The zero frequency value of the psd of a stationary process is equal to the total area under the graph of the autocorrelation function

By putting $\omega = 0$ in $S_X(\omega)$

$$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau)\,d\tau$$

Property 2:

The mean square value of a stationary process is equal to the total area under the graph of the psd (i.e.,)

$$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(\omega)\,d\omega$$

Property 3:

The power spectral density of a stationary process is always non negative. (i.e.,)

$S_X(\omega) \geq 0$ for all ω

Property 4:

The power spectral density of a real valued random process is an even function of frequency (i.e.,)

$S_X(-\omega) = S_X(\omega)$

Property 5:

The power spectral density appropriately formalized has the properties usually associated with probability density function (i.e.,)

$$P_X(\omega) = \frac{S_X(\omega)}{\int_{-\infty}^{\infty} S_X(\omega)\,d\omega}$$

TWO MARKS

1. What is meant by random experiment?

The mathematical technique for dealing with the result of an experiment, whose outcomes are not known in advance, is called random experiment. i.e., an experiment whose outcome is not known in advance

2. What is meant by sample space?

The set of all possible outcome of a random experiment is called sample space.

3. What is mean by random variable?

A function which takes on any value from the sample space and it's range is some set of real numbers is called a random variable. A (real) random variable is a mapping from the sample space Ω to the set of real numbers.

4. Give suitable example for random variable

Consider the example of tossing a fair coin twice. The sample space is S= {HH, HT, TH, TT} and all four outcomes are equally likely. Then we can define a random variable X as follows

Sample Point Ω	Value of the random Variable X = x	P{X = x}
HH	0	¼
HT	1	¼
TH	2	¼
TT	3	¼

5. Define random process.

The random process x(t) is defined as an ensemble of time functions together with a probability rule that assigns a probability to any meaningful event associated with an observation of one of the sample functions of the random process.

6. Express the mean of a statistical average in terms of random process

The statistical average of a random processes mean is defined as

$\mu_x(t) = E(x(t))$

The expected values are taken with respect to the appropriate probability density function.

7. Express the auto correlation function in terms of random process.

The statistical average of a random processes auto correlation function $R_{xx}(t_1,t_2)$ is defined as the expected values of the random processes with respect to the appropriate probability density function.

$$R_{xx}(t_1,t_2) = E(x(t_1) , x(t_2))$$

8. When a random process is said to be ergodic?

A random process is said to be ergodic if the time averages are equal to ensemble averages if

1) $E(\mu_x) = E(x(t))$ is said to be ergodic and the variance of $(\mu_x) \to 0$ as $T \to \infty$

2) In the auto correlation function if $E(R_{XX}(\tau)) = R_{XX}(\tau)$ and the variance $(R_{XY}(\tau)) \to 0$ as $T \to \infty$

9. Define power spectral density of stationary random process.

The auto correlation function $R_{XX}(\tau)$ of a stationary random process is such that $\int_{-\infty}^{\infty} | R_{XX}(\tau) | d\lambda < \infty$ and its fourier transform $G_X(f)$ is given by

$$G_X(f)= \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-2\pi jf\tau}\, d\lambda$$

is called power density spectrum or the power spectral density of λ (t).

10. Define mean.

The expected value of a random variable indicates its average or central value . It is denoted by E(X) or μ.

The mean of the random process is given by

$$\mu_{X(t)} = E[X(t)] = \int_{-\infty}^{\infty} x f_{x(t)}(x)\, dx$$

11. Define variance.

The variance of a random variable is a non-negative which gives an idea of how widely spread the values of random variables are like to be, the larger the variance, the more scattered the observations on average.

It is denoted by Var(X) or σ^2

The variance of a random variable is

$Var(X) = \sigma^2 = E[X-E(X)]^2$

$= E(X^2)-[E(X)]^2$

12. **Difference between random process and random variable**.

Random process	Random variable
The outcome of a random experiment is mapped into a waveform that is a function of time.	The outcome of a random experiment is mapped into a number.

13. When a random process is called stationary?

When the statistical properties of a random process do not change with time is called stationary random process.

14. When the random process is said to be strict sense or strictly stationary?

The strictly stationary random process possesses following characteristics:

1) The statistical properties do not change with shift of time origin.
2) A truly stationary process should start at $t = -\infty$ and should not stop till $t = \infty$.

15. When a random process called deterministic?

When the future values of any sample function can be predicted from a knowledge of past values then the random process is called deterministic random process.

16. **Consider the following statement: “The mean of a Poisson distribution is 5 while the standard deviation is 4”.**

For a Poisson distribution mean and variance are same. Hence the statement is not true

17. Comment the following : " The mean of a binomial distribution is 3 and variance is"

In binomial distribution , mean > variance but variance < mean

Since variance = 4 and mean = 3, the given statement is wrong