Unit-IV
NOISE CHARACTERIZATION

With reference to an electrical system, noise may be defined as any unwanted form of energy which tends to interfere with proper reception and reproduction of wanted signal. Noise is random, undesirable electrical energy that enters the communications system via the communicating medium and interferes with the transmitted message. However, some noise is also produced in the receiver.

4.1 Noise

An unwanted signal that corrupts the desired signal is called as noise.

Noise may be **predictable or random**

Predictable noise

It can be easily eliminated by proper engineering design. These kinds of noise are manmade and it can be easily reduced and not vary with time.

(eg) Radiation pick up generated by electrical appliances.

Random noise

This noise varies with time and is unpredictable. In the absence pof random noise message signal is received at the receiver perfectly.

Sources of noise

Noise sources are classified into **external noise and internal noise**

External noise

Natural disturbances(random noise) and Man made noie (Predictable noise) are referred to as external noise

Internal noise

Shot noise and Thermal noise are referred to as internal noise

It is the noise arised in electronic devices such as transistors, diodes, triodes , because of the discrete nature of current flow in these devices.

4.1.1Shot noise in photodetector

In photodetector circuit current pulse is generated every time an electron is emitted by the cathode due to light incident from a source of constant intensity.

The total current flowing through the photodetector is the sum of current pulse

$(i.e) X(t) = \sum_{k=-\infty}^{\infty} h(t-\tau_k)$ 4.1

Where

$X(t)$ is the total current

$h(t-\tau_k)$ is the current oulse generated at time $t=\tau_k$

The mean of the number of electrons (μ) is

$E(\mu)=\lambda t_o$ 4.2

Where μ is the rate of process

The total number of electrons emitted in the intreval $(t, t+t_o)$is

$\mu = N(t+t_o) - N(t)$ 4.3

The probability of K electrons emitted in the interval $(t+t_o)$ is

$P(\mu = K) = \frac{(\lambda t_o)^k}{K!}$ 4.4

Mean of shot noise

Mean of shot noise is $\mu_x = \lambda \int_{-\infty}^{\infty} h(t)dt$ 4.5

Where

h(t) is the waveform of a pulse representing shot noise.

λ is the rate of process

Auto covariance

The auto covariance of shot noise is given by

$C_x(\tau) = \lambda \int_{-\infty}^{\infty} h(t)h(t+\tau)dt$ 4.6

Power density spectrum of shot noise in diodes

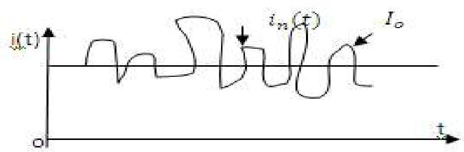


Fig.4.1 . Representation of current

In the above figure the current i(t) fluctuates about the mean value I_o. Th time varying component i_n(t) of the current i(t) is known as shot noise.

The total current

$i(t) = I_o + i_n(t)$ 4.7

Where I_o is the mean value

and i_n(t) is shot noise current

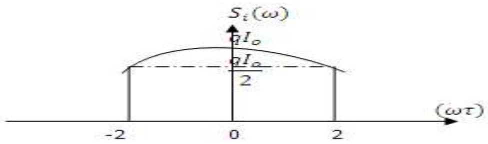


Fig. Power Density Spectrum

Fig.4.2 . Power density spectrum of shot noise in diodes

The power density spectrum of shot noise current is

$$S_i(\omega) = qI_o$$ 4.8

Where q is the electron charge(1.59×10^{-19}C)
and I_o is the mean value of current in Amperes.

Shot Noise in Triodes

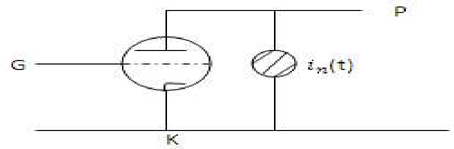


Fig.4.3 . Shot noise in triodes

The power density spectrum of noise current in the triode is similar to the diode which is given as

$$S_i(\omega) = 2KT(2.5g_m) \quad 4.9$$

Wher g_m is the transconductance of the triode
The noise current in triode is caused by noise voltage

$$V_n(t) = \frac{i_n(t)}{g_m} \quad 4.10$$

The power density spectrum of voltage source is

$$S_v(\omega) = S_i(\omega)\frac{1}{g_m^2} \quad 4.11$$

$$S_v(\omega) = 2KT(2.5g_m)\frac{1}{g_m^2} \quad 4.12$$

$$S_v(\omega) = 2KT\frac{2.5}{g_m} \quad 4.13$$

$$S_v(\omega) = 2KTR_{eq} \quad 4.14$$

Where

$$R_{eq} = \frac{2.5}{g_m} \quad 4.15$$

4.1.2 Thermal Noise (Resistor Noise)

This is the electrical noise which is arising from the random motion of electrons in a conductor.

Fig.4.4 .Representation of thermal noise in equivalent circuit

The intensity of random motion is proportional to thermal energy applied. So this noise is known as thermal noise. The net motion of all electrons give rise to an electric current flow through the resistor, causing the noise.

The mean square value of the thermal noise voltgae across the terminals of a resistor is

$E[V_{TN}^2] = 4KTR\Delta f\ V^2$ 4.16

Where K is Boltzman constant (1.38x10^{-3} J per degree kelvin)
T is absolute temperature (in degree kelvin)
R is Resistance in ohm
Δf is bandwidth in hertz
The mean square value of noise current is

$E[I_{TN}^2] = \frac{1}{R^2} E[V_{TN}^2]$ 4.17

$E[I_{TN}^2] = \frac{1}{R^2} 4KTR\Delta f$ 4.18

$E[I_{TN}^2] = \frac{1}{R} 4KT\Delta f$ 4.19

$E[I_{TN}^2] = 4KTG\Delta f\ amp^2$ 4.20

Power density spectrum of resistor noise

If the random motion between free electrons which contributes thermal noise are assumed to be independent then the thermal noise is Gaussian distributed with zero mean.
The power density spectrum of current contributing thermal noise is

$S_i(\omega) = \frac{2KTG}{1+(\frac{\omega}{\alpha})^2}$ 4.21

Where α is the average number of collisions per second per electron.
The variation of power density spectrum with frequency is shown in fig

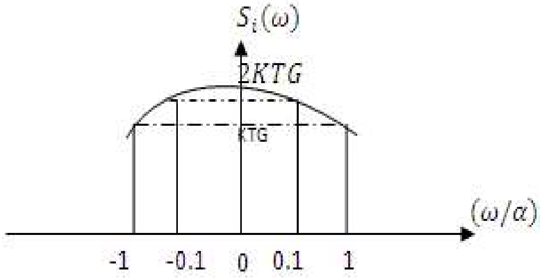


Fig.4.5 . Power density spectrum

When $(\omega/\alpha) << 1$

$S_i(\omega) = 2KTG$ 4.22

The power density spectrum of voltage source is

$S_v(\omega) = R^2 S_i(\omega)$ 4.23

$S_v(\omega) = R^2 2KTG$ 4.24

$S_v(\omega) = R^2 2KT\frac{1}{R}$ 4.25

$S_v(\omega) = 2KTR$ 4.26

Power of thermal noise voltage

The noise powe r P_n is given by

$P_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_v(\omega) d\omega$ 4.27

$P_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2KTR d\omega$ 4.28

$P_n = \frac{2KTR}{2\pi}\int_{-\infty}^{\infty} d\omega$ 4.29

$P_n = \frac{KTR}{\pi}(\omega)_{-\infty}^{\infty}$ 4.30

$P_n = \infty$ 4.31

The noise power for a finite bandwidth is

$P_n = S_v(\omega)2\Delta f$ 4.32

$P_n = (2KTR)2\Delta f$ 4.33

$P_n = 4KTR\Delta f$ 4.34

$P_n = V_n^2 = 4KTR\Delta f$ 4.35

$V_n = \sqrt{4KTR\Delta f}$ 4.36

Where Δf *is one sided bandwidth*

4.1.3 White Noise

White light contains all colour frequencies. Similarly white noise contains all frequencies in equal amount. The power density spectrum of white noise is independent of frequency which is given as

$S_W(\omega) = \frac{N_o}{2}$ 4.37

Where N_o is in watts per Hz.

When the probability of occurrence of white noise is specified by a Gaussian distribution function, it is known as white Gaussian noise. N_o may be expressed as $N_o = KT_o$.

Where T_o is the equivalent noise temperature of the receiver and K is a boltzmann constant

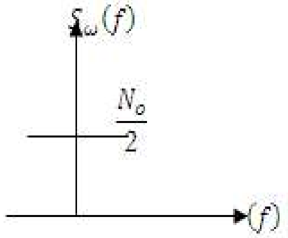


Fig.4.6 .Power Spectral Density

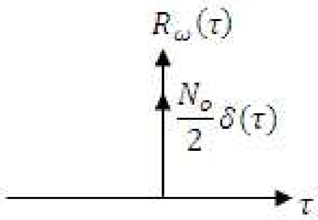


Fig.4.7 .Auto correlation function

Autocorrelation of white noise

Autocorrelation of white noise is given by

$R_\omega(\tau) = \int_{-\infty}^{\infty} S_w(\omega) \exp(j2\pi\omega\tau)\, d\omega$ 4.38

$R_\omega(\tau) = \int_{-\infty}^{\infty} \frac{N_o}{2} \exp(j2\pi\omega\tau)\, d\omega$ 4.39

$R_\omega(\tau) = \frac{N_o}{2} . \delta(\tau)$ 4.40

Where $\delta(\tau) = \int_{-\infty}^{\infty} \exp(j2\pi\omega\tau)\, d\omega$ 4.41

is a delta function.

A **delta function** is a function which contains the entire frequency component in equal amount.

4.2 Noise Figure

It is defined as the total noise power spectral density (S_{no}) at the output of the two port network to the noise power spectral density (S'_{no}), at the output assuming zero noise.

$(i.e) F = \frac{Power\ density\ of\ the\ total\ noise\ at\ the\ output\ of\ the\ network}{Power\ density\ at\ the\ output\ assuming\ zero\ noise}$

$F = \frac{S_{no}}{S'_{no}}$ 4.42

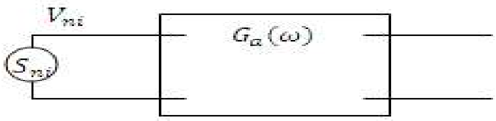


Fig. 4.8 .Two port network with input and output

Where S_{no} is the power density of total noise and S'_{no} is the power density at the output assuming zero noise.

Total output

$S_{no} = S'_{no} + S''_{no}$ 4.43

S''_{no} is the power density of noise in the network

Therefore

$F = \frac{S'_{no}+S''_{no}}{S'_{no}}$ 4.44

$F = 1 + \frac{S''_{no}}{S'_{no}}$ 4.45

The above expression gives expression for noise figure

Noise figure interms of power gain

For the above network input power density is S'_{no}

Therefore the gain of the network

$G_a(\omega)=\frac{S'_{no}}{S_{ni}}$ 4.46

$S'_{no} = G_a(\omega)\, S_{ni}$ 4.47

$S'_{no} = G_a(\omega)\, \frac{N_o}{2}$ 4.48

$S'_{no} = G_a(\omega)\, \frac{KT}{2}$ 4.49

For a noiseless network F=1

Noise figure interms of network transfer function

The relation between S'_{no} and S_{ni} interms of network transfer function is

$S'_{no} = |H(\omega)|^2 S_{ni}.$ 4.50

Figure of merit

$F = \frac{S_{no}}{S'_{no}}$ 4.51

$F = \frac{S_{no}}{|H(\omega)|^2 S_{ni}}$ 4.52

Noise Figure interms of equivalent input noise temperature

The expression for noise figure interms of temperature is

$F = 1 + \frac{T_e}{T}$ 4.53

Where T_e is the noise temperature of the network and t is the noise temperature of the source.

Average noise figureGeneral expression for noise figure is

$F = \frac{\int_{\omega_1}^{\omega_2} G_a(\omega) F(\omega)\, d\omega}{\int_{\omega_1}^{\omega_2} G_a(\omega)\, d\omega}$ 4.54

Where $G_a(\omega)$ is the gain of the network and $F(\omega)$ is the noise figure

NOISE FIGURE INTERMS OF $(SNR)_O$

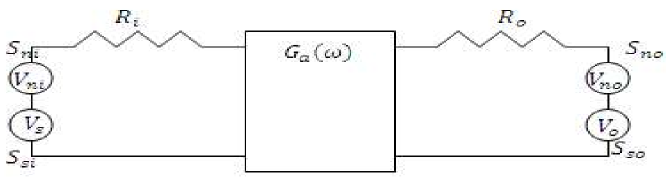


Fig.4.9 .Two Port Network

In above network S_{si} and S_{so} indicates the power spectral density of the message signal.

Where S_{si} and S_{so} is the power density of noise at the input and output

We know that

$F = \frac{S_{no}}{S'_{no}}$ 4.55

$S_{no} = F S'_{no}$ 4.56

$S_{no} = F G_a(\omega)\, S_{ni}$ 4.57

$S_{no} = F \frac{S_{so}}{S_{si}} S_{ni}$ 4.58

$F = \frac{S_{no} S_{si}}{S_{so} S_{ni}}$ 4.59

$F = \frac{S_{no}/S_{si}}{S_{so}/S_{ni}}$ 4.60

The above expression is referred to as spot noise figure.

Power contributed by two port network

We know that $F = \frac{S_i/N_i}{S_o/N_o}$ 4.61

S_o/N_o is the output signal to noise ratio and S_i/N_i is the input signal to noise ratio

$\frac{S_i}{N_i} = F.\frac{S_o}{N_o}$ 4.62

$\frac{S_i}{N_i} = F.G_a(\omega)\ \frac{S_i}{N_o}$ 4.63

$F = \frac{1}{G_a(\omega)} \frac{N_o}{N_i}$ 4.64

$F = \frac{G(_a(\omega))N_i + N_{tp}}{G_a(\omega)N_i}$ 4.65

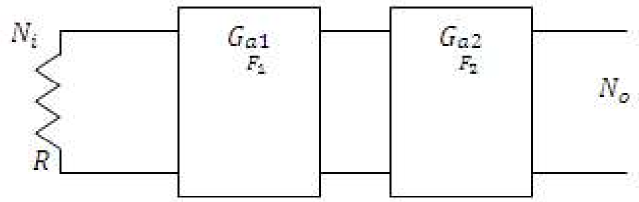
Where N_{tp} *is noise in network*

$F = 1 + \frac{N_{tp}}{G_a(\omega)N_i}$ 4.66

$N_{tp} = G_a(\omega)\,(F-1)\,N_i$ 4.67

The above expression gives power contributed by two port network

Noise Figure of cascaded Amplifier



Noise power N_o interms of the gain of network is

$N_{o1} = N_i G_{a1} G_{a2}$ 4.68

Power contributed by first network is

$N_1 = G_{a1}(F_1 - 1)\,N_i$ 4.69

The amplified noise at the second stage is

$N_{o2} = G_{a1}\,(F_1 - 1)\,N_i G_{a2}$ 4.70

Power contributed by second stage is

$N_{o3} = G_{a2}\,(F_2 - 1)\,N_i$ 4.71

Therefore total noise power

$N_o = N_{o1} + N_{o2} + N_{o3}$ 4.72

$N_o = N_i G_{a1} G_{a2} + G_{a1}(F_1 - 1)\, N_i G_{a2} + G_{a2}(F_2 - 1)\, N_i$ 4.73

$N_o = N_i(G_{a1} G_{a2} + G_{a1}(F_1 - 1)\, G_{a2} + G_{a2}(F_2 - 1))$ 4.74

$\frac{N_o}{N_i} = (G_{a1} G_{a2} + G_{a1}(F_1 - 1)\, G_{a2} + G_{a2}(F_2 - 1))$ 4.75

$\frac{N_o}{N_i} = G_{a1} G_{a2}(1 + (F_1 - 1) + \frac{1}{G_{a1}}(F_2 - 1))$ 4.76

$\frac{N_o}{N_i}\frac{1}{G_{a1} G_{a2}} = 1 + (F_1 - 1) + \frac{1}{G_{a1}}(F_2 - 1)$ 4.77

$\frac{N_o}{N_i}\frac{1}{G_a} = 1 + (F_1 - 1) + \frac{1}{G_{a1}}(F_2 - 1)$ 4.78

$F = F_1 + \frac{1}{G_{a1}}(F_2 - 1)$ 4.79

The above expression gives the noise figure if two networks are connected in cascade.
If more than two networks are connected in cascade then

$F = F_1 + \frac{1}{G_{a1}}(F_2 - 1) + \frac{1}{G_{a1} G_{a2}}(F_3 - 1) + \frac{1}{G_{a1} G_{a2} G_{a3}}(F_4 - 1) + \cdots$ 4.80

The above expression is known as Friss formula.

4.3 Noise Temperature

The noise power always depends on temperature of the network and independent of the resistance.
Representing noise power interms of temperature is referred to as noise temperature.
Assume the noise power present in network or amplifier as

$P_{na=(F-1)KTB}$ 4.81

Where F is noise figure
K is Boltzmann constant
T is Temperature and B is Bandwidth
The noise power interms of equivalent temperature is

$P_{na=KT_eB}$ 4.82

Equating equations (4.81) and (4.82)

$(F - 1)KTB = KT_eB$

$(F - 1)T = T_e$

The above expression gives the relation between equivalent temperature of network and total temperature.
We kno w that friss formula is

$F = F_1 + \frac{1}{G_{a1}}(F_2 - 1) + \frac{1}{G_{a1} G_{a2}}(F_3 - 1) + \frac{1}{G_{a1} G_{a2} G_{a3}}(F_4 - 1) + \cdots$ 4.83

In general friss formula is used to find out overall equivalent noise temperature.
Equation () implies

$F-1=F_1-1+\frac{1}{G_{a1}}(F_2-1)+\frac{1}{G_{a1}G_{a2}}(F_3-1)+\frac{1}{G_{a1}G_{a2}G_{a3}}(F_4-1)+\cdots$ 4.84

Therefor e

$T_e=T_{e1}+\frac{1}{G_{a1}}(T_{e2})+\frac{1}{G_{a1}G_{a2}}(T_{e3})+\frac{1}{G_{a1}G_{a2}G_{a3}}(T_{e4})+\cdots$ 4.85

The above expression gives equivalent noise temperature.

4.4 Narrow Band Noise

Band Pass Filter has narrow bandwidth (i.e) bandwidth is small compared to entire frequency. The noise output from this kind of filter is referred to as narrow band noise

Representation of narrow band noise interms of inphase and quadrature phase components

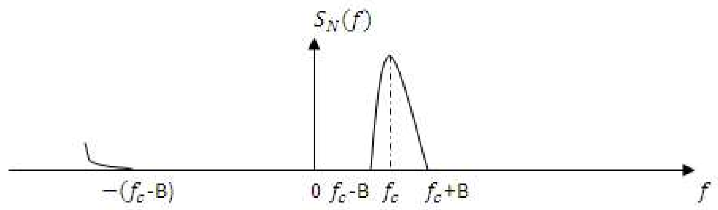


Fig. Power Spectral Density

Consider a narrowband noise n(t) of bandwidth 2B and centered on frequency f_c, which can be represented as

$n(t)=n_I(t)\cos 2\pi f_c t-n_Q(t)\sin 2\pi f_c t$ 4.86

Where $n_I(t)$ and $n_Q(t)$ are the inphase and quadrature component of n(t)

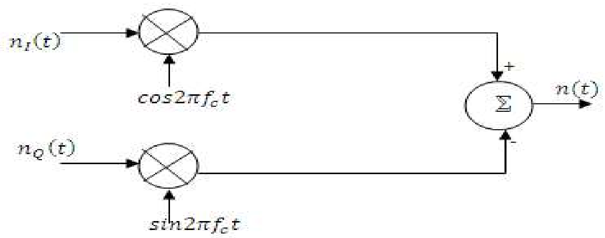


Fig. Generation of Noise n(t)

Properties of inphase and quadrature phase components of narrow band noise

1. $n_I(t)$ and $n_Q(t)$ have zero mean
2. If $n(t)$ is gaussian and stationary then $n_I(t)$ and $n_Q(t)$ are jointly gaussian and stationary

3. $n_I(t)$ and $n_Q(t)$ have same variance as n(t)

4. $Both\ n_I(t)$ and $n_Q(t)$ have same power spectral density which is related to the power spectral density $S_N(f)$ of the narrow band noise n(t) as

$$S_{N_I}(f) = S_{N_Q}(f) = \{S_N(f - f_c) + S_N(f + f_c)\} \quad 4.87$$

$0;\ otherwise$

Representation of narrowband noise interms of envelope and phase components

Noise n(t) can be represented interms of envelope and phase component as

$n(t) = R(t)\cos\,[2\pi f_c t + \theta(t)]$

Where $R(t) = \sqrt{n_I^2(t)} + \sqrt{n_Q^2(t)}$ is the **envelope** of n(t)

and $\theta(t) = tan^{-1}(\frac{n_Q(t)}{n_I(t)})$ is the **phase** of n(t)

4.5 NOISE IN AM RECEIVER USING COHERENT DETECTION

In full AM signal both sidebands and signals are transmitted.

Therefore

$S(t)=A_c\ (1+K_a m(t))\cos 2\pi f_c t$ 4.88

Where **$\cos 2\pi f_c t$** is the carrier signal

m(t) is the message signal and

K_a is the modulation index

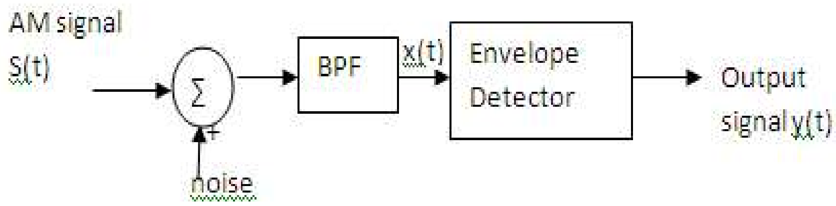


Fig. Model of AM Receiver

In the transmission medium the AM signal gets added with noise. So the resultant signal at the receiver output is **S(t) + n(t).**

Where **$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$**

$n_I(t)$ and **$n_Q(t)$** are the inphase and quadrature component of n(t) with respect to the carrier.

The received signal is given as input to BPF.The output of BPF is

$x(t)= A_c\ (1+K_a m(t))\cos 2\pi f_c t+ n_I(t)\cos 2\pi f_c t-n_Q(t)\sin 2\pi f_c t$ 4.89

The output of BPF ie x(t) is given as input to the envelope detector.The componenet of signal x(t) can be represented by means of phasors.

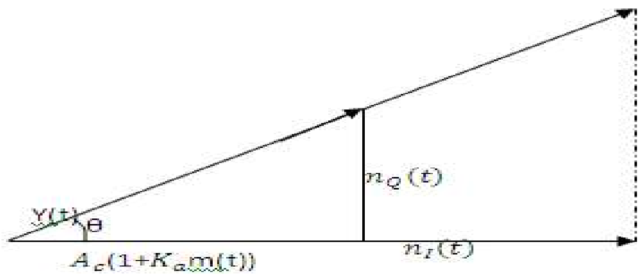


Fig. Phasor Diagram

The receiver output y(t)=envelope of x(t)

$\mathbf{Y(t)=\{[\ A_c\ (1+K_a m(t))+\ n_I(t)]^2+(\ n_Q(t))^2\}^{1/2}}$ **4.90**

The signal y(t) defines the output of an ideal envelope detector .When the average carrier power is large compared with the average noise power, the above expression becomes

$\mathbf{y(t) =\{[\ A_c\ (1+K_a m(t))+\ n_I(t)]^2\}^{1/2}}$ **4.91**

$\mathbf{=[\ A_c\ (1+K_a m(t))+\ n_I(t)]}$ **4.92**

$\mathbf{=\ A_c+\ A_c\ K_a m(t)+\ n_I(t)}$ **4.93**

The DC component AC can be removed by means of a blocking capacitor.Therefore

$\mathbf{y(t)=\ A_c\ K_a m(t)+\ n_I(t)}$**,**which is the signal at receiver output.

Figure of Merit

$\mathbf{(SNR)}_C$ = Average modulated power/Average noise power

$\mathbf{S(t)=\ A_c\ \cos 2\pi f_c t\ +\ A_c\ K_a m(t)\cos 2\pi f_c t}$ **4.94**

Average power of carrier component =$\mathbf{A_c^2/2}$

Average power of information bearing component= $\mathbf{(A_c\ \ K_a m(t)\cos 2\pi f_c t)^2}$ **4.95**

$\mathbf{=\ A_c^2 K_a^2\ m(t)^2\ \cos^2\ 2\pi fct}$ **4.96**

$\mathbf{=(\ A_c^2 K_a^2\ m(t)^2)((1+\ \cos 4\pi f_c t)/2)}$ **4.97**

$\mathbf{=[(\ A_c^2 K_a^2\ m(t)^2)/2]+[(\ A_c^2 K_a^2 m(t)^2\ \cos 4\pi f_c t)\ /2]}$**4.98**

$\mathbf{=[(\ A_c^2 K_a^2\ m(t)^2)/2]}$ **4.99**

$\mathbf{=[(\ A_c^2 K_a^2\ P)/2]}$ **4.100**

Average modulated power $\mathbf{=\ A_c^2/2+[(\ A_c^2 K_a^2\ P)/2]}$

$\mathbf{=\ (A_c^2+\ A_c^2 K_a^2\ P)/2}$ **4.101**

$\mathbf{=[\ A_c^2(1+\ K_a^2\ P)]/2}$ **4.102**

Average Noise power=$\mathbf{2W(N_O/2)}$

$= \mathbf{WN_O}$

Therfore $(SNR)_C = \frac{[A_c^2(1+K_a^2 P)]/2}{WN_O}$

$$(SNR)_C = \frac{[A_c^2(1+K_a^2 P)]}{2WN_O} \quad 4.105$$

$(SNR)_O$=Average power of modulating signal/Average Noise power

Average power of modulating signal= $A_c^2 K_a^2 m(t)^2$
$= A_c^2 K_a^2 (P/2)$
Average Noise power=$2W(N_O/2)$
$= WN_O$

Therefore $(SNR)_O = \frac{A_c^2 K_a^2 (P)/2}{WN_O}$

$$= \frac{[A_c^2 K_a^2 P]}{2WN_O} \quad 4.106$$

Figure of Merit=$(SNR)_O/(SNR)_C$

$$= \frac{[A_c^2 K_a^2 P)]/2WN_O}{[A_c^2(1+K_a^2 P)]/2WN_O}$$

$$= \frac{K_a^2 P}{1+K_a^2 P} \quad 4.107$$

Figure of Merit is less than unity,Which is due to the wastage of transmitted power which results from transmission of carrier as a component of AM wave

4.6 NOISE IN LINEAR RECEIVERS USING COHERENT DETECTION

Consider a DSBSC signal transmitted as
$s(t)=CA_c\, m(t)\cos 2\pi f_c t$
Where $A_c \cos 2\pi f_c t$ is the carrier signal
m(t) is the message signal and
C is a system dependent scaling factor(s(t) is measured in the same units as the additive noise component n(t)

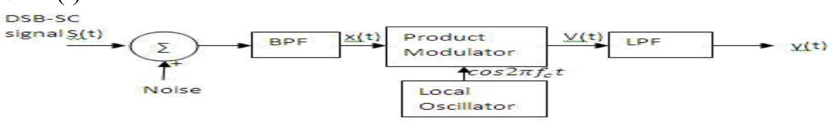


Fig. Model of DSB-SC Receiver using coherent Detection

In the transmission medium the transmitted signal gets added with noise, so that the signal at the receiver input is **s(t)+n(t)**

Where $n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$

$n_I(t)$ and $n_Q(t)$ are the inphase and quadrature component of n(t) with respect to the carrier

The received signal is given as input to BPF.The output of BPF is

$$x(t)= CA_c\, m(t)\cos 2\pi f_c t + n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t \qquad 4.108$$

The output of BPF ie x(t) is given as input to the product modulator.Another input to the product modulator is the carrier signal **$\cos 2\pi f_c t$.** The output of the product modulator is

$v(t)=x(t)\cos 2\pi f_c t$

$= CA_c\, m(t)\cos^2 2\pi f_c t + n_I(t)\cos^2 2\pi fct - n_Q(t)\sin 2\pi f_c t\cos 2\pi f_c t$

$$= CA_c\, m(t)[(1+\cos 4\pi f_c t)/2] + n_I(t)\,[(1+\cos 4\pi f_c t)/2] - n_Q(t)\sin 2\pi f_c t\cos 2\pi f_c t \qquad 4.109$$

The signal v(t) is given as input to LPF.It removes all high frequency components.

So that the output of LPF is $y(t)= [CA_c\, m(t)]/2 + n_I(t)/2$

Above equation indicates

1.The message signal m(t) and in-phase noise component $n_I(t)$ of the filtered noise n(t) appear additively at the receiver output.

2. The quadrature component $n_Q(t)$ of the noise n(t) is completely rejected by the coherent detector.

In coherent detection the component appears additively with the message signal

Figure of Merit

$(SNR)_C$ = Average modulated power/Average noise power

$s(t)= CA_c\, m(t)\cos 2\pi f_c t$

Average modulated power= $(C^2 Ac^2 m^2(t)\cos^2 2\pi f_c t)$

$= [C^2 Ac^2 m^2(t)(1+\cos 4\pi f_c t)/2]$

$=[C^2 Ac^2 P(1+\cos 4\pi f_c t)/2]$

$= [(C^2 Ac^2 P)/2 + C^2 Ac^2 P(\cos 4\pi f_c t)/2]$

$= (C^2 Ac^2 P)/2$

Average Noise power=$2W(N_O/2)$

$= WN_O$

Therfore $(SNR)_C = \dfrac{[C^2 A_c^2 P]/2}{WN_O}$

$$= \frac{[C^2 A_c^2 P]}{2WN_O} \qquad 4.110$$

$(SNR)_O$=Average power of modulating signal/Average Noise power

Average power of modulating signal= $(A_c^2 K_a^2 m^2(t))/4$

$= (A_c^2 K_a^2 P)/4$ **4.111**

Average Noise power= $\frac{2W(N_O/2)}{2}$

Therefore $(SNR)_O = \frac{A_c^2 K_a^2 (P)/4}{(WN_O)/2}$

$= \frac{[A_c^2 K_a^2 P]}{2WN_O}$

Figure of Merit= $(SNR)_O/(SNR)_C$

$= \frac{[A_c^2 K_a^2 P)]/2WN_O}{[A_c^2 K_a^2 P)]/2WN_O}$

$=1$ **4.112**

Figure of Merit is unity, Which is exactly same for SSBSC. Neither DSB-SC nor SSB modulation is not having improved noise performance and increased channel bandwidth, when high quality of reception is required.

4.7 NOISE IN FM RECEIVER

The received signal s(t) has a carrier frequency f_c and transmission bandwidth B_T.The BPF has a midband frequency f_c and transmission bandwidth B_T to pass the signal without distortion.

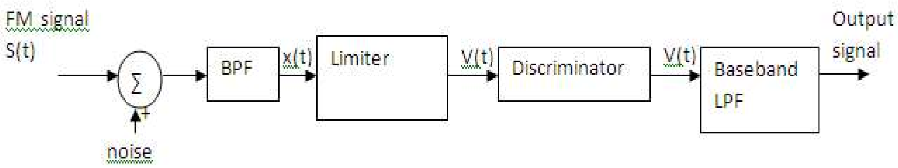


Fig. Block Diagram

In an FM systems the message signal is transmitted by variations of the instantaneous frequency of a sinusoidal carrier wave and its amplitude remains constant. Any variations of the carrier amplitude at the receiver input must result from noise or interference.

The **amplitude limiter** following the BPF in the receiver model is used to remove amplitude variations by clipping the modulated wave at the filtered output. The resulting rectangular wave is rounded off by another BPF that is an integral part of the limiter there by eliminating the harmonics of the higher frequency. Thus the filter output is again sinusoidal.

The **discriminator** consists of two components

1. **Slope network or differentiator**: It produces a hybrid modulated wave in which both amplitude and frequency vary in accordance with the message signal.
2.**Envelope Detector**: It recovers the amplitude variations and reproduces the message signal.
The Slope network or differentiator and Envelope Detector are usually implemented as integral part of a single physical unit.
The post detection filter ie the baseband LPF has enough bandwidth to accommodate high frequency component of message signal. The filter helps to keep the output noise to a minimum.The filtered noise n(t) is defined interms of inohase and quadrature components

ie **$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$** **4.113**

$= r(t)\cos(2\pi f_c t + u(t))$ **4.114**

Where r(t)= $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$

$$\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$$

The incoming FM signal is

$$s(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau]$$

Where A_c is the carrier amplitude ,f_c is the carrier frequency ,K_f is frequency sensitivity and m(t) is the message signal.

Assume $\varphi(t) = 2\pi K_f \int_0^t m(\tau)d\tau$

Therefore $s(t) = A_c \cos[2\pi f_c t + \varphi(t)]$

The noisy signal at the output BPF is $x(t) = s(t) + n(t)$

$x(t) = s(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau] +$ **$r(t)\cos[2\pi f_c t + u(t)]$** **4.115**

The phasor representation of x(t) is

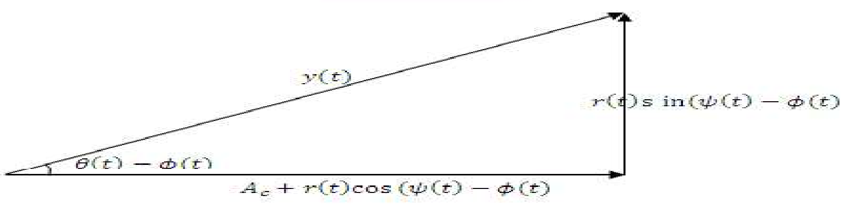


Fig. Phasor diagram

$$\tan(\theta(t) - \varphi(t)) = \frac{r(t)\sin(\psi(t) - \varphi(t))}{A_c + r(t)\cos(\psi(t) - \varphi(t))}$$

$$\theta(t) - \varphi(t) = \tan^{-1}\left(\frac{r(t)\sin(\psi(t) - \varphi(t))}{A_c + r(t)\cos(\psi(t) - \varphi(t))}\right)$$

$$\theta(t) = \varphi(t) + \tan^{-1}\left(\frac{r(t)\sin(\psi(t) - \varphi(t))}{A_c + r(t)\cos(\psi(t) - \varphi(t))}\right)$$

After simplification the above expression becomes

$$\theta(t) = \varphi(t) + \left(\frac{r(t)\sin(\psi(t) - \varphi(t))}{A_c}\right)$$

The discriminator output is

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

$$v(t) = \frac{1}{2\pi}\frac{d[\varphi(t) + (\frac{r(t)\sin(\psi(t) - \varphi(t))}{A_c}]}{dt}$$

$$v(t) = \frac{1}{2\pi}\frac{d[2\pi K_f \int_0^t m(\tau)d\tau + (\frac{r(t)\sin(\psi(t) - \varphi(t))}{A_c}]}{dt}$$

$$v(t) = \frac{1}{2\pi}\frac{d[2\pi K_f m(t) + (\frac{r(t)\sin(\psi(t) - \varphi(t))}{A_c}]}{dt}$$

$$v(t) = \frac{1}{2\pi}2\pi K_f m(t) + \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin(\psi(t) - \varphi(t))\right] \quad 4.116$$

$$v(t) = K_f m(t) + n_d(t)$$

Where $n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin(\psi(t) - \varphi(t))\right]$

Assume noise present in the discriminator output depends on carrier signal and narrow band noise and independent of message signal

Therefore $n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin(\psi(t)\right]$

$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\left[n_Q(t)\right] \quad 4.117$$

The above equation indicates the noise in discriminator output depends on carrier signal and quadrature noise component.

Figure of Merit

(SNR)$_C$ = Average modulated power/Average noise power

Average modulated power= $\mathbf{A_c^2/2}$

Average Noise power=$\mathbf{2W(N_O/2)}$
= $\mathbf{WN_O}$

Therfore $\mathbf{(SNR)_C = \dfrac{[A_c^2]/2}{WN_O}}$

$$= \frac{A_c^2}{2WN_O}$$

$\mathbf{(SNR)_O}$=Average power of modulating signal/Average Noise power

The power spectral density of $n_o(t)$ appearing at the receiver output is $S_{N_o}(f) = \begin{cases} \dfrac{N_o}{A_c^2}; |f| \le W \end{cases}$

Average Noise power= $\dfrac{N_o}{A_c^2}\int_{-W}^{w} f^2 df$

$$= \frac{N_o}{A_c^2}\left(\frac{f^3}{3}\right)_{-W}^{W}$$

$$= \frac{2N_oW^3}{3A_c^2}$$

Average power of modulating signal= $\mathbf{K_f^2\, m^2(t)}$
$\mathbf{= K_f^2\, P}$

Therefore $\mathbf{(SNR)_O = \dfrac{K_f^2\, P}{\frac{2N_oW^3}{3A_c^2}}}$

$$\mathbf{= \frac{3A_c^2\, K_f^2\, P}{2W^3N_O}}$$

Figure of Merit=$\mathbf{(SNR)_O/(SNR)_C}$
$\mathbf{=(3\, K_f^2\, P)/W^2}$ **4.118**

When the carrier to noise ratio is high an increase in transmission bandwidth provides a corresponding increase in the output signal to noise ratio or figure of merit of the FM system

4.8 FM THRESHOLD EFFECT

When a carrier to noise ratio becomes less than unity an impulse of noise is generated. This noise impulse appears at the output of the FM discriminator in the form of click sound. But

when the carrier to noise ratio is further decreased the spikes are generated rapidly and the clicks merge into a sputtering sound. This phenomenon is known as threshold effect in FM.
To minimize this threshold effect the minimum carrier to noise ratio should not deviated from the predicted signal to noise formula, assuming a small signal power..
The threshold effect is more severe in FM than AM.

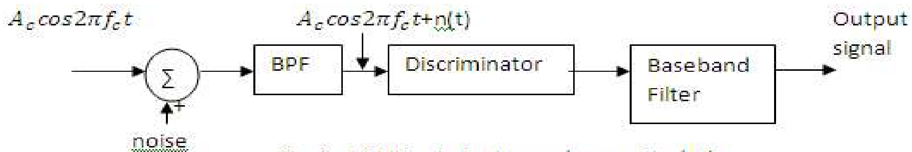


Fig. An FM Discriminator and associated Filter

An FM discriminator consists of IF amplifier, Discriminator and base band filter.
Assume input signal is unmodulated carrier along with white noise.The white noise is filtered and shaped by BPF.The composite signal at the frequency discriminator input is

$x(t) = [A_c + n_I(t)] \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$

The phasor diagram representation of the given expression is

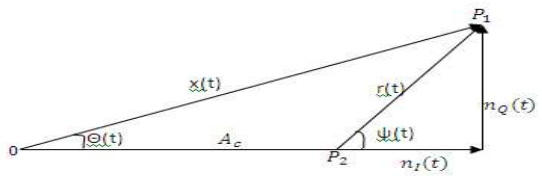


Fig. Phasor Representation

As the amplitude and phase of $n_I(t)$ and $n_Q(t)$ change with time in a random manner, the point P_1 is away from point P_2.When the carrier to noise ratio is large $n_I(t)$ and $n_Q(t)$ are smaller than A_c and hence point P_1 come close to point P_2.
When $r(t) >> A_c$

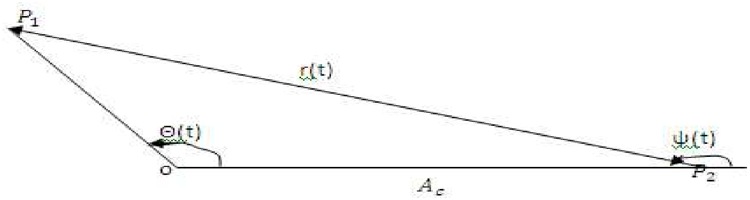


Fig. Phasor Diagram for $r(t) >> A_c$

The discriminator output is

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

When $\theta(t)$ changes by 2π then $\frac{d\theta}{dt}$ appears as a sharp spike with an area of 2π

The area under each spike is **2π**

Area= $\int_{t_1}^{t_2} (d\theta / dt)dt = (\theta)_{t_1}^{t_2} = 2\pi$

Each spike has different height depending on carrier to noise ratio.These spike behave like shot noise.

Positive going click

It occurs when the noise noise envelope r(t) and phase $\psi(t)$ of the narrow band noise n(t) satisfy the following condition

r(t)>A_c

$$\psi(t) < \pi \le \psi(t) + d\psi(t)$$

$$\frac{d\psi(t)}{dt} > 0$$

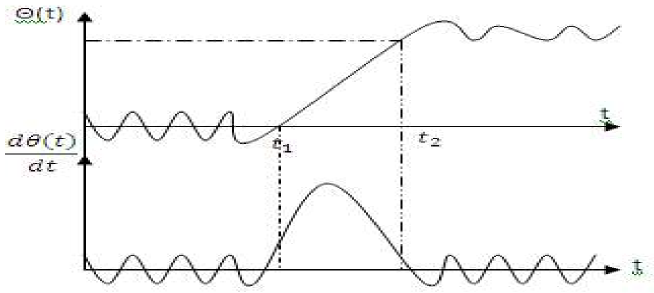


Fig. A plot of $\frac{d\theta(t)}{dt}$ as a function of time t

Negative going click

It occurs when the noise noise envelope r(t) and phase $\psi(t)$ of the narrow band noise n(t) satisfy the following condition

r(t)>A_c

$$\psi(t) > -\pi > \psi(t) + d\psi(t)$$

$$\frac{d\psi(t)}{dt} < 0$$

CARRIER TO NOISE RATIO

The carrier to noise ratio is given by the expression $\rho = \frac{A_c^2}{2B_T N_o}$

If the carrier to noise ratio decreases the average number of clicks per unit time increases.When this number increases ,threshold occurs .

The frequency of spike is given by the expression $f = \frac{B_T}{2\sqrt{3}} erfc(\sqrt{\rho}$

In practice threshold effect occurs when the input carrier to noise is below13dB(ie20).If ρ is above 20 the average number of spike generates per second is small and threshold may be avoided.

4.9 PRE-EMPHASIS AND DE-EMPHASIS

In FM noise has greater effect on the higher modulating frequencies.If we boost the amplitude of higher frequency modulating signals artificially then it will be possible to improve the immunity at higher modulating frequencies.The artificial boosting of higher modulating frequencies is called as Pre-emphasis.In the pre-emphasis and de-emphasis method, a simple R-C networks are used to improve the threshold.

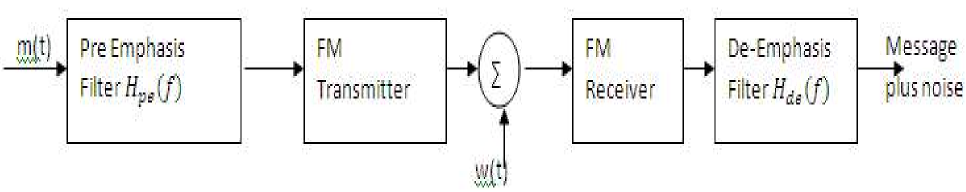


Fig. Use of Pre-Emphasis and De-Emphasis in an FM system

PRE-EMPHASIS

Pre emphasis is a circuit which boosts the signal amplitude of higher frequency component in the message band at the transmitter before modulation.

DE-EMPHASIS

The artificial boosting given to the higher modulating frequencies in the process of Pre emphasis is nullified or compensated at the receiver by a process called De-Emphasis.De-Emphasis circuit is used to bring back the articially boosted high frequency signal to their original amplitude.

Let $H_{pe}(f)$ denotes the frequency response of the Pre emphasis filter and $H_{de}(f)$ denotes the frequency response of the De emphasis filter. Then the relation between Pre-Emphasis and De-Emphasis filter is

$H_{pe}(f)=1/ H_{de}(f)$;$-W\leq f\leq W$

To determine the average noise power consider the power spectral density of the noise $n_d(t)$ at the discriminator output, which is given by

$$S_{N_o}(f) = \begin{cases} \frac{N_o}{A_c^2}; |f| \leq W \end{cases}$$

The modified power spectral density at the De-Emphasis filter output is

$$|H_{de}(f)|^2 S_{Nd}(f) = \begin{cases} \frac{N_o}{A_c^2} |H_{de}(f)|^2 ; |f| \leq W \end{cases}$$

Therefore the average noise power at the output of De-Emphasis is

$$= \begin{cases} \frac{N_o}{A_c^2} \int_{-W}^{W} |H_{de}(f)|^2 f^2 df = 3\int_{-W}^{W} f^2 |H_{de}(f)|^2 df \end{cases}$$

At the pre-emphasis and de-emphasis circuit output the average message power is unaffected.

IMPROVEMENT FACTOR

The improvement factor (I) in output signal to noise ratio produced by the use of pre-emphasis in the transmitter and deemphasis in the receiver is define by

$$I = \frac{AverageouputNoisepowerwithoutpre-emphasisandDe-emphasis}{AverageouputNoisepowerwithpre-emphasisandDe-emphasis}$$

The average output noise power without pre-emphasis and deemphasis is $\frac{2N_oW^3}{3A_c^2}$

Therefore
$$I = \frac{\frac{2N_oW^3}{3A_c^2}}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 df}$$

4.10 Problems

1. Consider a waveform h(t) which consist of a rectangular pulse of amplitude A and duration T. find Auto Covariance.

Solution:

Mean $\mu_x = \lambda \int_{-\infty}^{\infty} h(t)dt$

$$\mu_x = \lambda \int_0^T A dt$$

$$\mu_x = \lambda A[t]_0^T$$

$$\mu_x = \lambda AT$$

Auto Covariance $C_x(\tau) = \lambda \int_{-\infty}^{\infty} h(t)\, h(t+\tau) dt$

$\boldsymbol{C_x(\tau) = \lambda A^2(T - |\tau|), \quad |\tau| < T}$

$\boldsymbol{0, \quad |\tau| \geq T}$

2. Find the thermal noise voltage developed across a resistor of 800 ohms. The bandwidth of the measuring instrument is 7MHz ambient temperature is 30°C

Solution:

Given R=800Ω

$\Delta f = 7\ MHz$

$T = 30^{\circ}$C=273+30=303K

The r.m.s value of noise voltage is

$V_{rms} = \sqrt{4KT(\Delta f)R}$

$V_{rms} = \sqrt{4x1.38x10^{-23}x303x7x10^6x800}$

$\boldsymbol{V_{rms} = 9.67\mu V}$

3. Consider a white noise w(t) of zero mean and PSD $N_o/2$ is applied to a ideal LPF of bandwidth B and pass band magnitude one. Find the PSD and autocorrelation function of noise n(t) appearing at the output of the filter.

Solution:

The auto correlation function is inverse fourier transform of PSD

$$\boldsymbol{R_N(\tau) = \int_{-B}^{B} S_N(f) e^{j2\pi f\tau} df}$$

$$R_N(\tau) = N_o/2 e^{j2\pi f\tau} df$$

$$R_N(\tau) = (N_o/2) e^{j2\pi f\tau} df$$

$$R_N(\tau) = \frac{N_o}{2}\left[\frac{e^{j2\pi f\tau}}{j2\pi\tau}\right]_{-B}^{B}$$

$$R_N(\tau) = \frac{N_o}{2\pi\tau}\left[\frac{e^{j2\pi B\tau} - e^{-j2\pi B\tau}}{j2}\right]$$

$$R_N(\tau) = \frac{N_o B}{2\pi B\tau} sin2\pi B\tau$$

$$R_N(\tau) = \frac{sin2\pi B\tau}{2\pi B\tau} N_o B$$

$$\boldsymbol{R_N(\tau) = N_o B\ sinc2\pi B\tau}$$