ¼ |
| TT | 3 | ¼ |

A (real) random variable is a mapping from the sample space Ω to the set of real numbers. A schematic diagram representing a random variable is given in Figure 4.1.

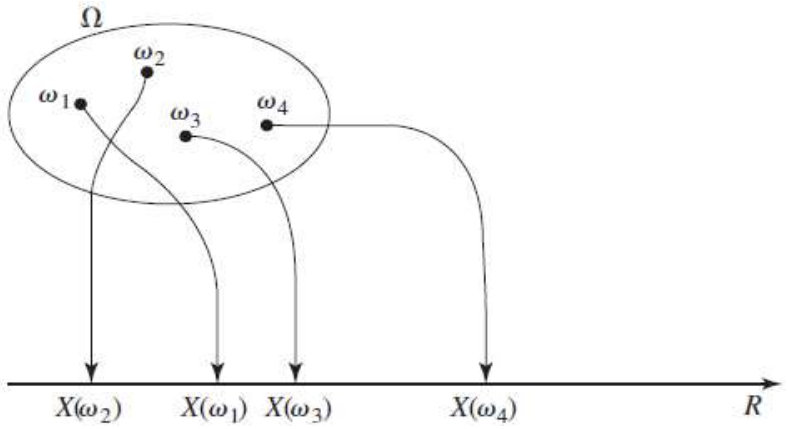



Figure 3.1 Random variable as a mapping from Ω to R.

Random variables are denoted by capital letters X, Y, etc.; individual values of the random variable X are X(ω).

**3.1.1 Discrete and Continuous Random Variables**

• A random variable X is called discrete if there exists a countable sequence of distinct real number $x_i$ such that $\sum_i P_m(x_i)=1.P_m(x_i)$ is called the probability mass function.

• A continuous random variable X can take any value from a continuous interval

• A random variable may also de mixed type. In this case the Random variable takes continuous values, but at each finite number of points there is a finite probability.

A random variable is discrete if the range of its values is either finite or countably infinite. This range is usually denoted by {xi}.The cumulative distribution function (CDF) of a random variable X is defined as FX (x) = P(ω ε Ω : X(ω) ≤ x) which can be simply written as FX (x) = P(X ≤ x) and has the following properties:

1. 0 ≤ FX (x) ≤ 1.
2. FX (x) is non-decreasing.
3. limx→−∞ FX (x) = 0 and limx→+∞ FX (x) = 1.
4. FX (x) is continuous from the right; i.e., limε↓0 F(x + ε) = F(x)
5. P(a < X ≤ b) = FX (b) − FX (a).
6. P(X = a) = FX (a) − FX (a−).

For discrete random variables FX (x) is a stair-case function. A random variable is called continuous if FX (x) is a continuous function. A random variable is called mixed if it is neither discrete nor continuous.

**3.1.2 Probability Distribution Function**

We can define an event {X ≤ x} = {s / X (s) ≤ x, s ε S}

The probability $F_X(x)$ =P {X≤ x} is called the probability distribution function.

Given $F_X(x)$, we can determine the probability of any event involving values of the random variable X

•$F_X(x)$ is a non-decreasing function of X.

•$F_X(x)$ is right continuous

=> F (x) approaches to its value from right.

•$F_X(-\infty) = 0$

•$F_X(\infty) = 1$

• $P\{x_1 < X \le x\} = F_X(x) - F_X(x_1)$

**3.1.3 Probability Density Function**

If $F_X(x)$ is differentiable $f_X(x) = \frac{d}{dx} F_X(x)$ is called the probability density function and has the following properties.

• $f_X(x)$ is a non- negative function

- $\int_{-\infty}^{\infty} f_X(x)dx = 1$
- $P(x_1<X\leq x_2) = \int_{-x1}^{x2} f_X(x)dx$

**3.1.4 Cumulative Distribution Function (CDF):**

The CDF of a random variable 'X 'is the probability that a random variable 'X 'takes a value less than or equal to x. The cumulative distribution function of a real-valued random variable X is the function given by

$F_X(x) = \mathrm{P}(X \leq x),$

where the right-hand side represents the probability that the random variable X takes on a value less than or equal to x. The probability that X lies in the semi-closed interval (a, b), where a < b, is therefore

$\mathrm{P}(a < X \leq b) = F_X(b) - F_X(a).$

The CDF of a continuous random variable X can be expressed as the integral of its probability density function $f_X$ as follows:

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt.$$

In the case of a random variable X which has distribution having a discrete component at a value b,

$$\mathrm{P}(X = b) = F_X(b) - \lim_{x \to b^-} F_X(x).$$

If $F_X$ is continuous at b, this equals zero and there is no discrete component at b.

**3.1.5 Important Random Variables:**

The most commonly used random variables in communications are:

**Bernoulli Random Variable:**

This is a discrete random variable taking two values one and zero with probabilities p and 1 − p. A Bernoulli random variable is

a good model for a binary data generator. Also, when binary data is transmitted over a communication channel, some bits are received in error. We can model an error by modulo-2 addition of a 1 to the input bit, thus changing a 0 into a 1 and a 1 into

a 0. Therefore, a Bernoulli random variable can be employed to model the channel errors.

Binomial Random Variable:

This is a discrete random variable giving the number of 1's in a sequence of n independent Bernoulli trials. The PMF is given by

$$P(X=k)=\begin{cases}\binom{n}{k}p^k(1-p)^{n-k}, & 0\le k\le n\\ 0, & \text{otherwise}\end{cases}$$

This random variable models, for example, the total number of bits received in error when a sequence of n bits is transmitted over a channel with bit-error probability of p.

**Uniform Random Variable:**

This is a continuous random variable taking values between a and b with equal probabilities over intervals of equal length. The density function is given by

$$f_X(x)=\begin{cases}\frac{1}{b-a}, & a<x<b\\ 0, & \text{otherwise}\end{cases}$$

This is a model for continuous random variables whose range is known, but nothing else is known about the likelihood of various values that the random variable can assume.

For example, when the phase of a sinusoid is random it is usually modeled as a uniform random variable between 0 and $2\pi$.

**Gaussian or Normal Random Variable:**

This is a continuous random variable described by the density function

$$f_X(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}$$

The Gaussian random variable is the most important and frequently encountered random variable in communications. The reason is that thermal noise, which is the major source of noise in communication systems, has a Gaussian distribution.

### 3.2 CENTRAL LIMIT THEOREM

The central limit theorem provides the mathematical justification for using a Gaussian Process as a model for a large number of different physical phenomena in which the observed random, at a particular instant of time, is the result of a large number of individual random events. Let $X_i$, i=1,2,...,N be a set of random variables that satisfies the following requirements.

1. $X_i$ are statistically independent
2. $X_i$ have the same probability distribution with mean $\mu_x$ and variance $\sigma^2_x$.

$X_i$ constitutes a set of independently and identically distributed random variables. Let these random variables be normalised as follows

$$Y_i=\frac{1}{\sigma_X}(X_i-\mu_X),\qquad i=1,2,\ldots,N$$

So that we have $E[Y_i]=0$ and $Var[Y_i]=1$

Define the random variable

$$V_N=\frac{1}{\sqrt{N}}\sum_{i=1}^{N}Y_i$$

The central limit theorem states that the probability distribution of $V_N$ approaches a normalised Gaussian distribution N(0,1) in the limit as the number of random variables approaches infinity.

### 3.3 RANDOM PROCESS

A random process is a indexed collection of random variables, or equivalently a non-deterministic signal that can be described by a probability distribution.

A random process is a function of two variables, namely s∈S and t ∈(−∞, ∞). As such, a better notation would be X(s,t). For convenience, we use the simplified notation X(t) to denote a random process.

Given a random process X(t), we can identify the following quantities:

X(t): The random process

$X_j(t)$: The sample function associated with the sample point $s_j$

$X(t_i)$: The random variable obtained by observing the process at $t=t_i$

$X_j(t_i)$: A real number, giving the value of $X_j(t)$ at $t=t_i$

We may distinguish between a random variable and a random process as follows:

- For a random variable, the outcome of a random experiment is mapped in to a number.
- For a random process, the outcome of a random experiment is mapped in to a waveform that is a function of time.

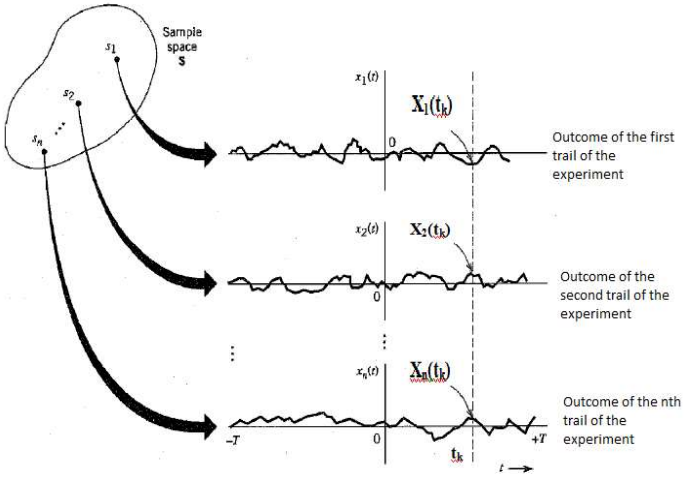



Figure 3.2 An ensemble of sample

### 3.4 STATIONARY PROCESS

The statistical characteristic of a random process is independent of the time at which observation of the process is initiated. If the random process is divided into number of time intervals, the various section of the process exhibits essentially the same statistical properties. Such a process is said to be stationary.

Let $X(t_1)$, $X(t_2)$, .....$X(t_n)$ be a random variable at time $t_1$, $t_2$, .....$t_n$ and the joint distribution function of random variable is $F_{X(t1),\ X(t2),\ .....X(tn)}(x_1,x_2,.....x_n)$ . Suppose we shift the observation

time by a fixed amount $t_0$, thereby obtaining a new set of random variables $X(t_1+ t_0)$, $X(t_2+ t_0)$, .....$X(t_n+ t_0)$ A random process is called strict-sense stationary if it satisfies the condition

$$F_{X(t1),\ X(t2),\ .....X(tn)}(x_1,x_2,.....x_n) = F_{X(t1+ t0),\ X(t2+ t0),\ .....X(tn+ t0)}(x_1,x_2,.....x_n)$$

In application, we encounter three methods of specification. The first (and simplest) is to state the rule directly. For this to be possible, the joint density function must depend in a known way on the time instants. For the second method, a time function involving one or more parameters is given. For example, $X(t)=A\cos(\omega_c t+\Theta)$ where A and $\omega_c$ are constants and $\Theta$ is a random variable with a known PDF. The third method of specifying a random process is to generate its ensemble by applying a stated operation to the sample functions of a known process.

## 3.5 MEAN, CORRELATION & COVARIANCE FUNCTIONS

### 3.5.1 Mean

The mean of the random process is given by

$$\mu_{X(t)} = E[X(t)] = \int_{-\infty}^{\infty} x f_{x(t)}(x)\, dx$$

where $f_{x(t)}(x)$ ⟶ 1st order probability density function of the process $f_{x(t)}(x)$ is independent of t.

The mean of a strictly stationary process is a constant. i.e., $\mu_{X(t)} = \mu_X$ for all t

### 3.5.2 Covariance Function

The covariance function is a number that measures the common variation of X and Y. It is defined as

$$\text{cov}(X, Y) = E[(X - E[X])(Y - E[Y])]$$
$$= E[XY] - E[X]E[Y]$$

The covariance is determined by the difference in E[XY ] and E[X]E[Y ]. If X and Y were statistically independent then E[XY ] would equal E[X]E[Y ] and the covariance would be zero. Hence, the covariance, as its name implies, measures the common variation. The covariance can be normalized to produce what is known as the correlation coefficient, ρ.

$$\rho = \frac{\text{cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

The correlation coefficient is bounded by $-1 \le \rho \le 1$. It will have value $\rho = 0$ when the covariance is zero and value $\rho = \pm 1$ when X and Y are perfectly correlated or anti-correlated.

### 3.5.3 Correlation

The numbers X(t1, e) and X(t2, e) are samples from the same time function at different times. This is a pair of random variables which we could write conveniently in terms of a doublet (X1,X2). It is described by a joint probability density function f(x1, x2; t1, t2). The

notation includes the times because the result surely can depend on when the samples are taken. A measure of particular interest is the correlation and covariance. The covariance is

$$C(t_1, t_2) = E[(X_1-\mu_1)(X_2-\mu_2)] = \int_{-\infty}^{\infty} (x_1-\mu_1)(x_2-\mu_2)f(x_1, x_2; t_1, t_2)dx_1dx_2$$

The correlation function is

$$R(t_1, t_2) = E[X_1X2] = \int_{-\infty}^{\infty} x_1x_2f(x_1, x_2; t_1, t_2)dx_1dx_2$$

$C(t_1, t_2) = R(t_1, t_2) - \mu_1\mu_2$

**3.5.4 Autocorrelation Function**

The autocorrelation1 function is very similar to the covariance function. It is defined as

$R(X, Y) = E[XY] = cov(X, Y) + E[X]E[Y]$

It retains the mean values in the calculation of the value. The random variables are orthogonal if $R(X, Y) = 0$.

**Properties of Autocorrelation Function**

1. The mean square value of the process may be obtained from $R_X(\tau)$ simply by putting $\tau$ =0 (i.e.,)

$R_X(0)=E[X^2(t)]$

2. The autocorrelation function $R_X(\tau)$ is an even function of $\tau$ (i.e.,)

$R_X(\tau)=R_X(-\tau)$

$=E[X(t)X(t-\tau)]$

3. The autocorrelation function $R_X(\tau)$ has its maximum magnitude at $\tau$ =0

$|R_X(\tau)| \leq R_X(0)$

4. If x(t) is a stationary and has a periodic component, then Rxx($\tau$) will have a periodic component with the same period.

5. If x(t) is a ergodic zero mean has no periodic component, then

$\lim |\tau| \to \infty \; R_X(\tau) = 0$

6. $R_X(\tau)$ cannot have an arbitrary shape

7. For a complex process $R_X(-\tau) = R^*_X(\tau)$

8. It is deterministic

9. If x(t) is a stationary process and it has no periodic component, then

$\lim |\tau| \to \infty \; R_X(\tau) = x^{-2}$

**3.5.5 Cross-correlation & Cross power Spectral Density**

**The Cross-correlation of the** two real random processes X(t) and Y(t) is defined as

$R_{XY}(t_1,t_2) = E[x(t_1)\, y(t_2)]$

Cross co variance of the two process X(t) and Y(t) is defined as

$C_{XY}(t_1,t_2) = R_{XY}(t_1,t_2) - \mu\, x(t_1)\;\; \mu\, y(t_2)$

Cross power Spectral Density

$$S_{XY}(w)= \int_{-\infty}^{\infty} R_{XY}(\tau)e^{-jw\tau}\, d\tau$$

Properties of **Cross-correlation function:**

1. The cross-correlation function for a jointly wss processes is defined as

$R_{XY}(\tau)=E\, X(t+\tau)Y(t)$

So that $R_{YX}(\tau)=E\, Y(t+\tau)X(t)$

$=E\, X(t)\, Y(t+\tau)$

$= R_{XY}(-\tau)$

Therefore,

$R_{YX}(\tau)=R_{XY}(-\tau)$

2. If X(t) and Y(t) are two WSS processes then

$$|R_{XY}(\tau)| = \sqrt{R_{XX}(0)R_{YY}(0)}$$

3. If the two process X(t) and Y(t) are orthogonal then

$R_{XY}(\tau)=0$

### 3.6 ERGODIC PROCESS

If the statistical average is equal to time average, then the process is ergodic process. The DC value of the signal X(t) by using time average

$$\mu_x(T) = \frac{1}{2T}\int_{-T}^{T} X(t)dt$$

$$E[\mu_x(T)] = \frac{1}{2T}\int_{-T}^{T} E[X(t)]dt$$

$$= \frac{1}{2T}\int_{-T}^{T} \mu_x dt = \frac{\mu x}{2T}\int_{-T}^{T} dt$$

$$= \frac{\mu x}{2T}(2T) = \mu_x$$

$\mu_x(T)$ represents the unbiased estimate of the ensemble average menu $\mu_x$. The process is ergodic in the mean if two conditions are satisfied. The time average $\mu_x(T)$ approaches the ensemble average $\mu_x$ in the limit as the observation time T approaches infinity. (i.e.,)

$$\lim_{T\to\infty} \mu_x(T) = \mu_x$$

The variance of $\mu_x(T)$ treated as random variable approaches zero on the limit as the observation time T approaches $\infty$.(i.e.,)

$$\lim_{T \to \infty} \mathrm{Var}[\mu_x(T)] = 0$$

Similarly time average autocorrelation function of a sample function X(t) is

$$R_X(\tau, T) = \frac{1}{2T} \int_{-T}^{T} X(t+\tau)\, X(t)\, dt$$

Therefore,

$$E[R_X(\tau, T)] = \frac{1}{2T} \int_{-T}^{T} E[X(t+\tau)\, X(t)]\, dt$$

$$= R_X(\tau)$$

This process is ergodic in the autocorrelation function (i.e.,) it satisfies the condition

$$\lim_{T \to \infty} R_X(\tau, T) = R_X(\tau)$$

$$\lim_{T \to \infty} \mathrm{Var}[R_X(\tau, T)] = 0$$

**3.7 GAUSSIAN PROCESS**

In probability theory and statistics, a Gaussian process is a stochastic process whose realizations consist of random values associated with every point in a range of times (or of space) such that each such random variable has a normal distribution. Moreover, every finite collection of those random variables has a multivariate normal distribution.

Gaussian processes are important in statistical modeling because of properties inherited from the normal distribution. For example, if a random process is modeled as a Gaussian process, the distributions of various derived quantities can be obtained explicitly. Such quantities include: the average value of the process over a range of times; the error in estimating the average using sample values at a small set of times.

A process is Gaussian if and only if for every finite set of indices *t*1, ..., *tk* in the index set *T*

$$\vec{\mathbf{X}}_{t_1,\ldots,t_k} = (\mathbf{X}_{t_1}, \ldots, \mathbf{X}_{t_k})$$

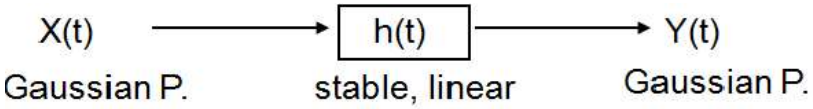
is a vector-valued Gaussian random variable. Using characteristic functions of random variables, the Gaussian property can be formulated as follows: { *Xt* ; *t* ∈ *T* } is Gaussian if and only if, for every finite set of indices *t*1, ..., *tk*, there are reals σ*l j* with σ*i i* > 0 and reals μ*j* such that

$$\mathrm{E}\left(\exp\left(i \sum_{\ell=1}^{k} t_\ell \mathbf{X}_{t_\ell}\right)\right) = \exp\left(-\frac{1}{2} \sum_{\ell,j} \sigma_{\ell j} t_\ell t_j + i \sum_{\ell} \mu_\ell t_\ell\right).$$

The numbers σ*l j* and μ*j* can be shown to be the covariances and means of the variables in the process.

**Properties of Gaussian process:**

Property 1.




If a Gaussian process X(t) is applied to a stable linear filter, then the random process Y(t) developed at the output of the filter is also Gaussian.

Property 2.

Consider the set of random variable or samples $X(t_1)$, $X(t_2)$, ···, $X(t_n)$ obtained by observing a random process X(t) at times $t_1, t_2, \cdots, t_n$.

If the process X(t) is Gaussian, then this set of random variables are jointly Gaussian for any n, with their n-fold joint p. d. f. being completely determined by specifying the set of means

$\mu_{x(ti)} = E[X(t_i)]$ , i = 1,2,….n

and the set of auto covariance functions

$C_X(t_k , t_i) = E[(X(t_k) - \mu_x(t_k))( X(t_i) - \mu_x(t_i))]$

Property 3.

If random variables $X(t_1)$, $X(t_2)$, ···, $X(t_n)$ from Gaussian process X(t) are uncorrelated, i. e.

$E[(X(t_k) - \mu_x(t_k))( X(t_i) - \mu_x(t_i))] = 0$, $i \neq k$

then these random variables are statistically independent

Property 4.

If a Gaussian process is stationary, then the process is also strictly stationary.

### 3.8 TRANSMISSION OF A RANDOM PROCESS THROUGH A LTI FILTER

A random process X(t) is applied as input to a linear time-invariant filter of impulse response h(t),It produces a random process Y (t) at the filter output as shown in Figure 1

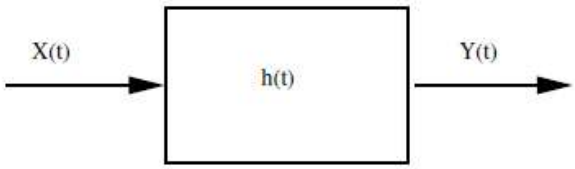



Figure 3.3: Transmission of a random process through a linear filter

Difficult to describe the probability distribution of the output random process Y (t), even when the probability distribution of the input random process X(t) is completely specified for $-\infty \leq t \leq +\infty$.

Estimate characteristics like mean and autocorrelation of the output and try to analyse its behaviour.

**Mean**

The input to the above system X(t) is assumed

stationary. The mean of the output random process Y (t) can be calculated.

$$\mu_Y(t) = E[Y(t)] = E\left[\int_{-\infty}^{+\infty} h(\tau)X(t-\tau)\,d\tau\right]$$

$$= \int_{-\infty}^{+\infty} h(\tau)E[X(t-\tau)]\,d\tau$$

$$= \int_{-\infty}^{+\infty} h(\tau)\,\mu_X(t-\tau)\,d\tau$$

$$= \mu_X \int_{-\infty}^{+\infty} h(\tau)\,d\tau$$

$$= \mu_X H(0)$$

where H(0) is the zero frequency response of the system.

**Autocorrelation**

The autocorrelation function of the output random process Y (t). By definition, we have

$R_Y$ (t; u) = E[Y (t)Y (u)]

where t and u denote the time instants at which the process is observed. We may therefore use the convolution integral to write

$$R_Y(t,u) = E\left[\int_{-\infty}^{+\infty} h(\tau_1)X(t-\tau_1)\,d\tau_1 \int_{-\infty}^{+\infty} h(\tau_2)X(t-\tau_2)\,d\tau_2\right]$$

$$= \int_{-\infty}^{+\infty} h(\tau_1)\,d\tau_1 \int_{-\infty}^{+\infty} h(\tau_2)E\left[X(t-\tau_1)X(t-\tau_2)\right]\,d\tau_2$$

When the input X(t) is a wide-stationary random process,

The autocorrelation function of X(t) is only a function of

the difference between the observation times t - $\tau_1$ and μ - $\tau_2$

Putting $\tau$ = t - μ , we get

$$R_Y(\tau) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\tau_1)h(\tau_2)R_X(\tau-\tau_1+\tau_2)\,d\tau_1\,d\tau_2$$

$R_Y$ (0) = E[$Y^2$(t)]

The mean square value of the output random process Y (t)

is obtained by putting $\tau$ = 0 in the above equation.

$$E[Y^2(t)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\tau_1)h(\tau_2)R_X(\tau_2-\tau_1)\,d\tau_1\,d\tau_2$$

### 3.9 POWER SPECTRAL DENSITY

By definition the impulse response of a linear time invariant filter is equal to the inverse Fourier transform of the frequency response of the system.

Using H ( $\omega$ ) to denote the frequency response of the system we may thus write,

$$E[Y^2(t)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\tau_1)h(\tau_2)R_X(\tau_2-\tau_1)\,d\tau_1\,d\tau_2$$

Where

$$h(\tau_1) = \frac{1}{2\pi}\left[\int_{-\infty}^{+\infty} H(\omega)\exp(j\omega\tau_1)\,d\omega\right]$$

$$E[Y^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} H(\omega)\exp(j\omega\tau_1)\,d\omega\right]h(\tau_2)R_X(\tau_2-\tau_1)\,d\tau_1\,d\tau_2$$

Changing the order of integration

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\omega)\,d\omega \int_{-\infty}^{+\infty} h(\tau_2)\,d\tau_2 \int_{-\infty}^{+\infty} R_X(\tau_2-\tau_1)\exp(j2\omega\tau_1)\,d\tau_1$$

Putting $\tau = \tau_2 - \tau_1$; $\tau_1 = \tau_2 - \tau$ ; $d\tau_1 = d\tau$

$$E[Y^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\omega)\,d\omega \int_{-\infty}^{+\infty} h(\tau_2)\exp(j\omega\tau_2)\,d\tau_2 \int_{-\infty}^{+\infty} R_X(\tau)\exp(-j2\omega\tau)\,d\tau$$

$$\text{Let } H^*(\omega) = \int_{-\infty}^{+\infty} h(\tau_2)\exp(j\omega\tau_2)\,d\tau_2$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\omega)\,d\omega\ H^*(\omega)\,d\omega \int_{-\infty}^{+\infty} R_X(\tau)\exp(-j\omega\tau)\,d\tau$$

Let $H(\omega)H^*(\omega) = |H(\omega)|^2$

$$= \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega \int_{-\infty}^{+\infty} R_X(\tau)\exp(-j2\omega\tau)\,d\tau$$

$$= \int_{-\infty}^{+\infty} |H(\omega)|^2 S_X(\omega)\,d\omega$$

Where $|H(\omega)|$ = Magnitude response of the filter
$S_X(\omega)$ = Power spectral density

This is simply the Fourier Transform of the autocorrelation function $R_X(t)$ of the input random process X(t). Let this transform be denoted by $S_X(\omega)$.

$$S_X(\omega) = \int_{-\infty}^{+\infty} R_X(\tau)\exp(-j\omega\tau)\,d\tau$$

$S_X(\omega)$ is called the power spectral density or power spectrum of the wide-sense stationary random process X(t).

The mean square value of the output of a stable linear time-invariant filter in response to a wide-sense stationary random process is equal to the integral over all frequencies of the power spectral density of the input random process multiplied by the squared magnitude of the transfer function of the filter.

**Properties of power spectral density**

The psd and autocorrelation function $R_X(\tau)$ of a stationary Random Process X(t) form a Fourier transform pair

$$S_X(\omega) = \int_{-\infty}^{+\infty} R_X(\tau)\exp(-j\omega\tau)\,d\tau$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(\omega)\exp(j\omega\tau)\,d\omega$$

These two relations together are called as Einstein Wiener-Khintchine Relation.

Property 1:

The zero frequency value of the psd of a stationary process is equal to the total area under the graph of the autocorrelation function

By putting $\omega = 0$ in $S_X(\omega)$

$$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau)\,d\tau$$

Property 2:

The mean square value of a stationary process is equal to the total area under the graph of the psd (i.e.,)

$$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(\omega)\,d\omega$$

Property 3:

The power spectral density of a stationary process is always non negative. (i.e.,)

$S_X(\omega) \geq 0$ for all $\omega$

Property 4:

The power spectral density of a real valued random process is an even function of frequency (i.e.,)

$S_X(-\omega) = S_X(\omega)$

Property 5:

The power spectral density appropriately formalized has the properties usually associated with probability density function (i.e.,)

$$P_X(\omega) = \frac{S_X(\omega)}{\int_{-\infty}^{\infty} S_X(\omega)\,d\omega}$$

**TWO MARKS**

**1. What is meant by random experiment?**

The mathematical technique for dealing with the result of an experiment, whose outcomes are not known in advance, is called random experiment. i.e., an experiment whose outcome is not known in advance

**2. What is meant by sample space?**

The set of all possible outcome of a random experiment is called sample space.

**3. What is mean by random variable?**

A function which takes on any value from the sample space and it's range is some set of real numbers is called a random variable. A (real) random variable is a mapping from the sample space Ω to the set of real numbers.