UNIT III

ANTENNA ARRAYS

N element linear array, Pattern multiplication, Broadside and End fire array – Concept of Phased arrays, Adaptive array, Basic principle of antenna Synthesis-Binomial array

3.1 N element linear array:

Consider an infinitesimal dipole of length dl kept at a point $(0,0,z_1^{'})$in free space . Let the z directed current in the dipole be I_1. The fields produced by the dipole are computed using the vector potential approach.

Since the dipole current is z directed , the vector potential also has only a z component which is given by

$$A_z = \frac{\mu}{4\pi} I_1 dl \frac{e^{-jk\,r_1}}{r_1}$$

Where $r_1$is the distance from the center of the current element to the field point (x,y,z). when the field point ia at a large distance , we can approximate $r_1$to

$$r_1 \approx r \qquad for\ amplitude$$

$$r_1 \approx r - z_1^{'} cos\theta \qquad for\ phase$$

Using the vector potential approach with these far-field approximations we get the electric field radiated by the dipole as

$$E_{\theta_1} = \frac{j\eta I_1 dl}{4\pi} sin\theta \frac{e^{-jkr}}{r} \; e^{-jk z_1' cos\theta}$$

Let us now consider N such infinitesimal , z directed current elements kept along the z axis at points $z_1', z_2', \dots \dots \dots z_N'$. Let the currents in these dipoles be $I_1, I_2, \dots \dots .. I_N$. respectively.

It is implied that all the currents have the same frequency. Using superposition , the field at any point can be written as a sum of the fields due to each of the elements.

$$E_\theta = E_{\theta_1} + E_{\theta_2} + E_{\theta_3} + \cdots \dots \dots + E_{\theta_N}$$

$$= \frac{j\eta dl}{4\pi} sin\theta \left[I_1 \frac{e^{-jk r_1}}{r_1} + I_1 \frac{e^{-jk r_2}}{r_2} + \cdots \dots \dots \dots . I_N \frac{e^{-jk r_N}}{r_N} \right]$$

Where $r_1, r_2 \dots \dots . r_N$ are respectively the distances from the dipoles 1, 2,…..N to the field point. In the far field region of these dipoles , the distance from the n^{th}dipole to the field point, r_n , is approximated to

$$r_n \approx r \qquad n = 1,2,3 \dots \dots . N \quad for\ amplitude$$

$$r_n \approx r - z_n' cos\theta \qquad n = 1,2,3 \dots \dots . N \qquad for\ phase$$

Where $z_n{}'$ is the location of the n^{th} dipole .

Now

$$E_\theta = \frac{j\eta dl}{4\pi} sin\theta \frac{e^{-jkr}}{r} \sum_{n=1}^{N} I_n \; e^{-jk z_n' cos\theta}$$

The term outside the summation corresponds to the electric field produced by an infinitesimal dipole excited by a unit current at the origin and is known as the element pattern. The remaining portion of the equation is called the array factor . thus the radiation pattern of an array of equi – oriented identical antenna elements is given by the product of the element pattern and array factor . This is known as the *pattern multiplication theorem.*

$$array\ pattern = element\ pattern\ \times array\ factor$$

The array factor (AF) is given by

$$AF = \sum_{n=1}^{N} I_n \; e^{-jk z_n' \cos\theta}$$

3.2 MULTIPLICATION OF PATTERN

Multiplication of pattern or simply pattern multiplication in general can be stated as follows:

" The total field pattern of an array of non- isotropic but similar sources is the multiplication of the individual source patterns and the pattern of array of isotropic point sources each located at the phase centre of individual source and having the relative amplitude and phase, whereas the total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources".

$$total\ field\ (E) = \{E_i(\theta,\phi) \times E_a(\theta,\phi)\} \times \{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)\}$$

Multiplication of Field pattern — Addition of phase pattern

Where $E_i(\theta,\phi)$ = Field pattern of individual source

$E_a(\theta,\phi)$ = Field pattern of array of isotropic point sources

$E_{pi}(\theta,\phi)$ = Phase pattern of individual source

$E_{pa}(\theta,\phi)$ = Phase pattern of array of isotropic point source

The angle θ *and* ϕ represent the polar and azimuth angles respectively. Theoretically,

$$\text{Resultant field pattern} = \begin{Bmatrix} \text{Individual} \\ \text{source} \\ \text{pattern} \end{Bmatrix} \times \begin{Bmatrix} \text{Pattern of array of 2 point} \\ \text{sources each located at the} \\ \text{phase centre of individual source} \end{Bmatrix}$$

Advantages of pattern multiplication

(i) It is a speedy method for sketching the pattern of complicated arrays just by inspection.
(ii) It provides to be useful tool in the design of antenna arrays.

RADIATION PATTERN OF 4-ISOTROPIC ELEMENTS FED IN PHASE AND SPACED $\frac{\lambda}{2}$ APART (UNIFORM LINEAR ARRAY)

Consider a four element array of antennas as shown in Fig in which the spacing between the elements in is $\frac{\lambda}{2}$ and the currents are in phase (α = 0). The pattern Can be obtained directly by adding the four electric fields due to the four antennas. However the same radiation pattern can be obtained by pattern multiplication in the following manner.

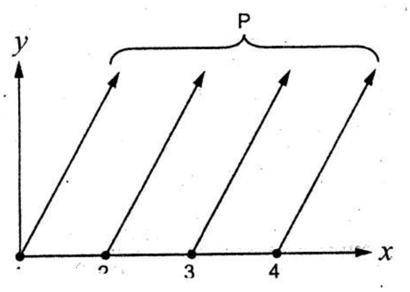


Fig 3.3: Linear array of 4 isotropic elements spaced $\frac{\lambda}{2}$ apart , fed in phase

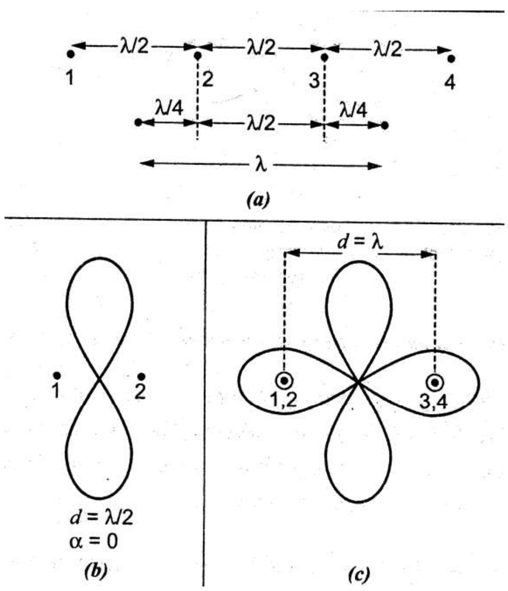


Fig :3.4

- ✓ Two isotropic point source spaced $\frac{\lambda}{2}$ apart fed in phase provides a bidirectional pattern as in Fig.3.4 (b)(as described in section 2.7.3).
- ✓ Now the elements 1 and 2 are considered as one unit and this new unit and new unit is considered to be placed between the midway of elements 1,2 and similarly the elements 3,4 as shown in Fig.3.4(a)
- ✓ ∴ 4 elements spaced $\frac{\lambda}{2}$ have been replaced by 2 units spaced λ and therefore the problem of determining radiation of 4elements has been reduced to find out the radiation pattern of 2 antennas spaced 'λ' apart as shown in Fig.3.4(a)

$$\begin{Bmatrix} \therefore Resultant\ radiation \\ pattern\ of\ 4\ elements \end{Bmatrix} = \begin{Bmatrix} Radiation\ pattern \\ of individual \\ elements \end{Bmatrix} \times \begin{Bmatrix} Array\ of \\ two\ unit \\ spaced `\lambda' \end{Bmatrix}$$

- ✓ **Here the width of the principal lobe is the same as the width of the corresponding lobe of the group pattern**. The number of secondary lobes can be determined from the number of nulls in the resultant pattern, which is sum of nulls in the unit and group patterns.

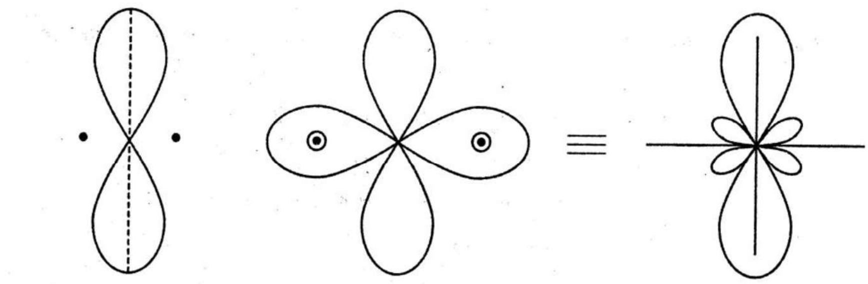


Fig 3.5: Resultant radiation pattern of 4 isotropic elements by pattern multiplication

RADIATION PATTERN OF 8 ISOTROPIC ELEMENTS FED IN PHASE AND $\frac{\lambda}{2}$ APART

The application of pattern multiplication can be extended to more complicated arrays. For example , the pattern of a broadside array of eight elements spaced $\lambda/2$ apart and fed phase would be obtained as follows. Consider four elements as one unit and another four elements as another similar unit. This is called unit pattern and it has radiation pattern as shown in fig 3.6(b). now the 8 element array has been reduced to 2 element array spaced

2λ apart as shown in fig 3.6(a). The resultant pattern for 8 element array is obtained as follows.

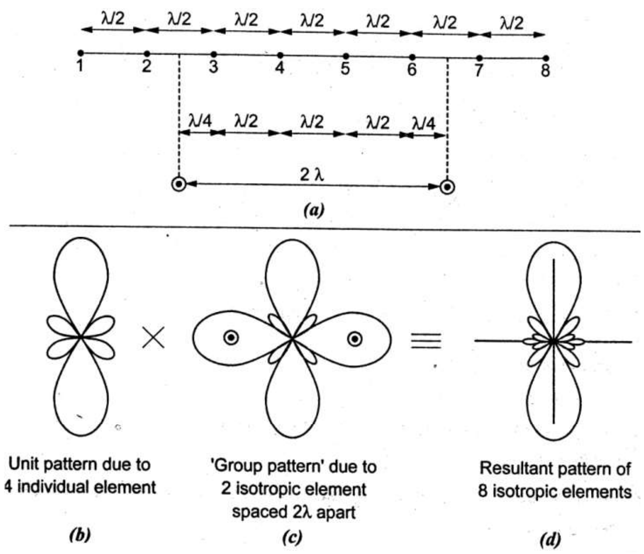


Fig 3.6 : Resultant radiation pattern of 8 - isotropic elements by pattern multiplication

3.3 n- element uniform uniform Array:

An array of n elements is said to be linear array , if all the individual elements are spaced equally along a line. An array is said to be uniform array if the elements in the array are fed with currents in the array are fed with currents with equal magnitudes and with uniform progressive phase shift along the line.

3.3 Broadside Array:

Consider the n number of identical radiators carry currents which are equal in magnitude and in phase . The radiators are equal space. Hence the maximum radiation occurs in the directions normal to the line of array. Hence such an array is known as uniform Broad side array.

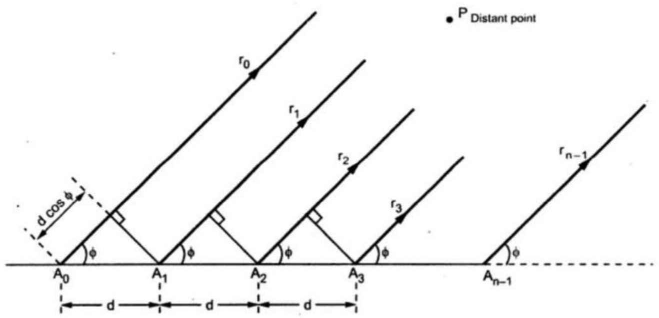


Fig :3.7

The current in the first element $I_1 = I_0$. The current in the n^{th} element can be written as $I_n = I_o e^{j(n-1)\alpha}$

The array factor of an N-element linear array along the z-axis is given by

$$AF = \sum_{n=1}^{N} I_n e^{jk z_n^{'} cos\theta} \qquad \ldots\ldots\ldots\ldots (1)$$

I_n = the current in n^{th} element

$z_n^{'} = (n-1)d$ is the location of the n^{th} element

$$AF = \sum_{n=1}^{N} I_o e^{j(n-1)\alpha} \; e^{jk(n-1)d\, cos\theta}$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)(kd\, cos\theta + \alpha)}$$

Where $\psi = kd\, cos\theta + \alpha$

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi}$$

$$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots\ldots\ldots\ldots + e^{j(N-1)\psi} \qquad \ldots\ldots\ldots\ldots (2)$$

Then AF is multiplied by $e^{j\psi}$

$$AF\, e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots\ldots\ldots\ldots + e^{Ne^{j\psi}} \qquad \ldots\ldots\ldots\ldots(3)$$

Subtracting equation (2) from (3)

$$AF(e^{jN\psi - 1}) = (e^{jN\psi - 1})$$

$$AF = \frac{(e^{jN\psi - 1})}{(e^{jN\psi - 1})}$$

Taking out $e^{jN\psi/2}$ from numerator $e^{j\psi/2}$ from denominator

$$AF = \frac{e^{jN\psi/2}\,(e^{jN\psi/2} - e^{-jN\psi/2})}{e^{j\psi/2}\,(e^{j\psi/2} - e^{-j\psi/2})}$$

$$= e^{\frac{j\psi(N-1)}{2}} \frac{2jsinN\psi/2}{2jsin\psi/2}$$

$$[\because e^{j\theta} - e^{-j\theta} = 2jsin\theta]$$

The magnitude of array factor is given by

$$AF = \left|\frac{sin\, N\psi/2}{sin\psi/2}\right|$$

Properties of Broadside array:

1. **Major Lobe:**
   In case of broadside array , the field is maximum in the direction normal to field is maximum in the direction normal to the axis of the array. Thus the condition for the maximum field is given by , $\psi = 0$,

$$i.e., kdcos\theta = 0$$
$$cos\theta = 0$$
$$\theta = 90^0 \; or \; 270^0$$

Thus $\theta = 90^0$ and $\theta = 270^0$ are called directions of priciple maxima.

2. **Magnitude of major lobe**

The maximum radiation occurs when $\psi = 0$. Hence we can write ,

$$|Major\ Lobe\ | = |AF| = \lim_{\psi=0}\left\{\frac{\frac{d}{d\psi}\left(\sin\left(\frac{N\psi}{2}\right)\right)}{\frac{d}{d\psi}\left(\sin\left(\frac{\psi}{2}\right)\right)}\right\}$$

$$= \lim_{\psi=0}\left\{\frac{\left(\cos\left(\frac{N\psi}{2}\right)\right)\left(\frac{N}{2}\right)}{\left(\cos\left(\frac{\psi}{2}\right)\right)\left(\frac{1}{2}\right)}\right\}$$

$$|Major\ Lobe\ | = N$$

Where N is the number of elements in the array

3. Nulls

To find the direction of minima , equating the ratio of magnitudes of the fields to zero.

$$\left|\frac{sin\ N\psi/2}{sin\psi/2}\right| = 0$$

Thus the condition of minima is given by

$$sin\frac{N\psi}{2} = 0\ ; but\frac{sin\psi}{2} \neq 0$$

Hence we can write

$$sin\frac{N\psi}{2} = 0$$

$$\psi = kd\ cos\theta$$

$$\sin(Nkd\ cos\theta\ ) = 0$$

Sub $k = \frac{2\pi}{\lambda}$

$$\sin(\frac{N}{2}\frac{2\pi}{\lambda}.dcos\theta) = 0$$

$$\frac{N\pi d}{\lambda}cos\theta_{\min} = \pm m\pi.$$

$$\theta_{\min} = \cos^{-1}\left(\pm\frac{m\lambda}{Nd}\right)$$

Where N = Number of elements in array

d = spacing between elements in meter.

λ = wavelength , m=1,2,3,……

This equation gives the directions of nulls.

4. Subsidary maxima (Side lobes)

The direction of the subsidary maxima or side lobes can be obtained if the equation AF becomes

$$sin\frac{N\psi}{2} = \pm 1$$

$$\frac{N\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \dots ....$$

$Hence \quad sin\frac{N\psi}{2} = \pm 1$ is not considered . Because if $\frac{N\psi}{2} = \frac{\pi}{2}$ then $sin\frac{N\psi}{2} = 1$ which is the ***direction of principle maxima***

Hence we can skip $\frac{N\psi}{2} = \frac{\pi}{2}$ value

Thus , we get

$$\psi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, \dots ....$$

Now $\psi = kd\, cos\theta = \left(\frac{2\pi}{\lambda}\right). dcos\theta$

Hence equation for ψ can be written as,

$$\left(\frac{2\pi}{\lambda}\right). dcos\theta = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, \dots ....$$

$$cos\theta = \left(\frac{\lambda}{2\pi d}\right)\left[\pm\frac{(2m+1)}{n}\pi\right] \qquad wherem = 1,2,3 \dots ..$$

$$\theta = \cos^{-1} \left[\pm\frac{\lambda(2m+1)}{2nd}\right]$$

The above equation represents the directions where certain radiation which is not maximum . hence it represents directions of subsidiary maxima or side lobes.

5. **Beam width of major lobe**

The beamwidth is defined as the angle between first nulls. Alternatively beamwidth is the angle equal to twice the angle between first null and major lobe maximum direction.

Hence the beamwidth between first nulls is given by,

$$BWFN = 2\gamma = , where\, \gamma = 90 - \theta$$

But

$$\theta_{\min} = \cos^{-1}\left(\pm\frac{m\lambda}{Nd}\right)$$

Also

$$90 - \theta_{\min} = \gamma \qquad i.e., 90 - \gamma = \theta_{\min}$$

Hence

$$90 - \gamma = \cos^{-1}\left(\pm\frac{m\lambda}{Nd}\right)$$

Taking cosine of angle on both the sides, we get

$$\cos(90 - \gamma) = \cos\left[cos^{-1}\left(\pm\frac{m\lambda}{Nd}\right)\right]$$

$$sin\gamma = \pm\frac{m\lambda}{Nd}$$

If γ is very small, $sin\gamma = \gamma$. Substituting in above equation, we get,

$$\gamma = \pm\frac{m\lambda}{Nd}$$

For first null , i.e., m = 1

$$\gamma = \frac{\lambda}{Nd}$$

But

$Nd = (N-1)d$,if N is very large. This N.d indicates the total length of array in meter. This is denoted by L .

$$BWFN = \frac{2\lambda}{L}\ rad = \frac{2}{\left(\frac{L}{\lambda}\right)}\ rad$$

Converting BWFN in degrees , we can write ,

$$BWFN = \frac{114.6\ \lambda}{L} = \frac{114.6}{\left(\frac{L}{\lambda}\right)}\ degrees$$

Now the half power beamwidth (HPBW) is given by

$$HPBW = \frac{BWFN}{2} = \frac{1}{\left(\frac{L}{\lambda}\right)}\ rad$$

Expressing HPBW in degrees we can write,

$$HPBW = \frac{57.3}{\left(\frac{L}{\lambda}\right)}\ degrees$$

6. **Directivity**

The directivity in case of broadside array is defined as

$$G_{max} = \frac{Maximum\ radiation\ intensity}{Average\ radiation\ intensity} = \frac{U_{max}}{U_{avg}}$$

$$U_{avg} = \frac{\pi}{nkd}$$

But

$$U_{max} = 1 \qquad at\ \theta = 90^0$$

$$D = \frac{1}{\frac{\pi}{Nkd}} = \frac{Nkd}{\pi}$$

$$here\ k = \frac{2\pi}{\lambda}$$

Hence

$$D = 2N\left(\frac{d}{\lambda}\right)$$

The total length of the array is given by

$$L = (N-1)d \ \approx N.d, if\ N\ is\ very\ large$$

Hence the directivity can be expressed interms of the total length of the array as

$$D = 2\left(\frac{L}{\lambda}\right)$$

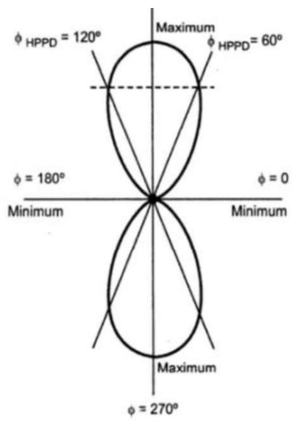


Fig 3.8 : Radiation Pattern

Sl. No	Parameter	Expression
1.	**Major Lobe:**	$\theta = 90^0\ or\ 270^0$
2	**Magnitude of major lobe**	N
3.	Nulls	$\theta_{\min} = \cos^{-1}\left(\pm\frac{m\lambda}{Nd}\right)$

4.	BWFN	$BWFN = \frac{2}{\left(\frac{L}{\lambda}\right)} \; rad = \frac{114.6}{\left(\frac{L}{\lambda}\right)} \; deg$
5.	**Beam width of major lobe**	$HPBW = \frac{1}{\left(\frac{L}{\lambda}\right)} \; rad = \frac{57.3}{\left(\frac{L}{\lambda}\right)} \; deg$
6.	Directivity	$D = 2N\left(\frac{d}{\lambda}\right) = 2\left(\frac{L}{\lambda}\right)$

3.4 Array with n elements with equal spacing and current equal in magnitude but progressive phase shift – End Fire Array / (Line of Array)

Consider n number of identical radiative with radiates with equal current which are not in phase . Assume that there is progressive phase lag of **kd radians** in each radiators.

The current in the first element $I_1 = I_0$. The current in the n^{th} element can be written as $I_n = I_o e^{j\,(n-1)\alpha}$

The array factor of an N-element linear array along the z-axis is given by

$$AF = \sum_{n=1}^{N} I_n e^{jk\, z_n' cos\theta} \qquad \dots\dots\dots\dots (1)$$

I_n = the current in n^{th} element

$z_n' = (n-1)d$ is the location of the n^{th} element

$$AF = \sum_{n=1}^{N} I_o e^{j\,(n-1)\alpha} \; e^{jk\,(n-1)d\; cos\theta}$$

$$AF = \sum_{n=1}^{N} e^{j\,(n-1)(kd\; cos\theta - kd)}$$

Where $\psi = kd\ cos\theta - kd$

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi}$$

$$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots\ldots\ldots\ldots + e^{j(N-1)\psi} \qquad \ldots\ldots\ldots\ldots(2)$$

Then AF is multiplied by $e^{j\psi}$

$$AF\ e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots\ldots\ldots\ldots + e^{Ne^{j\psi}} \qquad \ldots\ldots\ldots\ldots(3)$$

Subtracting equation (2) from (3)

$$AF(e^{jN\psi -1}) = (e^{jN\psi -1})$$

$$AF = \frac{(e^{jN\psi -1})}{(e^{jN\psi -1})}$$

Taking out $e^{jN\psi/2}$ from numerator $e^{j\psi/2}$ from denominator

$$AF = \frac{e^{jN\psi/2}\ (e^{jN\psi/2} - e^{-jN\psi/2})}{e^{j\psi/2}\ (e^{j\psi/2} - e^{-j\psi/2})}$$

$$= e^{\frac{j\psi(N-1)}{2}} \frac{2jsinN\psi/2}{2jsin\psi/2}$$

$$[\because e^{j\theta} - e^{-j\theta} = 2jsin\theta]$$

The magnitude of array factor is given by

$$AF = \left|\frac{sin\ N\psi/2}{sin\psi/2}\right|$$

Properties of Endfire array:

1. **Major Lobe:**
   In case of Endfire array , where currents supplied to the antennas are equal to amplitude but the phase changes progressively through array , the phase angle is given by

$$\psi = kd\ cos\theta - kd$$

In case of the endfire array, the condition of principle maxima is given by,
$\psi = 0$

$$i.e., kd\,cos\theta - kd = 0$$
$$\theta = 0^0 \; or \; 180^0$$

Thus $\theta = 0^0$ and $\theta = 180^0$ are called directions of priciple maxima.Also it indicates that the maximum radiation is along the axis of array or line of array.

2. Magnitude of major lobe

The maximum radiation occurs when $\psi = 0$. Hence we can write ,

$$|Major\ Lobe\ | = \ |AF| = \lim_{\psi=0} \left\{ \frac{\frac{d}{d\psi}\left(\sin\left(\frac{N\psi}{2}\right)\right)}{\frac{d}{d\psi}\left(\sin\left(\frac{\psi}{2}\right)\right)} \right\}$$
$$= \lim_{\psi=0} \left\{ \frac{\left(\cos\left(\frac{N\psi}{2}\right)\right)\left(\frac{N}{2}\right)}{\left(\cos\left(\frac{\psi}{2}\right)\right)\left(\frac{1}{2}\right)} \right\}$$
$$|Major\ Lobe\ | = N$$

Where N is the number of elements in the array

3.Nulls

To find the direction of minima , equating the ratio of magnitudes of the fields to zero.

$$\left|\frac{sin\ N\psi/2}{sin\psi/2}\right| = 0$$

Thus the condition of minima is given by

$$sin\frac{N\psi}{2} = 0\ ; but\frac{sin\psi}{2} \neq 0$$

Hence we can write

$$sin\frac{N\psi}{2} = 0$$
$$\psi = \ kd\ cos\theta - kd$$
$$\sin(\frac{Nkd\ (cos\theta_{min}\ - 1}{2})\ = 0$$

Sub $k = \frac{2\pi}{\lambda}$

$$\sin(\frac{2\pi}{\lambda}.\frac{Nd\ (cos\theta_{min}\ - 1}{2}) = 0$$

$$\frac{N\pi d}{\lambda} cos\theta_{\min} \quad - 1 = \pm m\pi.$$

$$\theta_{\min} \quad = \cos^{-1}\left(\pm\frac{m\lambda}{Nd} + 1\right)$$

Note that value of $\cos\theta \ - 1$ is always less than 1 . Hence it is always negative . Hence only considering negative values, we get

$$\theta_{\min} \quad = \cos^{-1}\left(1 - \frac{m\lambda}{Nd}\right)$$

Where N = Number of elements in array
d = spacing between elements in meter.
λ = wavelength , m=1,2,3,……
This equation gives the directions of nulls.
Consider the below equation

$$\cos\theta_{min} - 1 = \pm\frac{m\lambda}{Nd}$$

Expressing term on L.H.S interms of half angles , we get

$$2\sin^2\theta_{min} = \pm\frac{m\lambda}{Nd}$$

$$\sin^2\frac{\theta_{min}}{2} = \pm\frac{m\lambda}{2Nd}$$

$$sin\frac{\theta_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2Nd}}$$

$$\frac{\theta_{min}}{2} = \sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2Nd}}\right]$$

$$\theta_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2Nd}}\right]$$

4.Subsidary maxima (Side lobes)

The direction of the subsidary maxima or side lobes can be obtained if the equation AF becomes

$$sin\frac{N\psi}{2} = \pm 1$$

$$\frac{N\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \dots \dots$$

Hence $\frac{N\psi}{2} = \frac{\pi}{2}$ is skipped because with this value of $\frac{N\psi}{2}$ we get $sin\frac{N\psi}{2} = 1$ which is the ***direction of principle maxima***

Hence we can write $\frac{N\psi}{2} = \pm(2m+1)\frac{\pi}{2}$

Putting the value of ψ, then we get

$$\frac{Nkd(cos\theta - 1)}{2} = \pm(2m+1)\frac{\pi}{2}$$

$$Nkd(cos\theta - 1) = \pm(2m+1)\pi$$

Put $k = \frac{2\pi}{\lambda}$, we get

$$N\frac{2\pi}{\lambda}d(cos\theta - 1) = \pm(2m+1)\pi$$

$$(cos\theta - 1) = \pm\frac{(2m+1)\lambda}{2Nd}$$

Note that value of $\cos\theta - 1$ is always less than 0 . Hence it is always negative . Hence only considering negative values, we get

$$cos\theta - 1 = -\frac{(2m+1)\lambda}{2Nd}$$

$$\theta = \cos^{-1}\left[1 - \frac{(2m+1)\lambda}{2Nd}\right]$$

The above equation represents directions of subsidiary maxima or side lobes

5. Beam width of major lobe

The beam width of the end fire array is greater than that of broadside array.
Beam width = 2* Angle between first nulls and maximum of the major lobe i.e., θ_{min} .

$$\theta_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2Nd}}\right]$$

$$sin\frac{\theta_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2Nd}}$$

If θ_{min} is very low, then we can write $sin\frac{\theta_{min}}{2} \approx \frac{\theta_{min}}{2}$. Using this property in above equation we get,

$$\frac{\theta_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2Nd}}$$

$$\theta_{min} = \pm\sqrt{\frac{4m\lambda}{2Nd}} = \pm\sqrt{\frac{2m\lambda}{Nd}}$$

But $L = Nd$. Length of the antenna array , so the above eqn becomes

$$\theta_{min} = \pm\sqrt{\frac{2m\lambda}{L}} = \pm\sqrt{\frac{2m}{L/\lambda}}$$

The beam width between first null is given by

$$BWFN = 2\theta_{min} = \pm 2\sqrt{\frac{2m}{L/\lambda}}$$

Expressing BWFN in degrees , we get

$$BWFN = \pm 114.6\sqrt{\frac{2m}{L/\lambda}}\ deg$$

For m=1,

$$BWFN = \pm 2\sqrt{\frac{2}{L/\lambda}}\ rad = 114.6\sqrt{\frac{2m}{L/\lambda}}\ deg$$

6.Directivity

Similar to the broadside array , the directivity for the end fire array is given by,

$$U_{avg} = \frac{\pi}{2Nkd}$$

But

$$U_{max} = 1$$

$$D = \frac{1}{\frac{\pi}{2Nkd}} = \frac{2Nkd}{\pi}$$

$$here\ k = \frac{2\pi}{\lambda}$$

Hence

$$D = 4N\left(\frac{d}{\lambda}\right)$$

The total length of the array is given by

$$L = (N-1)d \approx N.d, if\ N\ is\ very\ large$$

Hence the directivity can be expressed interms of the total length of the array as

$$D = 4\left(\frac{L}{\lambda}\right)$$

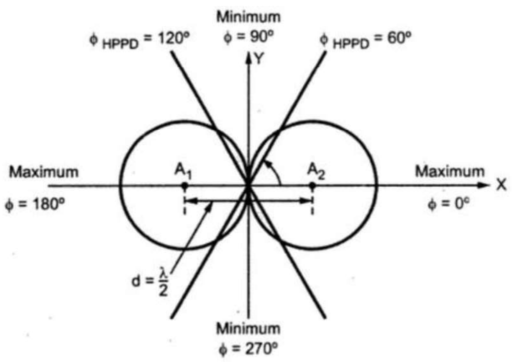


Fig 3.8 : Radiation pattern

Sl. No	Parameter	Expression
1.	**Major Lobe:**	$\theta = 0^0\ or\ 180^0$
2	**Magnitude of major lobe**	N
3.	Nulls	$\theta_{\min} = \cos^{-1}\left(1 - \frac{m\lambda}{Nd}\right)$
4.	BWFN	$BWFN = \pm 2\sqrt{\frac{2}{L/\lambda}}\ rad = 114.6\sqrt{\frac{2m}{L/\lambda}}\ deg$
5.	**Beam width of major lobe**	$HPBW = \pm\sqrt{\frac{2}{L/\lambda}}\ rad = 57.3\sqrt{\frac{2m}{L/\lambda}}\ deg$
6.	Directivity	$D = 4N\left(\frac{d}{\lambda}\right) = 4\left(\frac{L}{\lambda}\right)$

3.5 Phased Arrays

In case of the broadside array and the end fire array , the maximum radiation can be obtained by adjusting the phase excitation between elements in the direction normal and along the axis of array respectively . That means in other words elements of antenna array can be phased in particular way. So we can obtain an array which gives maximum radiation in any direction by controlling phase excitation in each element. Such an array is commonly called phased array.

The array in which the phase and the amplitude of most of the elements is variable,provided that the direction of maximum radiation (beam direction) and pattern shape along with the side lobes is controlled, is called as phased array.

Suppose the array gives maximum radiation in direction $\phi = \phi_0$ where $0 \leq \phi_0 \leq 180^0$, then the phase shift that must be controlled can be obtained as follows.

$$\psi = kdcos\theta + \alpha \quad at \ \phi = \phi_0 \ = 0$$

$$\therefore kdcos\phi_0 \ + \alpha = 0$$

$$\alpha = \ -kdcos\phi_0 \qquad \ldots\ldots.(1)$$

Thus from equation (1) it is clear that the maximum radiation can be achieved in any direction if the progressive phase difference between the elements is controlled. The electronic phased array operates on the same principle.

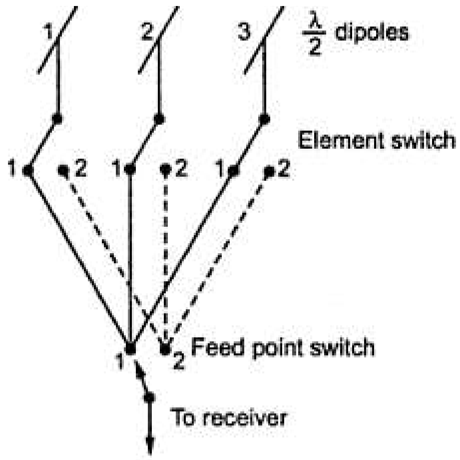


Fig3.9 : Phased array with mechanical switches at elements and feed point

Consider a three element array as shown in fig. the element of array is considered as $\frac{\lambda}{2}$ dipole. All the cables used are of same length. All the three cables are brought together at common feed point. Here mechanical switches are used. Such switch is installed one at each antenna and one at a common feed point. All the switches are ganged together. Thus by operating switch, the beam can be shifted to any phase shift.

To make operation reliable and simple, the gaged mechanical switch is replaced by **PIN dipole** which acts as **electronic switch.** But for precision in results, the number of cables should be minised.

In many application phase shifter is used instead of controlling phase by switching cables. It can be achieved by using ferrite device. The conducting wires are wrapped around the phase shifter. The current flowing through these wires controls the magnetic field within ferrite and then the magnetic field in the ferrite controls the phase shift.

The phased array for specialized functional utility are recognized by different names such as frequency scanning array, retroarray and adoptive array.

The array in which the phase change is controlled by varying the frequency is called frequency scanning array. This is found to be the simplest phased array as at each element separate phase control is not necessary. A simple transmission line fed frequency scanning array as shown in fig.

Each element of the scanning array is fed by a transmission line via directional coupler. Note that the directional couplers are fixed in position, while the beam scanning is done with a frequency change. To avoid reflections and to obtain pure form of the travelling wave, the transmission line is properly terminated of the load.

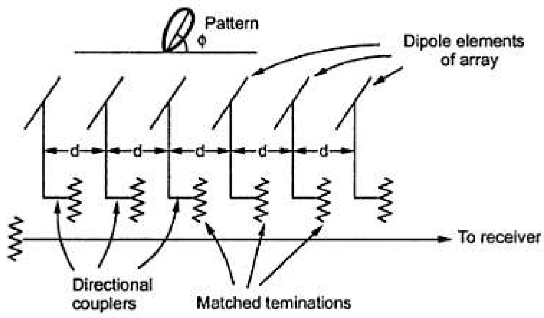


Fig3.10 : Frequency scanning line fed phased array

The main advantage of the frequency scanning array is that there are no moving pairs and no switches and phase shifters are required.

The array which automatically reflects an incoming signal back to the source is called retro array . it acts as a retro reflector similar to the passive square corner reflector. That means the wave incident on the array is received and transmitted back in the same direction . In other words , each element of the retro array reradiates signal which is actually the conjugate of the received one. Simplest form of the retro array is the Van Atta array shown in fig, in which 8 identical $\frac{\lambda}{2}$ dipole elements are used, with pairs formed between elements 1 and 8,2 and 7,3 and 6,4 and 5 using cables of equal length. If the wave arrives at angle ϕ say then it gets transmitted in the same direction.

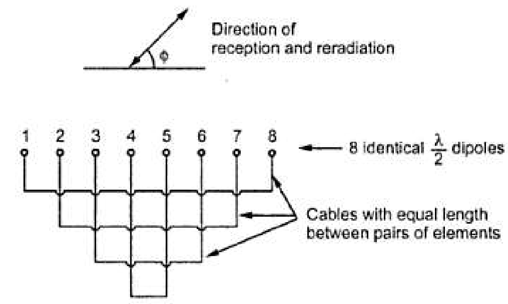


Fig3.11 : Van Atta retro array

An array which automatically turn the maximum beam in the desired direction while turn the null in the undesired direction is called adoptive array. The adoptive array adjust itself in the desired direction with aeareness of its environment . In modern adoptive arrays, the output of each element in the array is sampled, digitized and then processed using computers. Such arrays are commonly called smart antennas.

3.6 ADAPTIVE ARRAYS

The antenna elements and their transmission –line interconnections produce a beam or beams in predetermined directions. Thus when receiving , these arrays look in a given direction regardless of whether any signals are arriving from the direction or not. However , by processing the signals from the individual elements, an array can become active and react intelligently to its environment , steering its beam toward a desired signal while simultaneously steering a null toward an undesired, interfering signal and thereby maximizing the signal-to-noise ratio of the desired signal . The term ***adaptive array*** is applied to this kind of antenna.

Also , by suitable signal processing , performance may be further enhanced , giving simulated patterns of higher resolution and lower side lobes. In adition , by appropriate sampling and digitizing the signals at the terminals of each element and processing themwith a computer, a very intelligent or smart antenna can , in principle , be built. For a given with a computer , such an antenna's capabilities are limited, mainly by the ingenuityof the programmer and the available computer power. Thus , for example, multiple beams may be simultaneously directed toward many signals arriving from different directions within the field of view of the antenna. These antennas are some times called *Digital Beam Forming (DBF) antennas.*

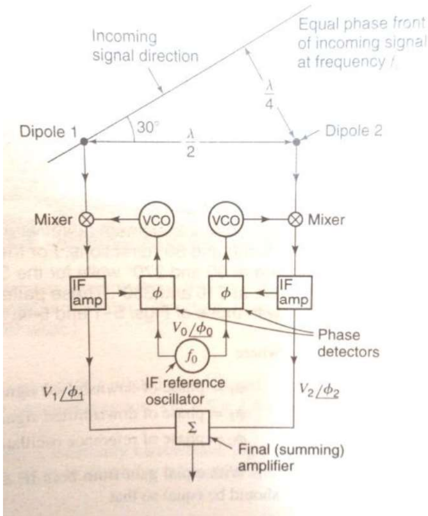


Fig 3.12 : Two element adaptive array with signal processing circuiry

A example of an adaptive array , a simple 2-element system is shown in fig. with $\frac{\lambda}{2}$ spacing between the elements at the signal frequency f_s. Let each element be a $\frac{\lambda}{2}$ dipole seen end-on in fig. so that the patterns of the elements are uniform in the plane of the page. With elements opertating in phase, the beam is broadside .

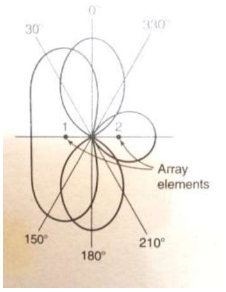


Fig 3.13: pattern of 2 element adaptive array for signals

Consider now the case of a signal at 30^0 from broadside , so that the wave arriving at element 2 travels $\frac{\lambda}{4}$farther than to element 1, thus retarding the phase of the signal by 90^0at element 2. Each element is equipped with its own mixer, VCO , intermediate frequency amplifier and phase detector. An oscillator at the intermediate frequency f_0 is connected to each phase detector as reference. The phase detector compares the phase of the downshifted signal with the phase of reference oscillator and produces a voltage proportioanl to the phase difference. This voltage, in turn , advances or retards the phase of the VCO output so as to reduce the phase difference to zero. The voltage for the VCO of element 1 would ideally be equal in magnitude but of opposite sign to the voltage for the VCO of element 2 so that the downshifted signals from both elements are locked in phase , making

$$\phi_1 = \phi_2 = \phi_0$$

Where ϕ_1 = phase of downshifted signal from element 1

ϕ_2 = phase of downshifted signal from element 2

ϕ_0 = phase of reference oscillator

With equal gain from both IF amplifiers the voltages V_1 and V_2 from both elements should be equal so that

$$V_1\angle\phi_1 = V_2\angle\phi_2$$

Making the voltages from the summing ampllifier proportional to $2V_1$(=V_2) and maximizing the response of the array to the incoming sifnal by steering the beam onto the incoming signal. In our example, 45^0 phase corrections of opposite sign whould be required by the VCOs (+ for element 1, - for element 2).

In our rudimentary 2 element example, the beam will be in the 0^0 direction for a signal from the 0^0 direction and at 30^0 for a signal from that direction , as shown by the patterns in fig. If interferingg signals are arriving from the 210^0 and 330^0 will suppress the interfreence. However , an interfering signal at 150^0 would be at a pattern maximum, the same as the desired signal at 30^0. To provide more effective adaptation to its environment, an array with more elements and more sophisticated signal processing is required. For example , the main beam may be steered towared the drsired signal by changing the progressive phase difference between elements , while ,independently, one or more nulls are steered toward interfering signals by modifying the array element amplitudes with digitally controlled attenuators.

3.7 BINOMAL ARRAYS

- ✓ So far we have seen only an array having current with equal amplitude. But binomial array deals with the non-uniform current.
- ✓ Here the amplitudes of the radiating sources are arranged according to the coefficients of successive terms of the binomial series and therefore it is named as binomial array.

Binomial Series

$$(a+b)^{n-1} = a^{n-1} + \frac{n-1}{1} a^{n-2}.b + \frac{(n-1)(n-2)}{2} a^{n-3} b^2 + \frac{(n-1)(n-2)(n-3)}{3} a^{n-4} b^3 \text{ …….}$$

$where\ n \rightarrow$ Number of radiation sources in the array.

Disadvantages of uniform linear array

- ✓ When the array length is increased to increase the directivity, secondary or minor lobes also appear.
- ✓ But this has to be reduced to a minimum desired level in comparison to principal (or) main lobes because considerable amount of power is wasted in these directions.

(In the case of radar, false target may be indicated).

Concept of Binomial array

It the array is arranged in such a way that radiating sources in the centre of the broad side array radiates more strongly than the radiating sources at the edges, minor lobes can be eliminated.

✓ The secondary lobes can be eliminated entirely if the following 2 conditions are satisfied.

(i)The space between the 2 consecutive radiating sources does not exceed $\frac{\lambda}{2}$ and

(ii) The current amplitudes in radiating sources (from outer towards centre source) are proportional to the coefficients of the successive terms of the binomial series.

Number of sources	Relative Amplitude
n=1	1
n=2	1,1
n=3	1,2,1
n=4	1,3,3,1
n=5	1,4,6,4,1
n=6	1,5,10,10,5,1
n=7	1,6,15,20,15,6,1
n=8	1,7,21,35,35,21,7,1
n=9	1,8,28,56,70,56,28,8,1
n=10	1,9,36,84,126,126,84,36,9,1

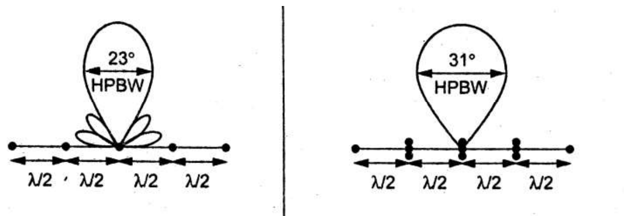
- ✓ These 2 conditions are necessarily satisfied in binomial arrays and the coefficients which corresponds to the amplitude of the sources are obtained by

  putting n=1,2,3,4,5,……in the above equation

  For example, the relative amplitudes for the arrays of 1to 10 radiating sources are as follows

This can be obtained from Pascal's triangle also where each internal integer is the sum of the above adjacent integers

- ✓ Here in this binomial array, elimination of secondary lobes takes place at the cost of directivity.
- ✓ HPBW (Half power Beam Width)of binomial array is more than that of uniform array for the same length of the array.
- ✓ For n=5,d = $\frac{\lambda}{2}$,HPBW of binomial array 1s $31^0$and HPBW of uniform array is23^0 as shown in Fig..
- ✓ Thus in uniform array, secondary lobes appear but principal lobe is sharp and narrow.
- ✓ In stone's binomial array, width of beam widens but without secondary lobes.



(a) Uniform array **(b) Binomial array with amplitude ratio 1 : 4 : 6 : 4 : 1**

Fig .3.14

- ✓ In derivation of binomial array, the principle of multiplicity of pattern is and its pattern has no minor lobes as shown in Fig.2.37 (a).
- ✓ The total electric field at distant point 'P' is

$$E = 2E_0 \cos\frac{\psi}{2} \qquad [\therefore d=\frac{\lambda}{2} \text{ and } E_0 = \frac{1}{2}]$$

$$E_{nor} = cos_{,}\left(\frac{\pi}{2}\cos\theta\right)$$

- ✓ If another identical array of 2 point sources is super imposed on the above array then the pattern will be as shown in Fig.3.15(b).The far field pattern according to principal of multiplication of pattern is $E_{nor} = cos^2\left(\frac{\pi}{2}\cos\theta\right)$.

  To principal of multiplication of pattern is $E_{nor} = cos^2\left(\frac{\pi}{2}\cos\theta\right)$

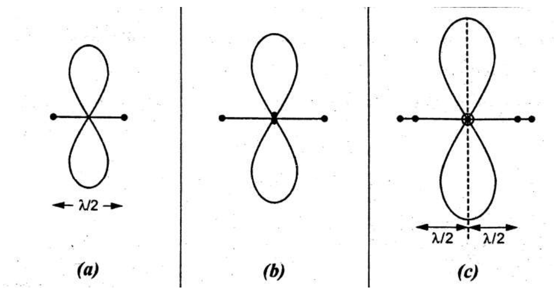
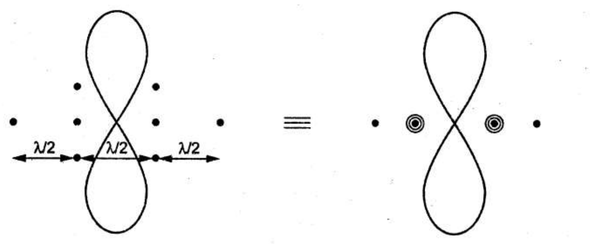


Fig.3.15

- ✓ The super imposition of 2 sources at the centre increases its current amplitude to just double in comparison to sources at the edges. Therefore the array now has the 3 effective
  Sources with amplitude ratio 1:2:1. (Although the centre source is shown separately one above the other for clarity but it should have been superimposed as shown in Fig. Similarly if the 3 source array is superimposed with another identical array, then an array of effective 4 sources with current amplitude ratio 1:3:3:1 is obtained as shown in Fig. and the far field is $E_{norm} = cos^2\left(\frac{\pi}{2}\cos\theta\right)$
- ✓ ∴ It is possible to have a pattern of any desired directivity without any minor lobes provided the current amplitude sources corresponds to the coefficients of the said binomial series .
- ✓ ∴ The far field pattern of 'n' sources is $cos^{n-1}\left(\frac{\pi}{2}\cos\theta\right)$



(a) 4 sources with amplitude ration 1 : 3 : 3 : 1 ***(b) 4 sources with super imposition of centre sources***

Fig 3.16

Disadvantages of Binomial Arrays

1. HPBW increases and hence the directivity decreases.
2. For the design of a large array, larger amplitude ratio of sources is required.

SOLVED PROBLEMS

Example 1 : A Broad side array consists of four identical half wave dipoles spaced 50 cm apart. If the wavelength is 0.1 m and each element carries r.f current of equal magnitude of 0.25 A and same phase, calculate power radiated and half power beam width of the major lobe.

Solution: Given: n = Number of elements = 4

λ = Wavelength = 0.1 m

d = Spacing between any two elements = 50 cm = 0.5 m

I = Current = 0.25 A

(i) Power radiated P_{rad} = n (I^2 R_{rad})

Where R_{rad} = Radiation resistance of antenna in the array

I,e., half wave dipole = 73

$\therefore$ P_{rad} = $4(0.25)^2 (73)$

= 18.25 watts

(ii) HPBW (Half Power Beam Width) = $\frac{BWFN}{2}$ (Beam width between first nulls)

BWFN = $\frac{2\lambda}{L} = \frac{2\lambda}{nd}$ radian

The length of the array (L) = n d = 4x 0.5 = 2meter

$\therefore$ BWFN = $\frac{2x0.1}{2}$ = 0.1 rad or 5.73^o

Hence HPBW = $\frac{BWFN}{2}$

= $\frac{0.1}{2}$ = 0.05 rad (or) 2.865^o

Example 2 : Find the minimum spacing between the elements in a broadside array of 10 isotropic radiators to have directivity of 7 dB.

Solution Given : $G_{D\ max}$ = 7dB

N = Number of elements =10

By definition, $G_{D\ max}$ in dB = 10 $\log_{10} | G_{D\ max} |$

7 = 10 $\log_{10} | G_{D\ max} |$

$$G_{D\ max} = 5.0118$$

The directivity of the broadside array is given by

$$G_{D\ max} = 2\frac{L}{\lambda} = 2\left[\frac{nd}{\lambda}\right]$$

$$5.0118 = 2\left(\frac{10\ x\ d}{\lambda}\right)$$

$\therefore$

$$\boxed{d = 0.25\,\lambda}$$

Hence to achieve directivity of 7 dB with a broadside array of 10 isotropic radiator, the minimum distance between the elements must be 0.25 λ

Example 3 : Calculate the directivity in dB for the broadside as well as and fire array consisting of 8 isotropic elements separated by $\frac{\lambda}{4}$ distance.

Solution: Given n = Number of elements = 8

d = Distance of separation = $\frac{\lambda}{4}$ metre

(i) For Broadside array : The directivity is given by

$$G_{D\ max} = 2\ \frac{nd}{\lambda} = \frac{2\times8\times\frac{\lambda}{4}}{\lambda} = 4$$

Hence $G_{D\ max}$ in dB = 10 log $_{10}$ [$G_{D\ max}$]

= 10 log $_{10}$ (4) = 6.021 dB

(ii) For End fire array : The directivity is given by

$$G_{D\ max} = 4\ \frac{n\ d}{\lambda} =$$

Hence $G_{D\ max}$ in dB = 10 log $_{10}$ [$G_{D\ max}$]

= 10 log $_{10}$ (8)

= 9.031 dB

Example 4 : Find the length and BWFN for broadside and end fire array if the directive gain is 15.

Solution : Given: $G_{D\ max}$ = 15

(i) For Broadside array:

$$G_{D\ max} = 2\ \frac{L}{\lambda}$$

$$15 = 2 \quad \frac{L}{\lambda}$$

$$\boxed{L = 7.5\,\lambda \text{ metre}}$$

(ii) By definition, BWFN is given by

$$\text{BWFN} = \frac{114.6}{(\frac{L}{\lambda})}$$

$$= \frac{114.6}{(\frac{7.5}{\lambda})} = 15.28^{o}$$

$$\boxed{\text{BWFN} = 15.28^{O}}$$

For End fire array:

(i) $$G_{D\ max} = 4 \left[\frac{L}{\lambda}\right]$$

$$15 = 4 \left[\frac{L}{\lambda}\right]$$

$$L = 3.75\,\lambda \text{ meter}$$

(ii) BY definition BWFN is given by

$$\text{BWFN} = 114.6 \sqrt{\frac{2}{L/\lambda}} \text{ degree}$$

$$= 114.6 \sqrt{\frac{2}{\frac{3.75\lambda}{\lambda}}}$$

$$\boxed{\text{BWFN} = 83.69O}$$

Example 5 : A uniform linear array consists of 16 isotropic point sources with a spacing of λ / 4. If the phase difference is 90^{o}, calculate

(i) HPBW (ii) Directivity (in dB)
(iii) Beam solid angle (iv) Effective aperture

Solution: The above mentioned array is an end fire array because of the phase shift.

Given: n = Number of total elements =16

D = Spacing between adjacent elements = $\lambda/4$

L = Total length of the array = (n-1) d

$= (16\text{-}1)\frac{\lambda}{4} = 15\frac{\lambda}{4}$

(i) The Half Power Beam Width (HPBW) for the end fire array is given by

HPBW $= 57.3\sqrt{\frac{2}{l/\lambda}}$ degree

If L $= 15\frac{\lambda}{4}$, we get

HPBW $= 57.3\sqrt{\frac{\frac{2}{15\lambda}}{4\frac{L}{\lambda}}} = 41.84^0$

(ii) The directivity for end fire array is given by

$$D = 4\left[\frac{L}{\lambda}\right] = 4\left[\frac{15\lambda}{\frac{1}{\lambda}}\right] = 15$$

The directivity can be expressed in decibel as

D(in dB) = 10 $\log_{10}$ 15 = 11.76 dB

(iii) Beam solid angle is given by

$$\psi/ = \frac{4\pi}{D} = \frac{4x\pi}{15} = 0.8377 \text{ Sr}$$

(iv) The effective aperture is given by

$$A_e = \frac{D\lambda 2}{4\pi} = \frac{15\ x\ \lambda}{4\pi} = 1.1936\ \lambda^2\ m^2$$

TWO MARK QUESTION

1. What is meant by uniform linear array?

An array is linear when the elements of the array are spaced equally along the straight. Line. If the elements are fed with currents of equal magnitude and having a uniform progressive phase shift along the line, then it is called uniform linear array.

2.What is Broad side array?

Broad side array is defined as an arrangement in which the principal direction of radiation is perpendicular to the array axis and the plane containing the array element, For Broad side array. The phase difference between adjacent element is =0.

3.Define End fire array.

End fire array is defined as an arrangement in which the principal direction of radiation coincides with the array axis.