UNIT – 5
ANALOG AND DIGITAL INSTRUMENTS

5.1 DIGITAL VOLTMETERS

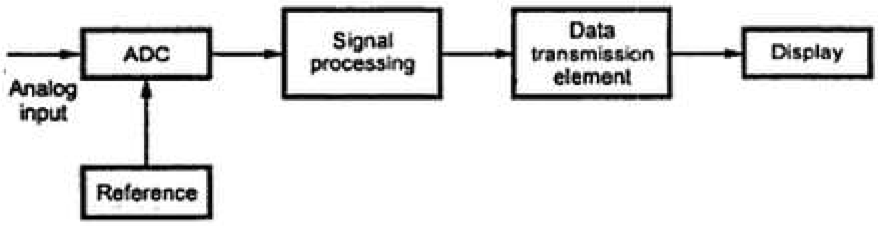
The digital voltmeters generally referred as DVM, convert the analog signals into digital and display the voltages to be measured as discrete numerical instead of pointer deflection, on the digital displays. Such voltmeters can be used to measure a. c and d. c voltages and also to measure the quantities like pressure, temperature, stress etc. using proper transducer and signal conditioning circuit.

5.1.1 Advantages of Digital Voltmeters

1. Due to the digital display, the human reading errors, interpolation errors and parallax errors are reduced.
2. They have input range from +1.000 V to +1000 V with the automatic range selection and the overload indication.
3. The accuracy is high up to ± 0.005 % of the reading
4. The resolution is better as 1 $_u$ V reading can be measured on 1 V range.
5. The input impedance is as high as 10 MΩ.
6. The reading speed is very high due to digital display.
7. They can be programmed and well suited for computerized control.
8. The output in digital form can be directly recorded and it is suitable for further processing also.
9. With the development of IC chips, the cost of DVMs, size power requirements of DVMs are drastically reduced.
10. Due to small size,are portable.

5.1.2 Basic Block Diagram of DVM

Any digital instrument requires analog to digital converter at its input. Hence first block in a general DVM is ADC as shown in the Fig.



Basic Block Diagram of DVM

Every ADC requires a reference. The reference is generated internally and reference generator circuitry depends on the type of ADC technique used. The output of ACD is decoded and single is processed in the decoding stage. Such a decoding is necessary to drive the seven segment display. The data from decoder is then transmitted to the display. The data transmission element may be a latches, counters etc. as per the requirement. A digital display shows the necessary digital result of the measurement.

5.1.3 Classification of Digital voltmeters

The digital voltmeters are classified mainly based on the technique used for the analog to digital conversion. Depending on this, the digital voltmeters are mainly classified as,

i) Non-integrating type and

ii) ii) Integrating type

The non-integrating type digital voltmeters are further classified as,

a) Potentiometric type : these are subclassified as,

1) Servo potentiometric type
2) Successive approximation type
3) Null balance type

b) Ramp type : These are subclassified as,

1) Liner type
2) Staircase type

The integrating type digital voltmeters are classified as,

a) Voltage to frequency converter type
b) Potentiometric type
c) Dual slop integrating type

Servo Potentiometric Type DVM

The input voltage to be measured is applied to of mechanical chopper type comparator after filtering and attenuation to suitable level. The reference voltage is applied at the two terminals of the potentiometer. The position of the sliding contact decides the value of the feedback voltage, which is used as the second input to the comparator. The comparator which is an error detector, compares the unknown voltage and the feedback voltage. The output of the comparator is a square wave single whose amplitude is a function of the difference in the two voltage connected to its two ends i.e error voltage. This output single from comparator is amplified and then fed to power amplifier. The power amplifier output is given to the servomotor which acts as a potentiometer adjustment device. The servo motor moves the sliding contact proportional to the error signal. The direction of the movement of the sliding contact proportional to the error signal. The direction of the movement of the sliding contact depends on the sign of the error i.e whether the feedback voltage is larger or smaller than the unknown input voltage . When the feedback voltage is same as the input voltage, the error is zero and therefore servomotor will not receive any single , which will stop the movement of the sliding contact. Thus the sliding contact will attain a stable position.

The servomotor also drives the mechanical readout. The voltage corresponding to the stable position of the sliding contact is indicated in the numerical form on the digital display.

The relation between the unknown input voltage and the reference voltage can be mathematically expressed as,

$$V_{in} = V_{ref}$$

Where V_{in} = voltage to be measured

V_{ref} = Reference voltage

X = Fraction depends on the position of slider

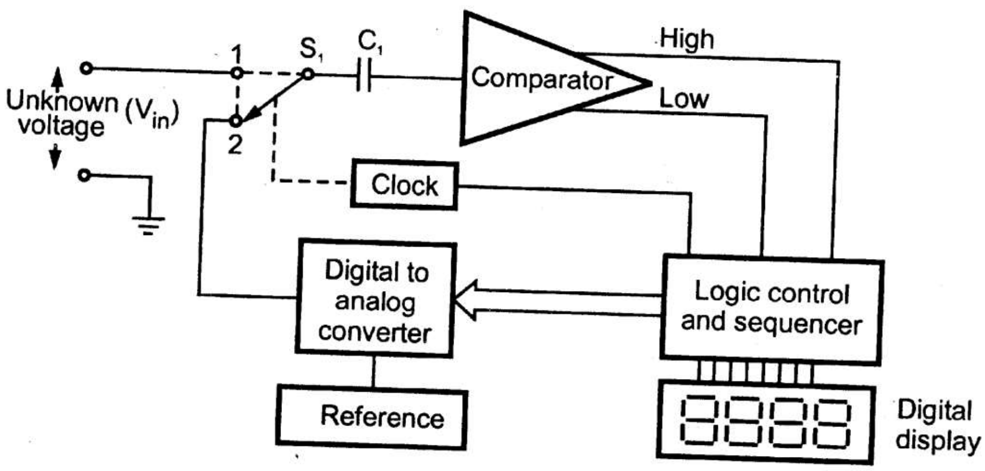
The potentiometer used in the servo balancing type DVM is a linear divider but in successive approximation type a digital divider is used. The digital divider is nothing but a digital to analog (D/A) converter. The servomotor is replaced by an electronic logic.

SUCCESSIVE APPROXIMATION TYPE DVM

In successive approximation type DVM, the comparator compares the output of digital to analog converter with the unknown voltage. Accordingly, the comparator provides logic high or low signals. The digital to analog converter successively generates the set pattern of signals.

The procedure continues till the output of the digital to analog converter becomes equal to the unknown voltage.

The capacitor is connected at the input of the comparator. The output of the digital to analog converter is compared with the unknown voltage, by the comparator. The output of the comparator is given to the logic control and sequencer, this unit generates the sequence of code which is applied to digital to analog converter. The position 2 of the switch s_1 receives the output from digital to analog converter. The unknown voltage is available at the position 1 of the switch s_1. The logic control also drives the clock which is used to alternate the switch s1 between the positions 1 and 2, as per the requirement.



Successive Approximation Type DVM

The set pattern of digital to analog converter is say 8-4-2-1. At the start, the converter generates 8 V and switch is at the position 2. The capacitor c_1 charge to 8 V .The clock is used to change the switch position. So during next time interval, switch position is 1 and unknown input is applied to the capacitor .As capacitor is change to 8 V which is more than the input voltage 3.7924 V, the comparator send HIGH signal to the logic control and sequencer circuit. This HIGH single resets the digital to analog resets the converter which generates its next step of 4 V. This again generates HIGH signal. This again resets the converter to generate the next step of 2 V.

Advantages

1. Very high speed of the order of 100 reading per second possible.
2. The method of ADC is inexpensive.
3. The resolution upto 5 significant digits is possible.
4. The accuracy is high.

Disadvantages

The disadvantages of successive approximation DVM are,

1. The circuit is complex.
2. The DAC is also required.
3. The input impedance is variable.
4. The noise can cause error due to incorrect decisions made by comparator.

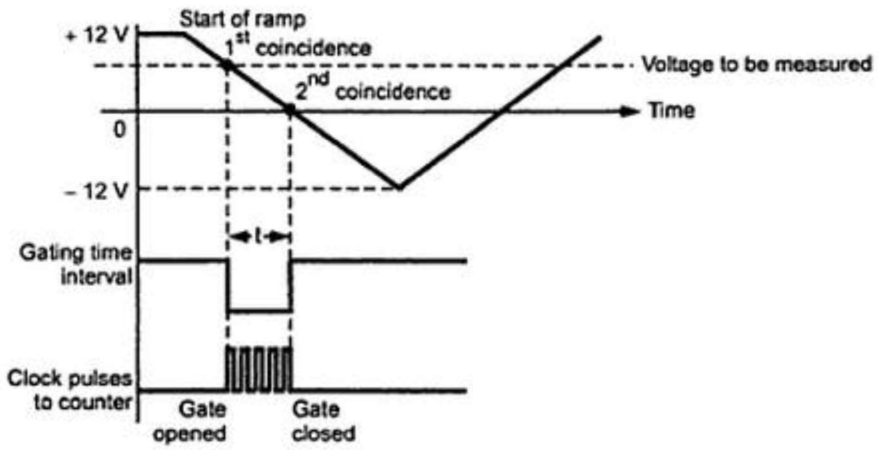
RAMP TYPE DVM

It uses a linear ramp technique or staircase ramp technique. The staircase ramp technique is simpler than the liner ramp technique.

Linear Ramp Type DVM

The basic principle of such measurement is based on the measurement of the time taken by a linear ramp to rise from 0 v to the level of the input voltage or to decrease from the level of the input voltage to zero. This time is measured with the help of electronic time interval counter and the count is displayed in the numeric from with the help of a digital display.

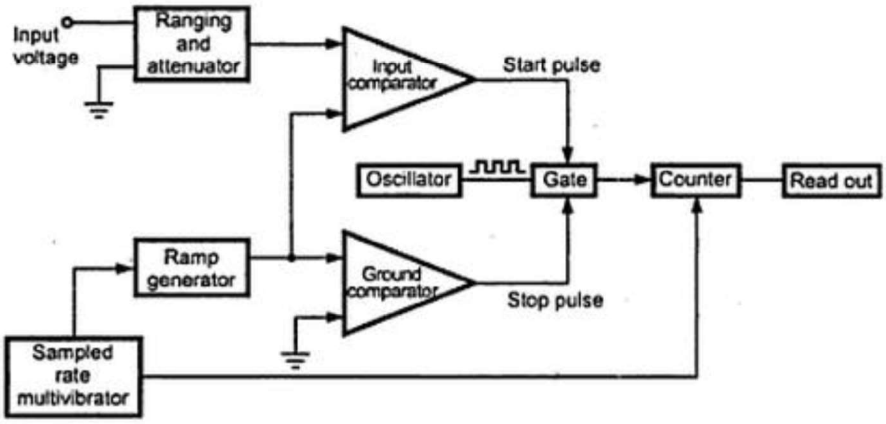
Basically it consists of a linear ramp which is positive going or negative. The range of the ramp ± 12V while the base range is ± 10 V.



Voltage to time conversion

At the start of measurement, a ramp voltage is initiated which is continuously compared with the input voltage. When these two these two voltages are same, the comparator generates a pulse which opens a gate i.e the input comparator a start pulse. The ramp continues to decrease and finally reaches to 0 V or ground potential. This is sensed by the second comparator or ground comparator. At exactly o V, this comparator produces a stop pulse which closes the gate. The number of clock pulses are measure by the counter. Thus the time duration for which the gate is opened, is proportional to the input voltage. In the time interval between start and stop pulses, the gate remains open and the oscillator circuit drives the counter. The magnitude of the count indicates the magnitude of the input voltage.

The block diagram of linear ramp DVM is shown



LinearRamp Type DVM

Properly attenuated input single is applied as one input to the input comparator. The ramp generator generates the proper linear ramp signal which is applied to both the comparators. Initially the logic circuit sends a reset signal to the counter and the readout. The comparators are designed in such a way that when both the input signals of comparator are equal then only the comparator changes its state. The input comparator is used to send the start pulse while the ground comparator is to send the stop pulse.

When the input and ramp are applied to the input comparator, and the point when negative going ramp becomes equal to input voltage the comparator sends pulse, due to which gate opens. The oscillator drives the counter starts counting the pulses received from the oscillator . Now the same ramp is applied to the ground comparator and it is decreasing . Thus when ramp becomes zero ,both the inputs of ground comparator becomes zero (grounded) i.e equal and it sends a stop pulse to the gate due to which gate gates closed. Thus the counter stop receiving the pulses from the local oscillator .A definite number of pulses will be counted by the counter, during the start and stop pulses which is measure of the input voltage .This is displayed by the digital readout.

The advantages of this technique are:

i) The circuit is easy to design

ii) The cost is low .

iii) The output pulse can be transferred over long feeder lines without loss of information.

iv) The input signal is converted to, time which is easy to digitize.

v) By adding external logic, the polarity of the input also can be displayed.

The disadvantages of this technique are:

i) The ramp requires excellent characteristics regarding its linearity.

ii) The accuracy depends on slop of the ramp and stability of the local oscillator.

iii) Large errors are possible if noise is superimposed on the input signal.

iv) The offsets and drifts in the two comparators may cause errors.

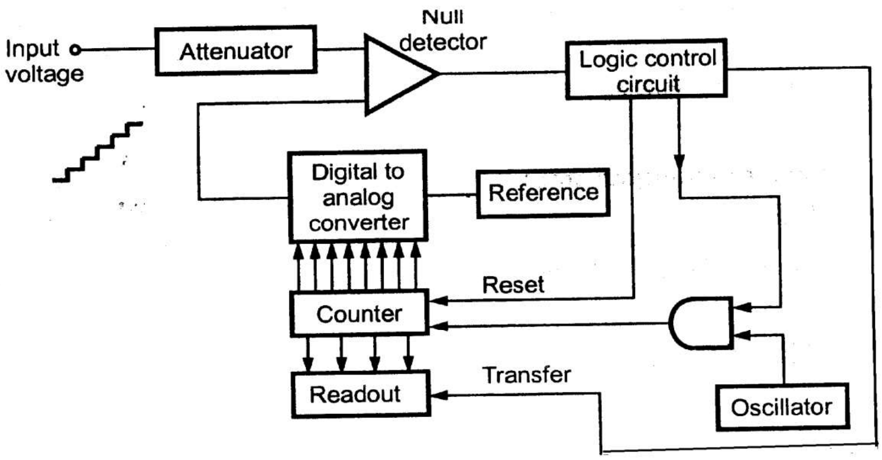
v) The speed of measurement is low.

vi) The swing of the ramp is ± 12 V, this limits the base range of measurement to ± 10 V.

Staircase Ramp Technique

In this type of DVM, instead of linear ramp, the staircase ramp is used. The staircase ramp is generated by the digital to analog converter.

The technique of using staircase ramp is also called null balance technique. The input voltage is properly attenuated and is applied to a null detector. The another input to null detector is the staircase ramp generated by digital to analog converter. The ramp is continuously compared with the input signal.



Staircase Ramp Type DVM

Initially the logical control circuit sends a reset signal. This signal resets the count. The digital to analog converter is also resetted by same signal.

At the start of the measurement, the logic control circuit sends a starting pulse opens the gate. The counter starts counting the pulses generated by the local oscillator.

The output of counter is given to the digital to analog converter which generates the ramp signal. At every count there is an incremental change in the ramp generated. The staircase ramp is generated at the output of the digital to analog converter. The given as the second input of the null detector. The increase in ramp continues achieves the voltage equal to input voltage.

When the two voltages are equal, the null detector generates a signal which initiates the logic control circuit. Thus logic control circuit. Thus logic control circuit sends a stop pulse, which the gate and the counter stops counting.

At the same time, the logic control circuit generates a transfer signal due to which counter information is transferred to the readout. The readout shows the digital the count.

The advantages of this technique are:

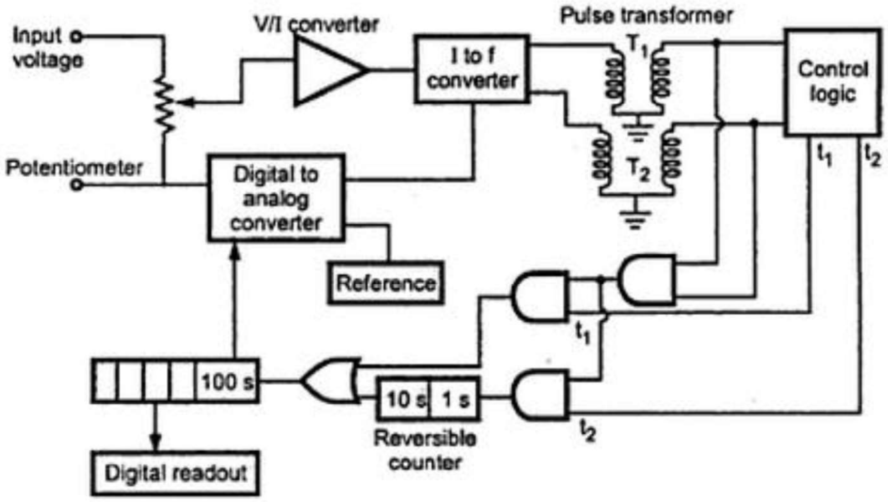
i. The greater accuracy is obtained than the linear ramp technique.
ii. The overall design is more simple hence economical.
iii. The input impedance of the digital to analog converter is high when compensation is reached.

The disadvantages of this technique are:

i. Though accuracy is higher than linear ramp, it is dependent on the digital to analog converter and its internal reference.
ii. The speed is limited upto 10 readings per second.

POTENTIOMETRIC INTEGRATING TYPE DVM

The block diagram of potentiometric integrating type DVM is shown in the potentiometer at the input side and each measurement consists of two sample t_1 and t_2 as decided by the control logic.



Potentiometric Integrating Type DVM

During the first sampling period $t_1$the output of digital to analog converter is zero. Hence the voltage to be measured directly applied to V/I converter and then I/f converter Thus the voltage to frequency conversion takes place during the period. The pulses produced by the pulse transfer arc fed into 100 s decade of the reversible counter.

The reversible counter counts these pulses which are proportional to the input voltage. This count is then transferred to digital to analog converter. The digital to analog convener produces a voltage corresponding to the counts. During the process of transfer, the count is retained in the counter

The input to V/f converter is flow the difference between the input voltage and the voltage produced by digital to analog converter Due to the small errors and reduced resolution the output of digital to analog converter is not exactly equal to the input Voltage. Hence there exists a small voltage at the input of V/f converter, which is the difference between input voltage and output of digital to analog converter.

Now the second sampling period, starts. During this period the V/f convener generat5 a train of pulses, the frequency of Which is proportional to the difference between the input and the output of digital to analog converter These pulses are given to the I *s* decade of the reversible counter. The carry is general when each hundredth pulse is general. This is then passed to 100 s decade. At the end of *the* period **t2,** the reading operation ends. The count is then transferred to the digital readout.

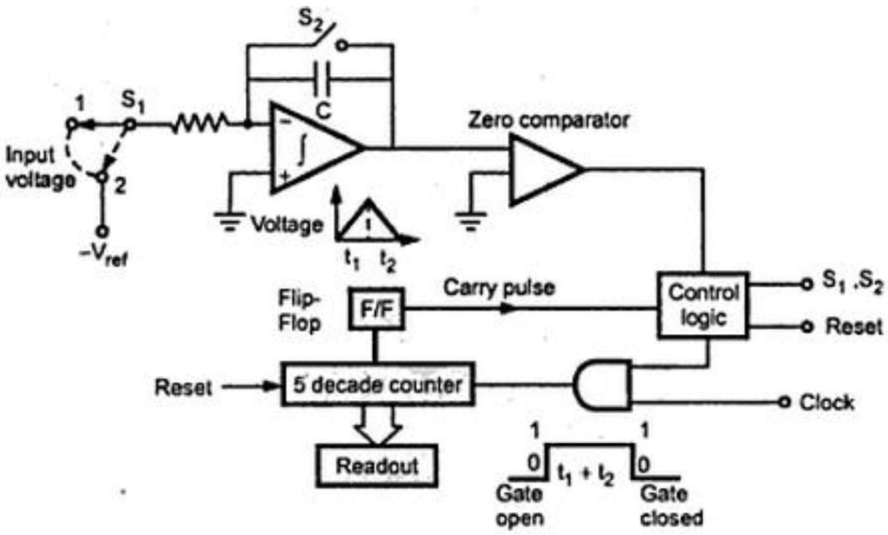
The advantages of this DVM are:

1. The accuracy is very high. It depends on the digital to analog Converter and its reference. The accuracy of V/f converter is of reduced importance.

2. The rejection of noise signals superimposed on input signal to be measured the high cost and less speed of operation are the two major limitations of this DVM

Dual Slope Integrating Type DVM

This is the most popular method of analog to digital Conversion In the ramp techniques, the noise *can* cause large errors but in dual slope method the noise is averaged out by the positive and negative ramps using the process of integration The basic principle of this method is that the input signal is integrated for a fixed interval of time. Then the same integrator is used to integrate the reference voltage with reverse Slope. Hence the name given to the technique *is* dual Slope integration technique.



Dual Slope Integrating Type DVM

It consists of five blocks, an op-amp used as an integrator a zero comparator clock *pulse* generator, a set of decimal counters and a block of control logic.

When the switch S1 is in position 1, the capacitor C starts charging from zero level. The rate of charging is proportional to the input voltage level. The output of the op-amp given by,

$$V_{out} = \frac{1}{R_1 C}\int_0^{t1} V_{in}\,dt$$

$$V_{out} = \frac{V_{in} t_1}{R_1 C}$$

Where, t_1=Time for which capacitor is charged

V_{in}= Input voltage

R_1 = Series resistance

C = Capacitor in feedback path

After the interval t1, the input voltage is disconnected and a negative voltage V_{ref} is connected by throwing the switch S_1 in position 2. In this position, the output of the op-amp is given by,

$$V_{out} = \frac{1}{R_1 C} \int_0^{t2} V_{ref} dt$$

$$V_{out} = \frac{V_{ref} t_2}{R_1 C}$$

Subtracting equation (1) from equation (2)

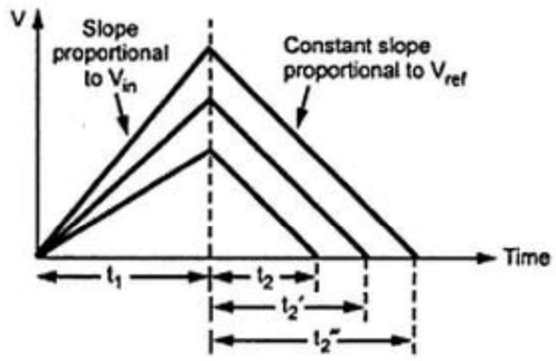
$$V_{out} = V_{out} = 0 = \frac{-V_{ref} t_2}{R_1 C} - \left(\frac{-V_{in} t_1}{R_1 C} \right)$$

$$\frac{V_{ref} t_2}{R_1 C} = \frac{V_{ref} t_1}{R_1 C}$$

$$V_{ref} t_2 = V_{in} t_1$$

$$V_{in} = V_{ref} \cdot \frac{t_2}{t_1}$$

Thus the input voltage is dependent on the time periods t_1 and t_2 and not on the values of R_1 and C.



Basic principle of dual slope method

At the start of the measurement, the counter is resetted to zero. The output of the flip-flop is also zero. This is given to the control logic. This control sends a signal so as to close an electronic switch to position1 and integration of the input voltage starts. It continues till the time period t_1. As the output of the integrator changes from its zero value, the zero comparator output changes its state. This provides a signal to control logic which inturn opens the gate and the counting of the clock pulses starts.

The counter counts the pulses and when it reaches to 9999, it generates a carry pulse and all digits go to zero. The flip-flop output gets activated to the logic '1'. This activates the control logic. This sends a signal which changes the switch S1 position from 1 to 2. Thus $-V_{ref}$ gets connected to op-amp. As V_{ref} polarity is opposite, the capacitor starts discharging. The integrator output will have constant negative slope as shown in the fig. The output decreases linearly and after the interval t_2, attains zero value, when the capacitor C gets fully discharged.

At this instant, the output of zero comparator changes its state. This inturn sends a signal to the control logic and the gate gets closed. Thus gate remains open for the period t_1+t_2. The counting operation stops at this instant. The pulses counted by the counter thus have a direct relation with the input voltage. The counts are then transferred to the readout.

The advantages of this technique are:

1. Excellent noise rejection as noise and superimposed a.c. are averaged out during the process of integration.

2. The RC time constant does not affect the input voltage measurement.

3. The capacitor is connected via an electronic switch. This capacitor is an auto zero capacitor and avoids the effects of offset voltage.

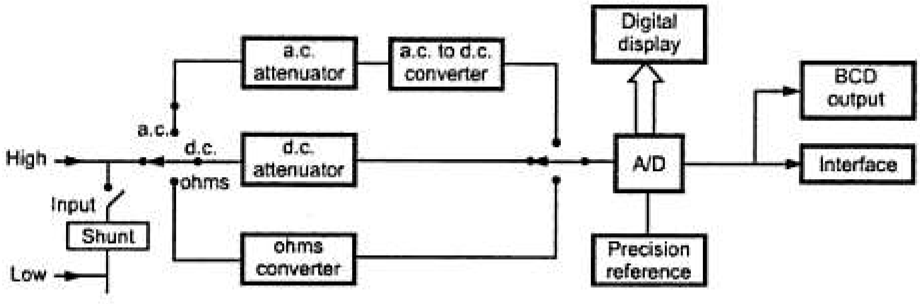
4. The integrator responds to the average value of the input hence sample and hold circuit is not necessary.

5. The accuracy is high and can be readily varied according to the specific requirements.

The only disadvantage of this type of DVM is its slow speed.

5.2 DIGITAL MULTIMETERS

The digital multimeter is an instrument which is capable of measuring a.c. voltages dc voltages. a.c. and d.c. currents and resistances over several ranges. The basic circuit of a digital multimeter is always a d.c. voltmeter as shown in the Fig.



Basic scheme of digital multimeter

The current is converted to voltage by passing it through low shunt resistance. The a.c quantities are converted to d.c. by employing various rectifier and filtering circuits. While for the resistance measurements the meter consists of a precision low current source that is applied across the unknown resistance while gives d.c. voltage. All the quantities are digitized using analog to digital converter and displayed in the digital form on the display. The analog multimeters require no power supply and they suffer less from electric noise and isolation problems but still the digital multimeters have following advantages over analog multimeters.

i) The accuracy is very high.

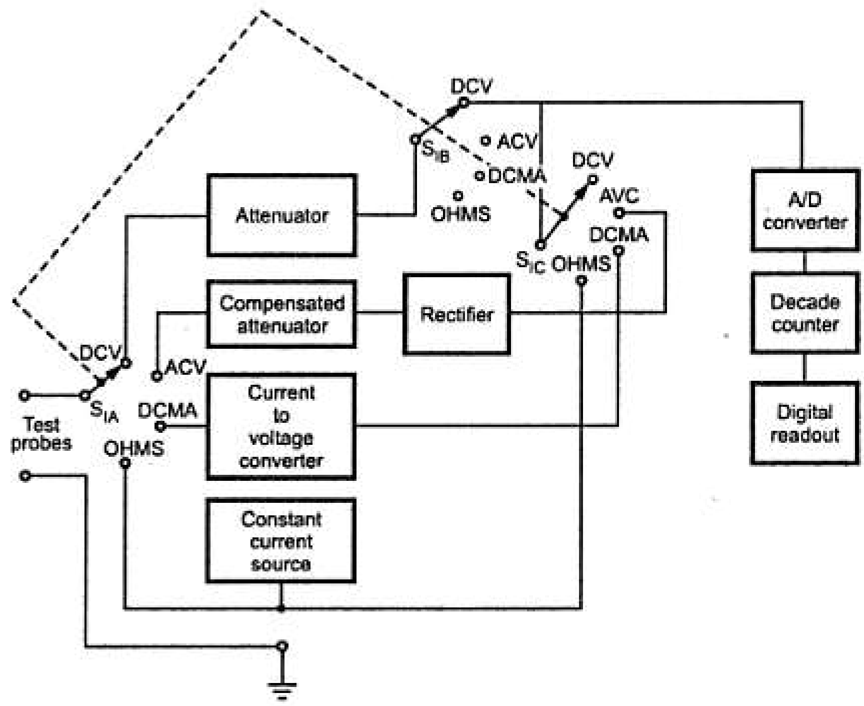
ii) The input impedance is very high hence there is no loading effect.

iii) An unambiguous reading at greater viewing distances is obtained.

iv) The output available is electrical which can be used for interfacing with external equipment.
v) Due to improvement in the integrated technology, the prices are going down.
vi) These are available in very small size.

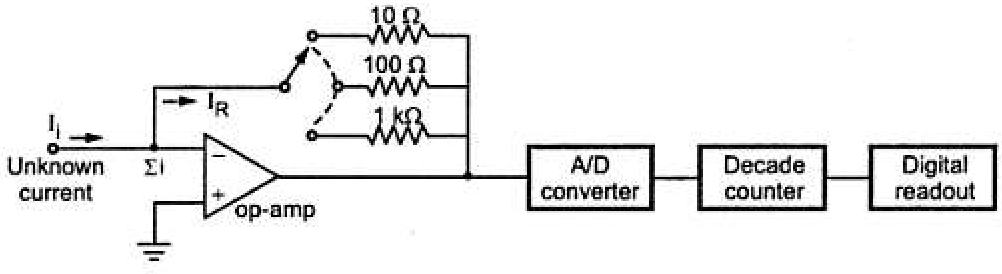
The requirement of power supply, electric noise and isolation problems are the limitations.

The basic building blocks of digital multimeter are several A/D converters, circuitry and an attenuation circuit. Generally dual slope integration type ADC is preferred in the multimeters. The single attenuator circuit is used for both a.c. and measurements in many commercial multimeters.



Block diagram of a digital multimeter

As mentioned above basically it is a d.c. voltmeter. In order to measure unknown currents, current to voltage converter circuit is implemented.



current to voltage converter

The unknown current is applied to the summing junction Li at the input of op-amp. As input current of op-amp is almost zero, the current I_g is almost same as I_i. This current I_g causes a voltage drop, which is proportional to the current to be measured. This voltage drop is the analog input to the analog to digital converter, thus providing a reading that is proportional to the unknown current.

In order to measure the resistances, a constant current source is used. The known current is passed through the unknown resistance. The voltage drop across the resistance is applied to analog to digital converter hence providing the display of the value of the unknown resistance. To measure the a.c. voltages, the rectifiers and filters are used. The ac. is converted to d.c and then applied to the analog to digital converter.

In addition to the visual display, the output from the digital multimeters can also be used to interface with some other equipments.

5.3 SPECIFICATIONS OF DIGITAL MULTIMETER

The Important specifications of a digital multimeter are as follows.

i) D.C. voltage

There are five ranges available from ± 200 mV to ± 1000 V.

The resolution is 10 iV on the lowest range.

The accuracy is ± 0.03 % of the reading '- two digits.

ii) A.C.voltage

There are five ranges from 200 mV to 750 V.

The resolution is 10 µV on the lowest range

The accuracy is frequency dependent but the best accuracy is 0.5 % + 10 digits between 45 Hz and I kHz on all the ranges.

iii) D.C. current

There arc five ranges from ± 200 µA to ± 2000 mA.

The resolution is ± 0.01 µA on the lowest range.

The accuracy is ± 0.3 % of reading. two digits.

iv) A.C. current

There are five ranges from 200 µA to 2000 mA.

The accuracy is frequency dependent but the best accuracy of ±1 % + ten digits between 45 Hz and 2 kHz on all the ranges.

PROBLEMS

Example 1

What is the resolution of a 3 = $\frac{1}{2}$ digit display on 1 V and 50 V ranges?

Solution : The number of full digits are n = 3

$$R = \frac{1}{10^3} = 0.001$$

Thus mater cannot distinguish between the values that differ from each other by less than 0.001 of full scale

Thus for 1 V range, the resolution is 1 × 0.001 = 0 = 001V

While for 50 V range, the resolution is 50 × 0.001 = 0.05V

Thus on 50 V range, the meter cannot distinguish between the readings that differ by less than 0.05V.

Example 2

A voltmeter uses 4-$\frac{1}{2}$ digit display i) Find its resolution ii) How would the 11.87 V be displayed on a 10V range? Iii) How would 0.5573 be displayed on 1 V and 10 V ranges?

Solution

i) For 4-$\frac{1}{2}$ digit display the full digits are n=4

$$R = \frac{1}{10^4} = 0.0001$$

ii) There are 5 digit places in 4-$\frac{1}{2}$ digit hence 1.87 would be displayed as 11.870

iii) Resolution on 1 V range is 1 V × 0.0001 = 0.0001V

Hence any reading upto 4th decimal can be displayed

Hence 0.5573 will be displayed as 0.5573

But resolution on 10V range is V × 0.0001 = 0.001V

Hence decimals upto the 3rd place can be displayed

Therefore on 10V range, the reading will be displayed as 0.557 rather than 0.5573

Example 3

A 3-$\frac{1}{2}$ digit DVM has an accuracy specification of 0.5% of the reading 1 digit i) What is the error in volts, when the reading is 5.00 V on its 10V range. Ii) What is the % error of reading, when the reading is 0.10V on its 10V range?

Solution : As number of digits n = 3

$$R = \frac{1}{10^3} = 0.001$$

For 10 V range, R = 0.001× 10 = 0.01V

1 digit = 0.01 V on 10V range

i) The reading is 5.00V

Error due to reading = ±0.5% of 5.00 = $\frac{0.5}{100}$ x 5 = 0.025V

and 1 digit error = 0.01V

Total error = 0.025 + 0.01 = 0.035V

ii) When reading is 0.10V

Error due to reading = ±0.5% of 0.1 = $\frac{0.5}{100}$ x 0.1 = ±0.0005V

and 1 digit error = ±0.01V

Total error =± 0.0105V

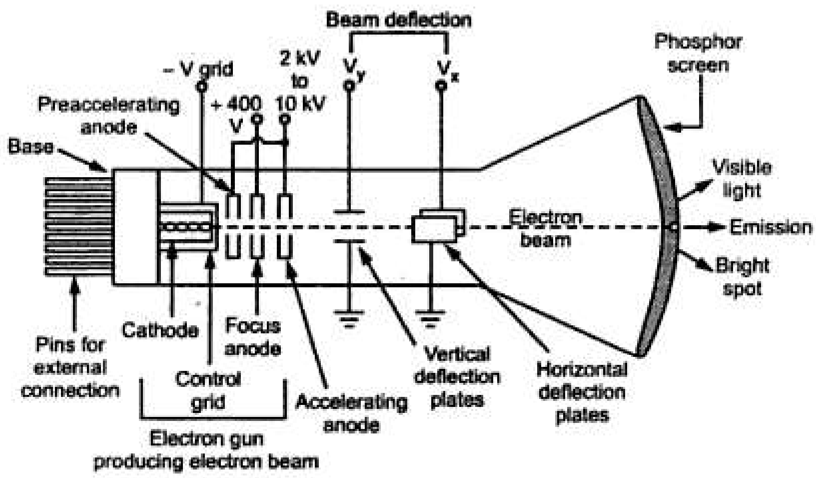
Error as % of reading = $\frac{0.0105}{0.1}$x100 = 10.5%

5.4 CATHODE RAY OSCILLOSCOPE

The device which allows, the amplitude of periodic or non- periodic signals, displayed primarily as a function of time is called cathode ray oscilloscope. The C.R.O gives the visual representation of the time varying signals. It is an integral part of the electronic laboratories.

5.4.1 CATHODE RAY TUBE (CRT)

The cathode ray tube (CRT) is the heart of the CRO. The CRT generate the electron beam, accelerates the beam, deflects the beam and also has a screen where beam becomes visible as a spot.



Cathode Ray Tube (CRT)

The main parts of the CRT are:

i. Electron gun

ii. Deflection system

iii. Fluorescent screen

iv. Glass tube or envelope

v. Base

A schematic diagram of CRT, showing its structure and main components is shown in the fig.

Electron Gun:

The electron gun section of the cathode ray tube provides a sharply focused electron beam directed towards the Fluorescent-coated screen. This section starts from thermally heated cathode, emitting the electrons. The control grid is given negative potential with respect to cathode. This grid controls the number of electrons in the beam, going to the screen.

The momentum of the electrons (their number x their speed) determines the intensity, or brightness, of the light emitted from the fluorescent screen due to the electron bombardment. The light emitted is usually of the green color. Because the electrons are negatively charged, a repulsive force is created by applying a negative voltage to control grid (in CRT, voltages applied to various grids are stated with respect to cathode, which is taken as common point).

This negative control voltage can be made variable. A more negative voltage results in less number of electrons in the beam and hence decreased brightness of the beam spot.

Since the electron beam consists of many electrons, the beam tends to diverge. This is because the similar (negative) charges on the electron repel each other. To compensate for such repulsion forces, an adjustable electrostatic field is created between two cylindrical anodes, called the focusing anodes. The variable positive voltage on the second anode is used to adjust the focus or sharpness of the bright beam spot.

The high positive potential is also given to the pre accelerating anodes and accelerating anodes, which results into the required acceleration of the electrons.

Both focusing and accelerating anodes are cylindrical in shape having small openings located in the center of each electrode, co-axial with the tube axis. The pre accelerating and accelerating anodes are connected to a common positive high voltage which varies between 2kV to 10 kV. The focusing anode is connected to a lower positive voltage of about 400v to 500v.

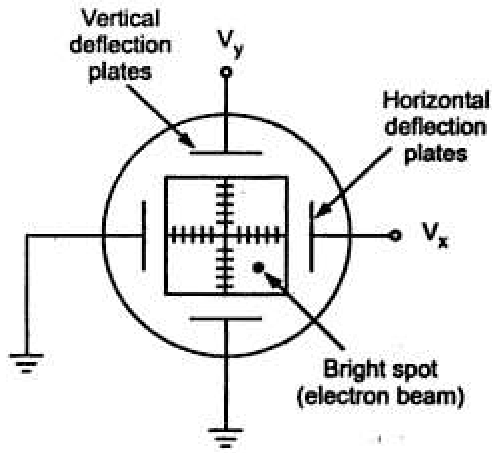
Deflection System:

When the electron beam is accelerated it passes through the deflection system with which beam can be positioned anywhere on the screen.

The deflection system of the cathode ray tube consists of two pairs of parallel plates, referred to as the vertical and horizontal deflection plates. One of the plates in each set is connected to ground (0v). To the other plate of each set, the external deflection voltage is applied through an internal adjustable gain amplifier stage.

A positive voltage applied to the Y input terminal (V_y) causes the beam to deflect vertically upward due to the attraction forces. While a negative voltage applied to the Y input terminal (V_y) will causes the electron beam to deflect vertically downward, due to the repulsion forces.

Similarly, a positive voltage applied to the X input terminal (V_x) causes the beam to deflect horizontally towards the right, while a negative voltage applied to the X input terminal (V_x) causes the beam to deflect horizontally towards the left of the screen. The amount of vertical or horizontal deflection is directly proportional to the applied voltage.



The horizontal deflection (x) produced will be proportional to the horizontal deflecting voltage , V_x applied to X-input.

$$\therefore \quad x \propto Vx$$

$$\therefore \quad x = K_x V_x$$

Where K_x is constant of proportionality.

Then $K_x = x/V_x$ where K_x expressed as cm/volts or division/volt, is called horizontal sensitivity of the oscilloscope.

Similarly, The vertical deflection (y) produced will be proportional to the vertical deflecting voltage , V_y applied to Y-input.

$$\therefore \quad y \propto V_y$$

$$\therefore \quad y = K_y V_y$$

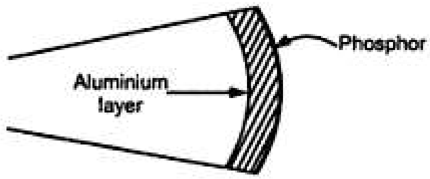
Where K_y is constant of proportionality.

Then $K_y = y/V_y$ where K_y expressed as cm/volts or division/volt, is called vertical sensitivity of the oscilloscope.

Fluorescent Screen:

The light produced by the screen does not disappear immediately when bombardment by electrons ceases, i.e. when the signal becomes zero. The time period for which the trace remains on the screen after the signal becomes zero is known as “persistence”.

Medium persistence traces are mostly used for general purpose applications. Long persistence traces are used in the study of transients. Short persistence is needed for extremely high speed phenomena.



The screen is coated with a fluorescent material called phosphor which emits the light when bombarded by electrons. There are various phosphors available which differ in colour, persistence and efficiency.

One of the common phosphor is Willemite, which is zinc, orthosilicate, $ZnO + SiO_2$, with traces of manganese. This produces greenish trace. Other useful screen materials include compounds of zinc, cadmium, magnesium and silicon.

The kinetic energy of the electron beam is converted into both light and heat energy when it hits the screen. The heat so produced gives rise to "phosphor burn" which is damaging and sometimes destructive.

The phosphor screen is provided with an aluminium layer called aluminizing the cathode ray tube.

Glass Tube:

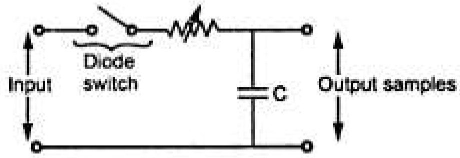
All the components of a CRT are enclosed in an evacuated glass tube called envelope. This allows the emitted electrons to move about freely from one end of the tube to the other end.

Base:

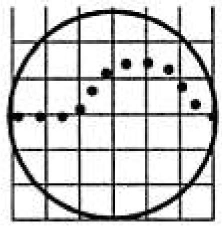
The base is provided to the CRT through which connections are made to the various parts.

5.4.2 SAMPLING OSCILLOSCOPE

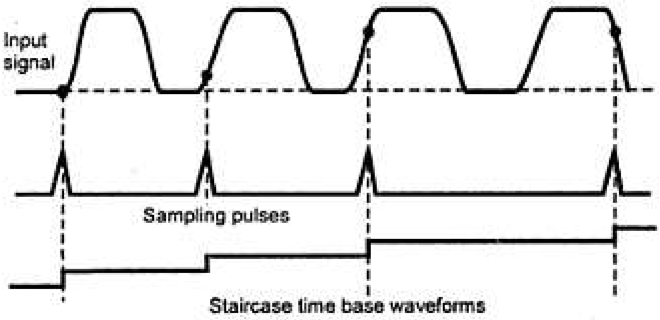
Using sampling technique higher frequency signals is converted to low frequency signal. In this technique, instead of monitoring the input signals continuously, it is sampled at regular intervals. These samples are presented on the screen in the form of dots. Many thousands of dots may be displayed on the screen. Such samples are merged to reconstruct the input signal.



Basic sampling circuit



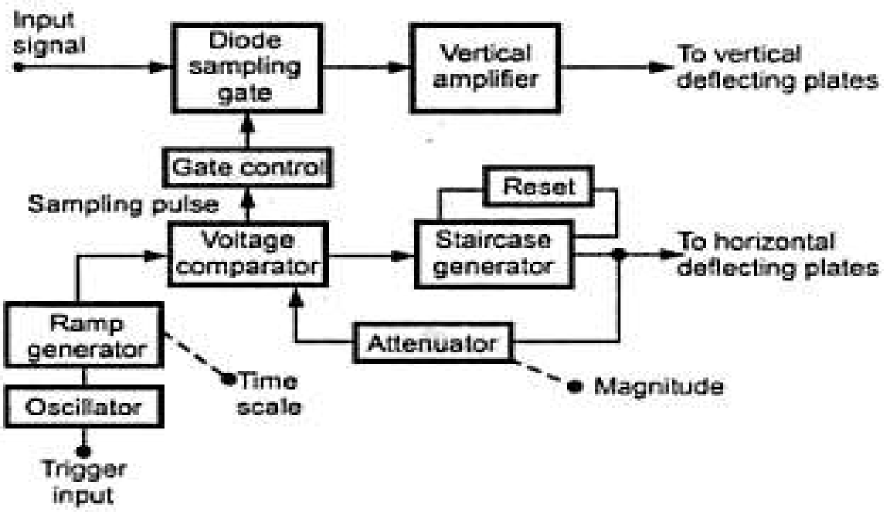
Display of dots



Sampling principle

Block Diagram of Sampling Oscilloscope

The block diagram of sampling oscilloscope is shown in the fig.



Block Diagram of Sampling Oscilloscope

The input signal is applied to the diode sampling gate. At the start of each sampling cycle a trigger input pulse is generated which activates the blocking oscillator. The oscillator output is given to the ramp generator which generates the linear ramp signal.

Since the sampling must be synchronized with the input signal frequency, the signal is delayed in the vertical amplifier.

The generator produces a staircase waveform which is applied to an attenuator. The attenuator controls the magnitude of the staircase signal and then it is applied to a voltage comparator. Another input to the voltage comparator is the output of the ramp generator. The voltage comparator compares the two signals and produces the output pulse when the two voltages are equal. This is nothing but a sampling pulse which is applied to sampling gate through the gate control circuitry.

This pulse opens the diode gate and sample is taken in this sampled signal is then applied to the vertical amplifier and the vertical deflecting plates.

The output of the staircase generator is also applied to the horizontal deflecting plates. During each step of staircase the spot moves on the screen. The comparator output advances the staircase output through one step.

After certain number of pulses about thousand or so, the staircase generator resets. The smaller the size of the steps of the staircase generator, larger is the number of samples and higher is the resolution of the image.

Advantages

The Advantages of the sampling oscilloscope are:

i) Very high frequency performance can be achieved.

ii) High speed electrical signals can be analyzed.

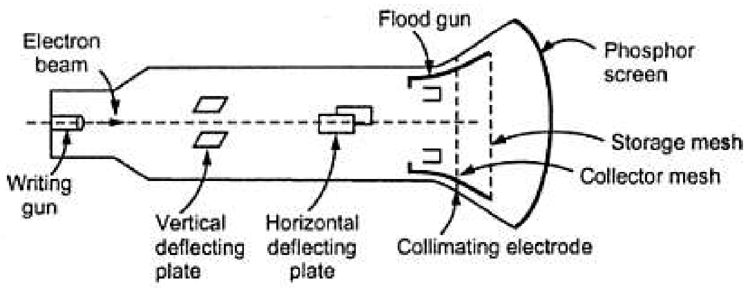
iii) The technique allows the design of the oscilloscope with wide bandwidth, high sensitivity even for low duty cycle pulses.

iv) A clear display is produced.

v) Controlling the size of the steps of the staircase generator, the number of samples and hence the resolution can be controlled.

The only limitation of the sampling oscilloscope is that is cannot be used to display the transient waveforms.

5.4.3 STORAGE OSCILLOSCOPE



Storage Oscilloscope

Two types of storage techniques are used in cathode ray tube which are,

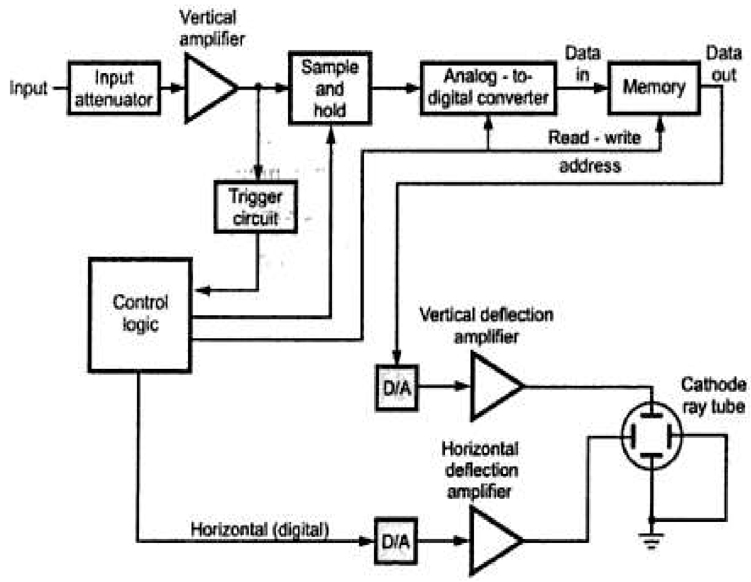
1. Mesh storage

2. Phosphor storage

5.4.4 DIGITAL STORAGE OSCILLOSCOPE

In this oscilloscope, the waveform to be stored is digitized and then stored in a digital memory. The conventional cathode ray tube is used in this oscilloscope hence the cost is less. The power to be applied to memory is small. Once the waveform is digitized then it can be further loaded into the computer and can be analyzed in detail.

Block Diagram



Block Diagram of digital storage oscilloscope

The input signal is applied to the amplifier and attenuator section. The attenuated signal is then applied to the vertical amplifier.

To digitize the analog signal, analog to digital converter is used. The output of the vertical amplifier is applied to the A/D converter section. The main requirement of A/D converter in the digital storage oscilloscope is its speed. The digitized output only needed in the binary form and not in BCD. The successive approximation type of A/D converter is most often used in the digital storage oscilloscope.

The digitizing the analog input signal means taking the samples at periodic intervals of the input signal. The rate of sampling should be at least twice as fast as the highest frequency present in the input signal, according to sampling theorem. This ensures no loss of information. The sampling rates as high as 100,000 samples per second is used.

The sampling rate and memory size are selected depending upon the duration and the waveform to be recorded.

Once the input signal is sampled, the A/D converter digitizes it. The signal is then captured on the memory. Once it is stored in the memory , many manipulations are possible as memory can be read out without being erased.

One important feature of digital storage oscilloscope is its mode of operation called **pretrigger view**. This mode means that the oscilloscope can display what happened before a trigger input is applied. This mode of operation is useful when a failure occurs.

The digital storage oscilloscope has three modes of operation.

(i) Roll mode

Very fast varying signals are displayed clearly in this mode. In this mode, the input signal is triggered at all. The fast varying signal is displayed as if it is changing slowly, on the screen in this mode.

(ii) Store mode

This is called refresh. In this case input initiates trigger circuit. Memory write cycle starts with trigger pulse. When the memory is full, write cycle stops. Then using digital to analog converter , the stored signal is converted to analog and displayed. When next trigger occurs the memory is refreshed.

(iii) Hold or save mode

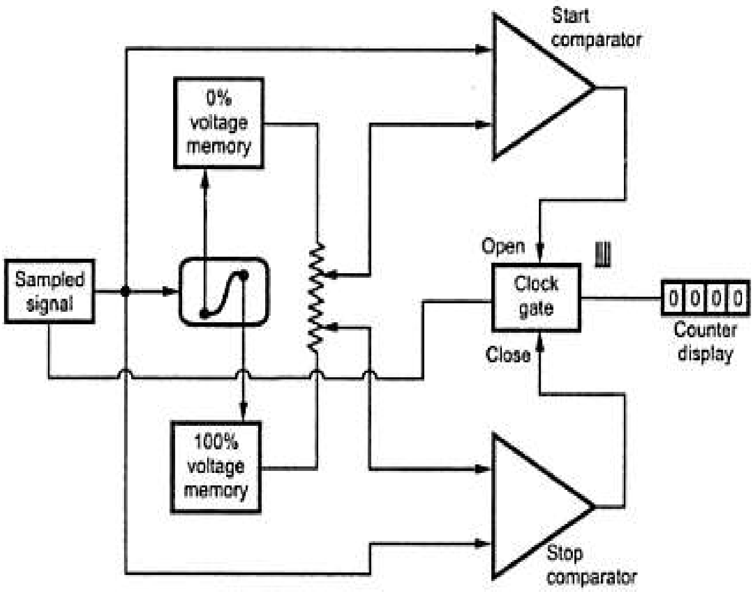
This is automatic refresh mode. When new sweep signal is generated by time base generator, the old contents get over written by new one. By pressing hold or save button, overwriting can be stopped and previously saved signal gets locked.

Advantages

1. It is easier to operate and has more capability.
2. The storage time is infinite.
3. The X-Y plots, B-H curve, P-V diagrams can be displayed.
4. The display flexibility is available.

5.4.5 DIGITAL READ OUT OSCILLOSCOPE

The digital read out oscilloscope consist of CRT display and counter display. The input signal is first sampled using sampling circuit. The sampled signal is given as one output to each start and stop comparators. When input is sampled, the sampling circuit advances the sampling position by a fixed amount. This process is called strobing. The selected sweep time per cm control and number of samples taken per cm decide the equivalent time between two samples.



Digital Read out Oscilloscope

The CRT trace is used to identify 0% and 100% zone positions. The positions can be shifted anywhere on the display.

The potential divider is used which taps the voltage between the 0% and 100% levels. The 0% level is used to produce a pulse for opening of gate while 100% level is used to produce a pulse for closing of gate.

The coincidence of any of the input waveforms with the selected percentage point is sensed by the voltage comparator. When 0% level gives the start pulse, clock gate opens and the counter starts counting the pulses. When 100% level gives the stop pulse, clock gate closes and the counter stops counting the pulses.

The number of pulses counted by a counter are proportional to actual sample taken. The digital read out is obtained using Nixie display tube.

5.5 BRIDGES

The bridges are used for the measurement of resistance, inductance, capacitance etc.

A bride circuit in its simplest form consists of a network of four resistance arms forming a closed circuit. A source of current is applied to two opposite junctions. The current detector is connected to other two junctions.

The bridge circuit use the comparison measurement methods and operate on null indication principle. The bridge circuit compares the value of an unknown component with that of an accurately known standard component.

In a bridge circuit, when no current flows through the null detector which is generally galvanometer, the bridge is said to be balanced. The relationship between the component values of the four arms of the bridge at the balancing is called balancing condition.

Advantages

1. The measurement accuracy is high.

2. The balance equation is independent of the sensitivity of the null detector.

3. The balance equation is independent of the magnitude of the input voltage or its source impedence.

4. The balance condition remains unchanged if the source and detector are interchanged.

5. The bridge circuit can be used in the control circuits.

5.6 TYPES OF BRIDGES

The two types of bridges are,

1. D.C bridges

2. A.C bridges

The d.c. bridges are used to measure the resistance. The d.c. bridges use the d.c. voltage as the excitation voltage.

The a.c. bridges are used to measure the impedances consisting capacitance and inductance. The a.c. bridges use the a.c. voltage as the excitation voltage.

5.7 D.C. BRIDGES

The two types of d.c. bridges are,

1. Wheatstone bridge

2. Kelvin bridge

The various types of a.c bridges are,

1.Capacitance comparison bridge

2. Inductance comparison bridge

3. Maxwell's bridge

4. Hay's bridge

5. Anderson bridge

6. Schering bridge

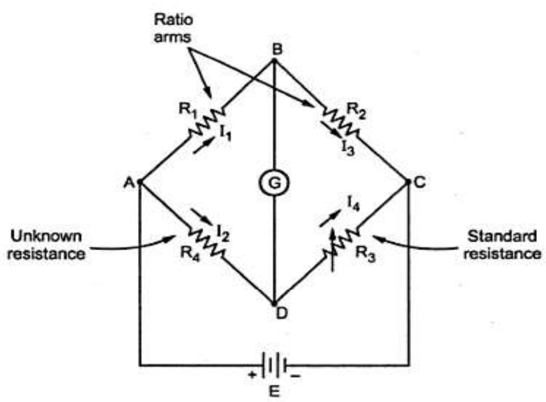
7. Wien bridge

WHEATSTONE BRIDGE

The bridge consists of four resistive arms together with a source of e.m.f. and a null detector. The galvanometer is used as a null detector.

The Fig. shows the basic Wheatstone bridge circuit.

The arms consist the resistance R_1 and R_2 are called ration arms. The arm consisting the standard known resistance R_3 is called standard arm. The resistance R_4 is the unknown resistance to be measured. The battery is connected between A and C while galvanometer is connected between B and D.



Wheatstone Bridge

Balance Condition

When the bridge is balanced, the galvanometer carries zero current and it does not show any deflection. Thus bridge works on the principle of null deflection or null indication.

To have zero current through galvanometer, the points B and D must be at the same potential. Thus potential across arm AB must be same as the potential across arm AD.

Thus $I_1 R_1 = I_2 R_4$ ... (1)

As galvanometer current is zero,

$I_1 = I_3$ and $I_2 = I_4$ ... (2)

Considering the battery path under balanced condition,

$$I_1 = I_3 = \frac{E}{R_1+R_2} \quad \text{... (3)}$$

and

$$I_2 = I_4 = \frac{E}{R_3+R_4} \quad \text{... (4)}$$

Using equation (3) and equation (4) in (1),

$$\frac{E}{R_1+R_2} x R_1 = \frac{E}{R_3+R_4} x R_4$$

$$\therefore R_1 (R_3+R_4) = R_4 (R_1+R_2)$$

$$\therefore R_1 R_3+R_4 = R_1 R_4+R_2+R_4$$

This is required balance condition of Wheatstone bridge.

The following points can be observed.

1. It depends on the ratio of R_1 hence these arms are called ration arms.
2. As it works on null indication, the results are not dependent on the calibration and characteristics of galvanometer.
3. The standard resistance R_3 can be varied to obtain the required balance.

Sensitivity of Wheatstone Bridge

When the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection the amount of deflection on the sensitivity of the galvanometer this sensitivity can be expressed as amount of deflection per unit current.

As the current is in microampere and deflection can be measured in mm, radians or degrees, the sensitivity is expressed s mm/μA, radians/μA or degrees/μA. more is the sensitivity of a galvanometer, more is its deflection for the same amount of current.

Another way of representing the galvanometer sensitivity is the amount of deflection per unit voltage across the galvanometer. This is called voltage sensitivity of the galvanometer.

While the bridge sensitivity is defined as the deflection of the galvanometer per unit fractional change in the unknown resistance it is denoted as S_B

Wheatstone Bridge under Small Unbalance

The bridge sensitivity can be calculated by solving the bridge for small unbalance.

At balance condition, $R_4 = R_3\frac{R_1}{R_2}$

i.e. $\frac{R_4}{R_3} = \frac{R_1}{R_2}$

Let the resistance R_4 is changed by ΔR creating the unbalance. Due to this, the e.m.f. appears across the galvanometer. To obtain this e.m.f., let us use The venin's method. Remove the branch of galvanometer and obtain the voltage across the open circuit terminals.

$$E_{AB} = I_1 R_1 \quad \dots (6)$$

$$I_1 = \frac{E}{R_1+R_2} \quad \dots (7)$$

$$E_{AD} = I_2 (R_4+\Delta R) \quad \dots (8)$$

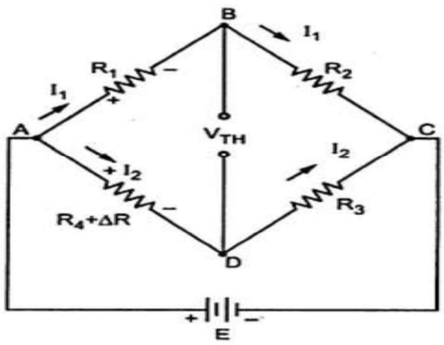
$$I_2 = \frac{E}{R_3+R_4\Delta R} \quad \dots (9)$$

$$V_{BD} = V_{TH} = E_{AD} - E_{AB} \quad \dots . (10)$$

$$\therefore \quad V_{TH} = \frac{E(R_4+\Delta R)}{R_3+R_4+\Delta R} - \frac{E}{R_1+R_2}R_1$$

$$\therefore \quad V_{TH} = E\left\{\frac{R_4+\Delta R}{R_3+R_4+\Delta R} - \frac{R_1}{R_1+R_2}\right\} \quad \dots (11)$$

As $\frac{R_4}{R_3} = \frac{R_1}{R_2}$ then $\frac{R_1}{R_1+R_2} = \frac{R_4}{R_4+R_3}$



Bridge under unbalance

Using above relation in equation (11).

$$V_{TH} = E\left\{\frac{R_4+\Delta R}{R_3+R_4+\Delta R} - \frac{R_4}{R_3+R_4}\right\}$$

$$= E\left\{\frac{R_3R_4+R_3\Delta R+R_4^2+R_4\Delta R-R_3R_4-R_4^2-R_4\Delta R}{(R_3+R_4)(R_3+R_4+\Delta R)}\right\}$$

$$= \frac{ER_3\Delta R}{(R_3+R_4)^2+(R_3+R_4)\Delta R}$$

But as ΔR is very small, $(R_3+R_4)\Delta R << (R_3+R_4)^2$

Now $S_B = \frac{\theta}{\Delta R/R}$ = bridge sensitivity

and $\Delta R/R = \Delta R/R_4$ as there is change in R_4.

From the galvanometer sensitivity S_v,

$\theta = S_v \times e$ where e = Voltage across galvanometer = V_g

Using θ in the expression of S_B

$$\therefore \quad S_B = \frac{S_vV_g}{\Delta R/R_4} = \frac{S_vER_3\Delta RR_4}{(R_3+R_4)^2} = \frac{S_vER_3R_4}{R_3^2+2R_3R_4+R_4^2}$$

Thus the bridge sensitivity depends on the bridge parameters, the supply voltage and the voltage sensitivity of the galvanometer.

Key point : Thus maximum sensitivity occurs when $\frac{R_3}{R_4} = 1$

Applications of Wheatstone Bridge

The Wheatstone bridge is basically a d.c. bridge and used to measure the resistances in the range 1Ω to low megaohm.

It is used to measure the d.c. resistance of various types of wires for the purpose of quality control of wire.

It is used to measure the resistance of motor winding, relay coils etc.

It is used by the telephone companies to locate the cable faults. The faults may be of the type line to line short or line to ground short.

Advantages of Wheatstone Bridge

The various advantages of Wheatstone bridge are,

1. The result are not dependent on the calibration and characteristics of galvanometer as it works on null deflection.
2. The source e.m.f. and inaccuracies due to the source fluctuations do not affect the balance of the bridge. Hence the corresponding errors are completely avoided.
3. Due to null deflection method used, the accuracy and sensitivity is higher than direct deflection meters.

Limitations of Wheatstone Bridge

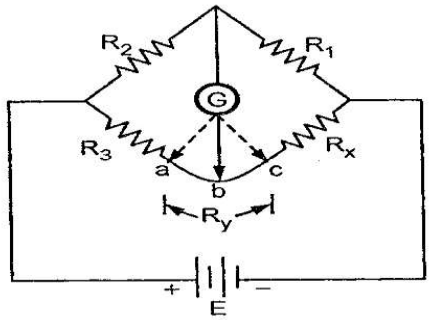
The effect of lead resistance and contact resistance is very much significant while measuring low resistances.

The bridge cannot be used for high resistance measurement i.e. measurement in high mega ohm range. This is because while such measurement the resistance presented by the bridge becomes so large that the galvanometer becomes insensitive to show any imbalance.

Similarly heating effect due to large current also plays a major role. The excessive currents may generate heat which may cause the permanent change in the resistance.

The resistance used must be very precise having tolerance upto 1 % or 0.1%, hence cost is high.

KELVIN BRIDGE



Kelvin Bridge

In the Wheatstone bridge, the bridge contact and lead resistance causes significant error, while measuring low resistances. Thus for measuring the values of resistance below 1 Ω, the modified form of Wheatstone bridge is used, known as Kelvin bridge. The consideration of the effect of contact and lead resistance is the basic aim of the Kelvin bridge.

The fig. is the basic circuit of the Kelvin bridge.

The resistance R_y represents the resistance of the connecting leads from R_3 to R_x. The resistance R_x is the unknown resistance to be measured.

The galvanometer can be connected to either terminal a,b or terminal c. When it is connected to a, the lead resistance R_y gets added to R_x hence the value measured by the bridge, indicates much higher value of R_x.

If the galvanometer is connected to terminal c, the R_y gets added to R_3. This results in the measurement of R_x much lower than the actual value.

The point b is in between the points a and c, in such a way that the ration of the resistance from c to be and that from a to be is equal to the ration of R_1 and R_2.

$$\therefore \quad \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \qquad \ldots (1)$$

Now the bridge balance equation in its standard form is,

$R_1R_3 = R_2R_x$ … (2)

But R_3 and R_x now are changed to R_3+R_{cb} respectively due to lead resistance.

∴ $R_1 (R_3+R_{ab}) = R_2(R_x+R_{cb})$ … (3)

∴ $(R_x+R_{cb}) = \frac{R_1}{R_2}(R_3+R_{ab})$

Now we have, $\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$

∴ $\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$ … adding 1 to both sides

$$\frac{R_{cb}+R_{ab}}{R_{ab}} = \frac{R_1+R_2}{R_2}$$

… (5)

But $R_{cb}+R_{ab} = R_y$ …. total lead resistance

Substituting in equation (5) we get,

$$\frac{R_y}{R_{ab}} = \frac{R_1+R_2}{R_2}$$

… (6)

∴ $R_{ab} = \frac{R_2R_y}{R_1+R_2}$ … (7)

Now $R_{cb}+R_{ab} = R_y$

∴ $R_{cb} = R_y - R_{ab}$ … (8)

Substituting equation (7) into equation (8),

$$R_{cb} = R_y - \frac{R_2R_y}{R_1+R_2}$$

$$= R_y\left[1 - \frac{R_2}{R_1+R_2}\right]$$

∴ $R_{cb} = \frac{R_1R_y}{R_1+R_2}$ … (9)

Substituting these values of R_{cb} and R_{ab} in the equation (4) we get,

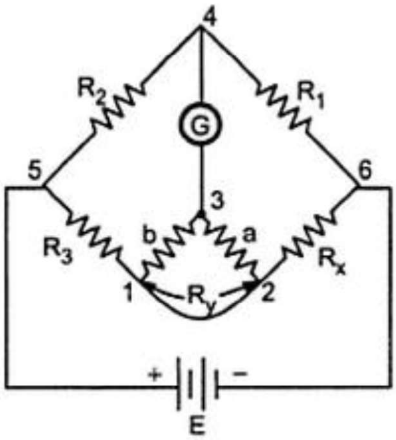
$$R_x + \frac{R_1 R_y}{R_1+R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1+R_2}\right)$$

$$\therefore \quad R_x + \frac{R_1 R_y}{R_1+R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1+R_2}$$

Thus equation represents standard bridge balance equation for the Wheatstone bridge. The effect of the connecting lead resistance is completely eliminated by connecting the galvanometer to an intermediate position 'b'.

This principle forms the basis of the construction of Kelvin's Double Bridge which is popularly called Kelvin Bridge.

KELVIN'S DOUBLE BRIDGE



Kelvin's Double Bridge

This bridge consists of another set of ration arms hence called double bridge. The Fig. shows the circuit diagram of Kelvin's Double Bridge.

The second set of ration arms is the resistances 'a' and 'be'. With the help of these resistances the galvanometer is connected to point '3'. The galvanometer gives null indication when the potential of the terminal '3' is same as the potential of the terminal '4'

Thus $E_{45} = E_{513}$ ... (11)

Here E_{45} = potential across R_2

E_{513} = potential across R_3 and b

The ratio of the resistances a and b is same as the ratio of R_1 and R_2

$$\therefore \quad \frac{a}{b} = \frac{R_1}{R_2} \quad \ldots (12)$$

Now $$E_{45} = R_2 \frac{E}{R_1+R_2} \quad \ldots (13)$$

Consider the path from 5-1-2-6 back to 5 through the battery E. The resistance between the terminal 1-2 is the parallel combination of R_y and (a+b).

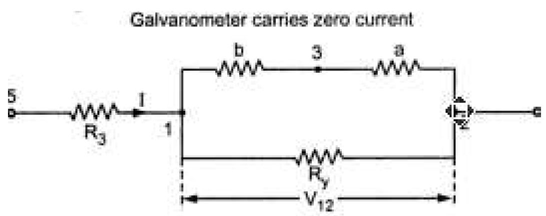
$$\therefore \quad E = Ix[R_3+R_y||(a+b)9+R_x]$$

$$\therefore \quad E = I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \quad \ldots (14)$$

Substituting in equation (13),

$$E_{45} = \frac{R_2}{R_1+R_2} x\, I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \quad \ldots (15)$$

For E_{153}, consider the path from the terminal 5 to 2 as shown in the Fig. 3.18.



Now from the Fig. 3.18 we can write,

$$V_{12} = Ix\left[\frac{R_y(a+b)}{R_y+a+b}\right]$$

and $$V_{13} = \frac{b}{a+b} . V_{12}$$

$$V_{13} = \frac{b}{a+b} . I\left[\frac{R_y(a+b)}{R_y+a+b}\right] \quad \ldots (16)$$

$\therefore \quad E_{513} = I R_3 + V_{13}$

$$\therefore \quad E_{513} = I R_3 + I \frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y+a+b}\right]$$

$$\therefore \quad E_{513} = I\left[R_3 + \frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y+a+b}\right]\right] \quad \ldots (17)$$

Now $\quad E_{45} = E_{513} \quad$ … for balancing

$$\therefore \quad \frac{IR_2}{R_1+R_2}\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] = I\left[R_3 + \frac{b}{a+b}\left\{\frac{R_y(a+b)}{a+b+R_y}\right\}\right]$$

$$\therefore \quad R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1+R_2}{R_2}\left[R_3 + \frac{b}{a+b}\left\{\frac{R_y(a+b)}{a+b+R_y}\right\}\right]$$

$$\therefore \quad R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left[1 + \frac{R_1}{R_2}\right]\left[R_3 + \frac{bR_y}{R_y+a+b}\right]$$

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = R_3 + \frac{R_1R_3}{R_2} + \frac{bR_y}{R_y+a+b} + \frac{R_1bR_y}{R_2(R_y+a+b)}$$

$$\therefore \quad R_x = \frac{R_1R_3}{R_2} + \frac{bR_y}{R_y+a+b} + \frac{R_1bR_y}{R_2(R_y+a+b)} - \frac{(a+b)R_y)}{(R_y+a+b)}$$

$$\therefore \quad R_x + \frac{R_1R_3}{R_2} = \frac{bR_1R_y}{R_2(R_y+a+b)} - \frac{aR_y}{(a+b+R_y)}$$

But $\quad \frac{a}{b} = \frac{R_1}{R_2} \quad$ thus $\quad \frac{R_1}{R_2} - \frac{a}{b} = 0$

This is the standard equation of the bridge balance. The resistances a,b and R_y are not present in the equation. Thus the effect of lead and contact resistances is completely eliminated.

Key Point : The important condition for this bridge balance condition is that the ratio of the resistances of ratio arms must be same as the ratio of the resistances of the second ratio arms.

In a typical Kelvin's double bridge, the range of a resistance covered is 1 Ω to 10μΩ with an accuracy of ±0.05% to ± 0.2%.

Practical Kelvin's Double Bridge

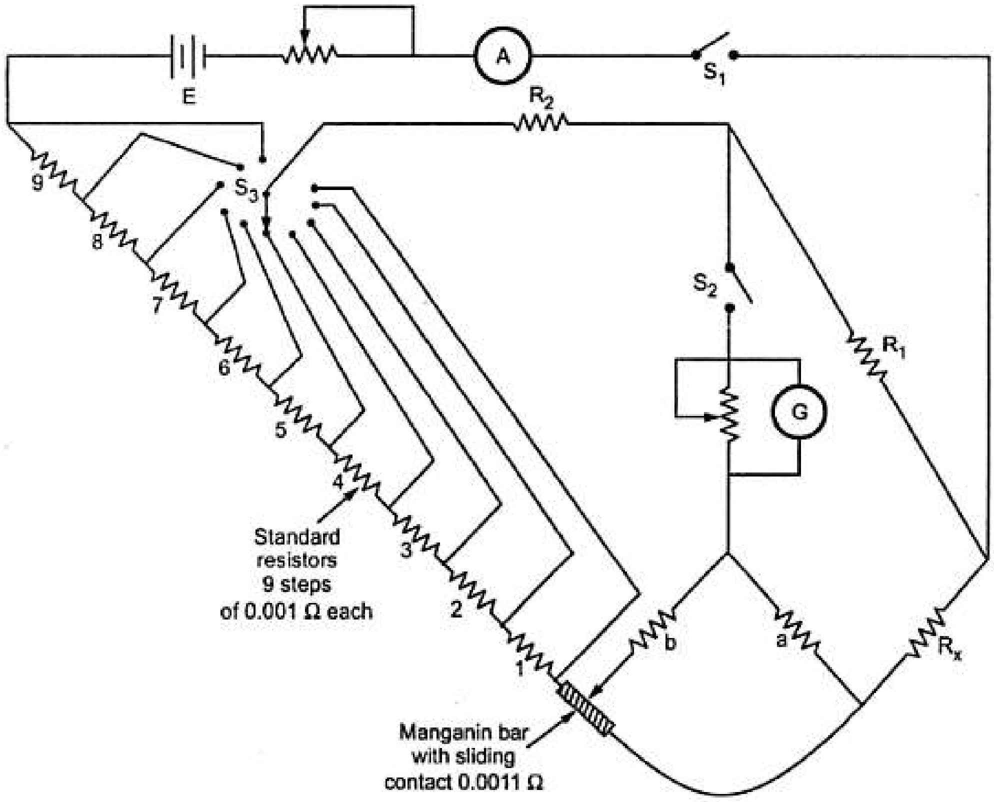
The Fig. shows a commercial Kelvin's double bridge, capable of measuring the resistances of very low range from 10Ω to 0.00001Ω.

The resistance R_3 is replaced by the standard resistance consisting on nine steps of 0.001Ω each, plus a calibrated manganin bar of 0.0011Ω with a sliding contact. The required resistance can be selected by the switch S_3. The total resistance of the R_3 arm amounts to 0.0101Ω and is variable in steps of 0.001Ω by the sliding contact.

When both the contacts are switched to select the proper value of standard resistance, the voltage drop between the ration arm connection points is changed but the total resistance around the battery circuit is unchanged.

With this arrangement any contact resistance can be placed in series with the relatively high resistance values of the ratio arms. Due to this, the effect of contact resistance becomes negligibly small.

The ratio of R_1 and R_2 is selected in such a way that the larger part of the variable standard resistance is used and hence R_x is determined to the largest possible number of significant. This increases the measurement accuracy.



Practical Kelvin's Double Bridge

MEASUREMENT OF HIGH RESISTANCE

The measurement of high resistance of the order of hundreds and thousands of mega ohms is often required in electrical equipment's. The examples of such resistances are,

i. Insulation resistance of components like machines, cables etc.

ii. Leakage resistance of capacitors.

But there are certain difficulties in measurements of such high resistances. Because of very high resistance, very small currents flow through the measuring circuits, which is very difficult to sense. The various other difficulties are:

i. Presence of leakage currents : The leakage currents are produced and are of comparable magnitude to the current being measured. Such currents cause errors.

These currents depend on humidity and hence are unpredictable. Hence leakage currents must be eliminated from the measurement.

ii. The stray charges may appear due to electrostatic effect. Such charges and alternating fields can also cause serious measurement errors.

iii. One point of the circuit may be connected to earth for accuracy in measurements.

iv. When the voltage is applied to the insulation resistance, it takes some time for charging and absorbing currents.

v. Very high voltage is required in order to raise the current magnitudes. The galvanometer should be very sensitive and proper steps must be taken to prevent the damage of galvanometer due to high voltages.

5.8 A.C. BRIDGES

An a.c bridge in its basic form consists of four arms, a source of excitation and a balance detector. Each arm consists of an impedance.The source is an a.c. supply which supplies a.c. voltage at the required frequency.

For the a.c. bridges commonly used detectors are as follows,

i. Headphones
ii. Vibration galvanometers
iii. Tuneable amplifier detectors

MAXWELL'S INDUCTANCE BRIDGE

Using this bridge, we can measure inductance by comparing with a standard variable self-inductance arranged in the bridge circuit.

Consider Maxwell's inductance bridge as shown in the Fig 3.23 (a). Two branches consist of non-inductive resistance R_1 and R_2. One of the arms consists variable induction with series resistance r. The remaining arm consists unknown inductance L_X.

At balance, we get condition as

$$\frac{R_1}{[(R_3+r)+j\omega L_3]} = \frac{R_2}{R_X+j\omega L_X} \quad \text{........ (1)}$$

$$R_1[R_X + j\omega L_X] = R_2[(R_3 + r) + j\omega L_3]$$

$$R_1 R_X + j\omega R_1 L_x = R_2 (R_3 + r) + j\omega R_2 L_3$$

Equating imaginary terms, we can write

$$R_1 L_X = R_2 L_3$$

Equating real terms, we can write,

$$R_1 L_X = R_2 (R_2 + r)$$

MAXWELL'S INDUCTANCE CAPACITANCE BRIDGE

Using this bridge we can measure inductance by comparing with a variable standard capacitor.

One of the ratio arms consists of resistance and capacitance in parallel. Hence it is simple to write the bridge equations in the admittance form.

The general bridge balance equation is,

$$\overline{Z_1 Z_X} = \overline{Z_2 Z_3}$$

$$\overline{Z_X} = \frac{\overline{Z_2 Z_3}}{\overline{Z_1}}$$

$$= \overline{Z_2 Z_3}\ \bar{Y}_1 \quad \ldots\ldots \quad (4)$$

Where $\bar{Y}_1 = \frac{1}{\bar{Z}_1}$ i.e. R_1 in parallel with C_1

$$\overline{Z_X} = R_2$$

$$\overline{Z_3} = R_3$$

$\overline{Z_X}$ = $R_x + j\,\omega L_1$, as L_X in series with R_x

Now $\bar{Y}_1$ = $\frac{1}{\bar{R}_1} + j\,\omega C_1$ …….. (5)

as $\overline{Z_1}$ = $R_1 \,||\, j\left(\frac{1}{\omega C_1}\right)$

as $\frac{1}{j} = -j$

Substituting all the values in equation (4) we get,

$$R_x + j\,\omega L_1 = R_2 R_3 \left[\frac{1}{R_1} + j\,\omega C_1\right]$$

$$R_x + j\,\omega L_1 = \frac{R_2 R_3}{R_1} + j\,R_2 R_3 C_1 \quad …….. (6)$$

Equating imaginary parts,

$$\omega L_x = R_2 R_3 \ \omega\, C_1$$

The resistances are expressed in ohms, the inductances in henries and capacitance in farads.

The quality factor of the coil is given by,

$$Q = \frac{\omega L_X}{R_X} = \frac{\omega R_2 R_3 C_1}{\left(\frac{R_2 R_3}{R_1}\right)}$$

$$Q = \omega R_1 C_1 \quad …….. (9)$$

The **advantages** of using standard known capacitor for measurement are:

1) The capacitors are less expensive than stable and accurate standard inductors.
2) The capacitors are almost lossless.
3) External fields have less effect on a capacitor. The standard inductor requires well shielding in order to eliminate the effect of stray magnetic fields.
4) The standard inductor will not present its rated value of inductance unless current flow through it is precisely adjusted.
5) The capacitors are smaller in size.

This bridge is also called **Maxwell Wien Bridge.**

Advantages of Maxwell Bridge

The advantages of the Maxwell bridge are:

1) The balance equation is independent of losses associated with inductance.
2) The balance equation is independent of frequency of movement.
3) The scale of the resistance can be calibrated to read the inductance directly.
4) The scale of R_1 can be calibrated to read the Q value directly.

Limitation of Maxwell Bridge

The limitations of the Maxwell bridge are:

1) It cannot be used for the measurement of high Q values. Its use is limited to the measurement of low Q value from 1 to 10. This can be proved from phase angle balance condition which says that sum of the angles of one pair of opposites arms must be equal.

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

2) There is an interaction between the resistance and reactance balances.

Getting the balance adjustment is little difficult.

3) It is unsuited for the coils with low Q values, less than one, because of balance convergence problem.

4) The bridge balance equations are independent of frequency.

Commercial Maxwell bridge measures the inductance from 1 -1000 H, with ± 2 % error.

ANDERSON BRIDGE

It is another important a.c. bridge used for the measurement of self- inductance in terms of a standard capacitor. Actually this bridge is nothing but modified Maxwell's bridge in which also the value of self -inductance is obtained by comparing it with a standard capacitor. This bridge is basically used for the precise measurement of inductance over a wide range of value.

One arm of the bridge consists of unkown inductor L_X resistance in series with L_X. This resistance R_1 includes resistance of the inductor. C is the standard capacitor with r, R_2, R_3 and R_4 are non-inductive known resistances.

The bridge balance equation are,

$$i_1 = i$$

$$i_2 = i_4 + i_C,$$

$$V_2 = i_2 R_2$$

$$v_3 = i_3 R_3$$

$$V_1 = V_2 + i_c r \text{ and } V_4 = V_3 + i_C r$$

$$V_1 = i_1 R_1 +_1 \omega L_1$$

$$V_4 = i_4 R_4$$

$$V = \bar{V}_2 + \bar{V}_4$$

$$= \bar{V}_1 + \bar{V}_3$$

To find balance equations transforming a star formed by R_2, R_4 and r into equivalent delta.

The elements in equivalent delta are given by,

$$R_5 = \frac{R_2 r + R_4 r + R_2 R_4}{R_4}$$

$$R_6 = \frac{R_2 r + R_4 r + R_2 R_4}{R_2}$$

$$R_7 = \frac{R_2 r + R_4 r + R_2 R_4}{r}$$

Now R_7 shunts the source, hence it does not affect the balance condition. Thus by neglecting R_7 and rearranging a network as shown in the Fig. 3.26 (b), we get a Maxwell inductance bridge.

Thus, balance equations are given by,

$$L_x = CR_3R_3 \text{ and}$$

$$R_1 = R_3\frac{R_5}{R_6}$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

If the capacitor used is not perfect, the value of inductance remains unchanged, but the value of R_1 changes.

This method can also be used to measure the capacitance of the capacitor C if a calibrated self -inductance is available.

Advantages

The advantages of Anderson's bridge are,

1. Can be used for accurate measurement of capacitance in terms of inductance.

2. Other bridges require variable capacitor but a fixed capacitor can be used for Anderson's bridge.
3. The bridge is easy to balance from convergence point of view compared to Maxwell's bridge in case of low values of Q.

Disadvantages

The disadvantages of Anderson's bridge are,

1. It is more complicated than other bridges.
2. Uses more number of components.
3. Balance equations are also complicated to derive.
4. Bridge cannot be easily shielded due to additional junction point, to avoid the effects of stray capacitances.

SCHERING BRIDGE

It is one of the most widely used a.c. bridges for the measurement of unknown capacitors, dielectric loss and power factor.

The Fig.3.29 shows the connections of Schering bridge. It can be used for low voltages. The C_x is perfect capacitor to be measured. R_x is series resistance. C_2 is standard air capacitor having very stable value. R_3 and R_4 are non-inductive resistances while C_4 is variable capacitor.

From the general balance equation,

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

Now $$Z_1 = R_X - j\frac{1}{\omega C_X}$$

$$Z_2 = -\frac{1}{\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = R_4 || \frac{-j}{\omega C_4}$$

$$= \frac{R_4\left(-\frac{j}{\omega C_4}\right)}{\left(R_4 - j\frac{1}{\omega C_4}\right)}$$

$$Z_4 = \frac{-j}{\omega R_4 C_4 - j}$$

$$= \frac{-j\, R_4(\omega R_4\, C_4 + j)}{(\omega R_4 C_4 - j)(\omega R_4 C_4 + j)}$$

$$= \frac{R_4 - j\omega R_4{}^2 C_4}{\omega^2 R_4{}^2 C_4{}^2 + 1}$$

$$Z_1 = \frac{Z_2 Z_3}{Z_4}$$

$$= \frac{\left(\frac{-j}{\omega C_2}\right)(R_3)}{\frac{R_4 - j\omega R_4{}^2 C_4}{1 + \omega^2 R_4{}^2 C_4{}^2}}$$

$$= \frac{(1 + \omega^2 R_2{}^4 C_4{}^2)\left(\frac{-j}{\omega C_2}\right)}{\left(R_4 - j\omega R_4{}^2 C_4\right)}$$

Rationalising, $Z_1 = R_3\left(1 + \omega^2 R_2{}^4 C_4{}^2\right)\left\{\frac{\frac{-j}{\omega C_2}\left(R_4 - j\omega R_4{}^2 C_4\right)}{R_4{}^2 + \omega^2 R_4{}^4 C_4{}^2}\right\}$

$$R - j\frac{1}{\omega C_X} = \frac{R_3(1+\omega^2 R_2{}^4 C_4{}^2)}{R_4{}^2(1+\omega^2 R_2{}^2 C_4{}^2)}\left\{\frac{R_4{}^2 C_4}{C_2} + \frac{-jR_4}{\omega C_2}\right\}$$

$$-j\frac{1}{\omega C_X} = -j\frac{R_3}{R_4{}^2} \times \frac{R_4}{\omega C_2}$$

$$= -j\left[\frac{1}{\frac{R_4}{R_3}\omega C_2}\right]$$

$$\omega C_X = \frac{R_4}{\omega C_2}\omega C_2$$

WIEN BRIDGE

Basically the bridge is used for the frequency measurement but it is also used for the measurement of the unknown capacitor with great accuracy.

Its one ratio arm consists of a series RC circuit i.e. R_1 and C_1. The second ratio arm consists of a resistance R_2. The third arm consists of the parallel combination of resistance and capacitor i.e. R_3 and C_3.

we can write,

$$Z_1 = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2$$

$$Z_3 = R_3 \,||\, C_3$$

$$Y_3 = \frac{1}{R_3} + j\,\omega C_3$$

and $Z_4 = R_4$

The balance condition is,

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

$$\overline{Z_2} = \frac{\overline{Z_1 Z_4}}{\overline{Z_3}}$$

$$= Z_1 \overline{Z_2 Z_3}$$

$$R_2 = \left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right] R_4 \left[\frac{1}{R_3} + j\omega C_3\right]$$

$$R_2 = R_4 \left[\frac{R_1}{R_3} + j\omega R_1 C_3 - j\frac{1}{\omega C_1 R_3} + \frac{C_3}{C_1}\right]$$

$$R_2 = R_4 \left[\frac{R_1}{R_3} + \frac{C_3}{C_1}\right] + jR_4 \left[\omega R_1 C_3 - \frac{1}{\omega C_1 R_3}\right]$$

Equating real parts of both sides,

$$R_2 = \frac{R_4 R_1}{R_3} + \frac{C_3 R_4}{C_1}$$

Equating real parts of both sides,

$$\omega R_1 C_3 - \frac{1}{\omega C_1 R_3} = 0$$

$$\omega^2 = \frac{1}{R_1 R_3 C_1 C_3}$$

$$\omega = \frac{1}{\sqrt{R_1 R_3 C_1 C_3}} \quad \text{........ (2)}$$

The equation (1) gives the resistance ratio while the equation (3) gives the frequency of applied voltage.

Generally in Wien bridge, the selection of the components is such that

$$R_1 = R_3 = R$$

and $$C_1 = C_3 = c$$

$$\frac{R_2}{R_4} = 2 \quad \text{........ (4)}$$

and $$f = \frac{1}{2\pi RC} \quad \text{........ (5)}$$

The equation (5) is the equation for the frequency of the bridge circuit.

Applications

The bridge is used to measure the frequency in audio range. The audio range is [20-200-2 k – 20 k] Hz. The resistances are used for the changing while the capacitors are used for fine frequency control.

The bridge can be used for capacitance measurement if the operating frequency is known.

The bridge is also used in a harmonic distortion analyzer, as a notch filter and in audio frequency and radio frequency oscillators as a frequency determining element. The accuracy of 0.5% - 1% can be readily obtained using this bridge.

5.9 MEASUREMENT OF RESISTANCE

The measurement of resistance is done by comparing the unknown resistance with the standard resistance.

The unknown resistance R and the known standard resistance R_s are connected on the secondary of the tapped ratio transformer. The silder position is adjusted till the detector D shows null.

$$I_1 = \frac{E1}{R} \text{ while } I_2 = \frac{E2}{Rs}$$

and $E_1 = K_1 N_1$ and $E_2 = K_1 N_2$

As detector does not carry any current, $I_1 = I_2$

$$\frac{K1\,N1}{R} = \frac{K1\,N2}{R}$$

$$R = \frac{N1}{N2} R_s$$

Thus knowing N_1 and N_2 the unknown resistance R can be measured.

If the resistance to be measured is very small, then the lead resistance can affect the resistance measurement. In such a case, two ratio transformers are used to form a type of kelvin's double bridge circuit.

The impedance Z_1, Z_2, Z_3, Z_4 and Z_5 are the lead impedances. Then the unknown resistance is given by,

$$R = R_s \frac{N1\,ZB+Z1}{(1-N1)\,ZB+ZA} + \frac{(1-Z2)ZA+Z3}{Z1+Z2+Z3+Z5} \times$$

$$\frac{N1ZB+Z1}{(1-N1)ZB+Z4} - \frac{N1ZA+Z3}{(1-N2)ZA+Z3}$$

But if the lead impedances are very small then,

$$R = \frac{N1}{N2} R_s$$

5.10 MEASUREMENT OF CAPACITANCE

The unknown capacitance can be measured by ratio transformer bridge in comparison with the standard capacitance.

The C represents the unknown capacitor while R represents the loss of the capacitor. The C_s is standard perfect capacitor while R_s is the standard resistance. For balance, the magnitude and phase of the current passing through the detector must be same. To achieve this, the standard resistance is kept variable.

At the balance condition,

$$C = \frac{N2}{N1} c_s$$

Thus the dissipation factor of the capacitor is given by

$$D = \tan \delta = \frac{1}{\omega\, CR} = \frac{1}{\omega Cs\, Rs}$$

PROBLEMS

Example 1 : A slide wire potentiometer has a battery of 4 V and negligible internal resistance. The resistance of slide wire is 100Ω and the length is 200 cm. a standard cell of 1.018 V is used for standardizing the potentiometer and the rheostat is adjusted so that balance is obtained when the sliding contact is at 101.8 c.m.

a. Find the working current and the rheostat setting.

b. If the slide wire has divisions marked in mm and each division can be interpolated to one fifth, calculate the resolution of the instrument

Solution : a) Since the d.c. potentiometer is standardized using standard cell of e.m.f. 1.018V with sliding contact at 101.8 cm, the portion of slide wire of the same length represents a voltage of 1.018 V.

∴ Resistance of 101.8 cm length of wire

$$= \left(\frac{101.8}{200}\right) \times 100 = 50.9\ \Omega$$

∴ Working current I $= \frac{1.018}{50.9} = 20$ mA

Total resistance in battery circuit = Resistance of rheostat + Resistance of slide wire

∴ Resistance of rheostat = Total resistance in battery circuit – Resistance of the slide wire

$$= \left[\frac{\text{Battery voltage}}{\text{Working current}}\right] - \text{Resistance of the slide wire}$$

$$= \left[\frac{4}{20 \times 10^{-3}}\right] - 100$$

$$= 100\ \Omega$$

b) The measurement range is nothing but total voltage across the slide wire.

∴ Range of voltage = (Working current) x (resistance of slide wire)

$$= (20 \times 10^{-3}) \times (100)$$

$$= 2\ \text{V}$$

∴ The length of 200 cm represents 2 V so 1 mm represent voltage given by,

$$V = \left(\frac{2}{200}\right)\left(\frac{1}{10}\right) = 1\ \text{mV}$$

Since it is possible to read $\frac{1^{th}}{5}$ of 1 mm. the resolution of instrument is given by, Resolution = [v] $\left[\frac{1}{5}\right] = \frac{1\times10^{-3}}{5}$ = 0.2 mV

Example 2 : The Wheatstone bridge is shown in the fig. 4.6. calculate the value of unknown resistance, assuming the bridge to be in balanced condition.

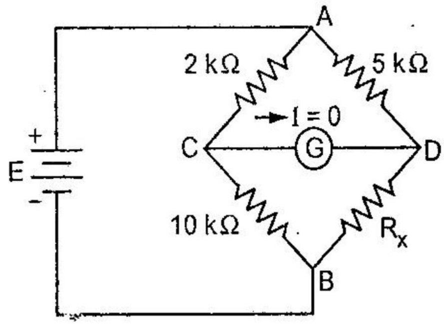


Fig. 4.6

Solution : As per the bridge shown in the Fig. 4.1 earlier,

R_1 = 10kΩ, R_2 = 2kΩ, R_3 = 5 kΩ and R_4 = R_x

Under balanced condition,

R_4 =

Thus unknown resistance is 25 kΩ

Example 3 : Calculate the current through the galvanometer for the bridge shown in the Fig. 4.7

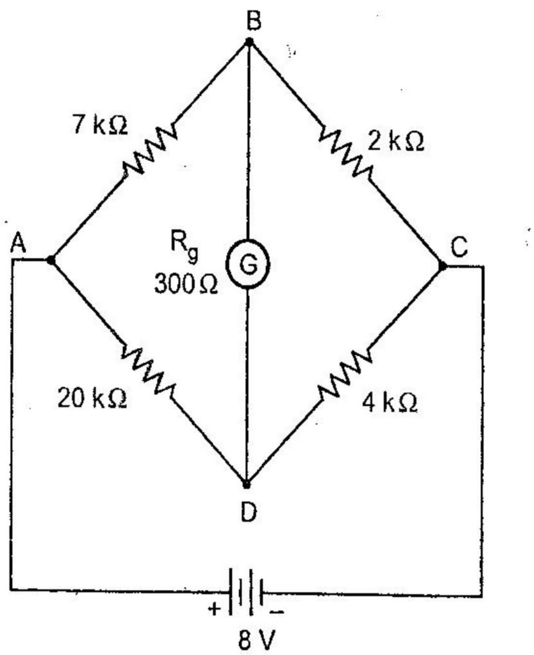


Fig. 4.7

Solution : From the Fig. 4.7

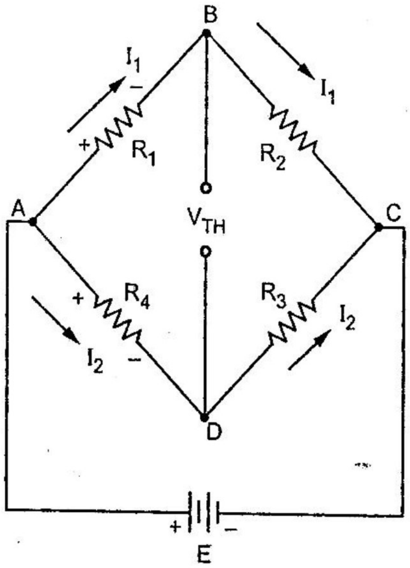


Fig. 4.8

$R_1 = 7\ k\Omega$, $R_2 = 2\ k\Omega$

$R_3 = 4\ k\Omega$, $R_4 = 20\ k\Omega$, E=8V.

Use Thevenin's equivalent for I_g

$$V_{TH} = V_{BD} = V_{AD} - V_{AB}$$

$$= I_2 R_4 - I_1 R_1$$

$$= \frac{E}{R_3+R_4} R_4 \frac{E}{R_1+R_2} R_1$$

$$= 8\left\{\frac{20}{20+4} - \frac{7}{7+2}\right\}$$

$$= 0.444\ V$$

Thus B is positive w.r.t D.

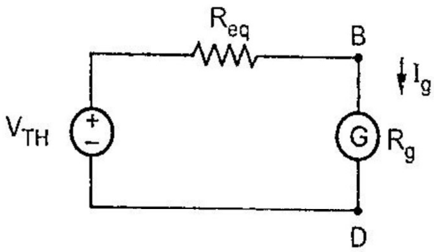


Fig. 4.9

Now R_{eq} =

$[R_1||R_2]+[R_3||R_4]$ .. with E shorted

$$= \frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4} = 4.888\ k\Omega$$

$$I_g = \frac{V_{TH}}{R_{eq}+R_g} = \frac{0.444}{4.888X10^3+300}$$

$$= 85.62\ \mu A$$

This is the current through the galvanometer.

Example 4: In a Kelvin's double bridge, there is error due to mismatch between the ratios of outer and inner arm resistances. The bridges uses,

Standard resistance = 100.03μΩ

Inner ratio arms = 100.31 Ω and 200 Ω

Outer ratio arms = 100.24 Ω and 200 Ω

The resistance of the connecting leads from standard to unknown resistance is 700μΩ. Calculate the unknown resistance under this condition.

Solution : From the given data,

R_3 = 100.03μΩ, R_2 = 100.24 Ω, R_1 = 200 Ω

b = 100.31μ, a = 200 Ω, R_y = 700μΩ

Thus unknown resistance is,

$$R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{[R_y + a + b]}\left\{\frac{R_1}{R_2} - \frac{a}{b}\right\}$$ .. Refer equation (18) of section 4.4.1

$$= \frac{200\text{x}100.03\text{x}10^{-6}}{100.24} + \frac{100.31\text{x}700\text{x}10^{-6}}{[700\text{x}10^{-6} + 200 + 100.31]}\left\{\frac{200}{100.24} - \frac{200}{100.31}\right\}$$

$$= 1.9958\text{x}10^{-4} + (2.3381\text{x}10^{-4})(1.3923\text{x}10^{-3})$$

$$= 1.999\text{x}10^{-4}\Omega \qquad = 199.905\mu\Omega$$

Example 5: *The arms of an a.c. bridge have impedances as shown in the Fig. 4.21. Determine whether the bridge is balanced or unbalanced.*

Solution : From the given bridge,

$Z_1 = 50 \angle 40^0$ Ω, $Z_2 = 100 \angle -90^0$ Ω, $Z_3 = 15 \angle 45^0$ Ω, $Z_4 = 30 \angle 30^0$ Ω

The bridge balance equation is,

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

Equating magnitudes, $|Z_1 Z_4| = |Z_2 Z_3|$

$|Z_1 Z_4| = 50 \times 30 = 1500$ and $|Z_1 Z_4| = 100 \times 15 = 1500$

Thus magnitude condition is satisfied.

Now $\theta_1 = 40^0, \theta_2 = -90^0, \theta_3 = 45^0, \theta_4 = 30^0$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

µ $\theta_1 + \theta_4 = 40 + 30 = 70^0$ and $\theta_2 + \theta_3 = -90^0 + 45^0 = -45^0$

Thus **angle condition is not satisfied.**

Hence the bridge is not balanced condition.

Example 6: *A capacitance comparison bridge is used to measure the capacitive impedance at a frequency of 3 kHz. The bridge constants at bridge balance are,*

$C_3 = 10\ \mu F$

$R_1 = 1.2\ k\Omega$

$R_2 = 100\ k\Omega$

$R_3 = 120\ k\Omega$

Find the equivalent series circuit of the unknown impedance.

Solution : *From the bridge balance equations,*

$$R_X = \frac{R_2 R_3}{R_1}$$

$$= \frac{100 \times 10^3 \times 120 \times 10^3}{1.2 \times 10^3}$$

$$= 10\ M\Omega$$

While $C_x = \frac{R_1 C_3}{R_2} = \frac{1.2 \times 10^3 \times 10 \times 10^{-6}}{100 \times 10^3}$

$= 0.12\ \mu F$

Example 7: *The arms of an a.c. Maxwell's bridge are adjusted as :*

Arm AB: nonreactive resistance of 700Ω

Arm CD : nonreactive resistance of 300 Ω

Arm AD : nonreactive resistance of 1200 Ω in parallel with capacitor of 0.5μF

If the bridge is balanced under this condition, find the components of the branch BC.

Solution: The bridge is shown in the Fig. 4.28.

From the bridge,

$C_1 = 0.5\ \mu F$, $R_1 = 1200\ \Omega$

$R_2 = 700\ \Omega$, $R_3 = 300\ \Omega$

From bridge balance equation,

$R_x = \frac{R_2 R_3}{R_1}$

$= \frac{700 \times 300}{1200}$

$=$ **175 Ω**

And $L_X = R_2 R_3 C_1$

$= 700 \times 300 \times 0.5 \times 10^{-6}$

$=$ **105 mH**

Example 8 :

The Schering bridge has the following constants :

Arm AB - Capacitor of 1 μF in parallel with 1.2 kΩ resistance

Arm AD - resistance of 4.7 kΩ

Arm BC - capacitor of 1 μF

Arm CD - unknown capacitor C_X and R_X

The frequency of supply is 0.5 kHz. Calculate the unknown capacitance and its dissipation factor.

Solution : From the given information,

$R_1 = 1.2\ k\Omega \quad C_1 = 1\ \mu F$

$R_2 = 4.7\ k\Omega \quad C_3 = 1\ \mu F$

From the balance equations,

$$R_x = \frac{R_2 C_1}{C_1}$$

$$= \frac{4.7 \times 10^3 \times 1 \times 10^{-6}}{1 \times 10^{-6}}$$

$$= 4.7\ k\Omega$$

$$C_X = \frac{R_1 C_3}{R_2}$$

$$= \frac{4.7 \times 10^3 \times 1 \times 10^{-6}}{4.7 \times 10^3}$$

The dissipation factor,

$$D = \omega C_X R_X$$

$$= 2\pi f C_X R_X$$

$= \quad 2\pi \times 0.5 \times 10^3 \times 0.255 \times 10^{-6} \times 4.7 \times 10^3$

$= \quad 3.765$

Example 9 : In a bridge shown in the Fig 4.78, find the constants of Z_x considering as series circuit.

Solution : From the bridge,

$Z_1 = 200\Omega$

$\mathbf{Z_2} = \mathbf{R_2}\ j\,(\frac{1}{\omega C2})$

$= 200 - j(\frac{1}{1X10^3 X2\pi X5X10^{-6}}) = 200\ j\ 31.83\Omega$

$Z_3 = R_3 - j\,(\frac{1}{\omega C_3}) = 500\ j\ (\frac{1}{2\pi X10^3 X0.2X10^6})$

$= 500 - j\ 795.77\ \Omega$

From the basic balance equation,

$Z_1 Z_4 = Z_2 Z_3$

$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{(200-j31.83)(500-j795.77)}{200}$

$= \frac{[202.517\angle-9.04^{\circ}][939.81\angle-57.85^{\circ}]}{200}$

$= 951.6413\angle-66.89^{\circ}\ \Omega$

$Z_x = 373.j\ 875.274\Omega$

Example 10 : An a. c bridge is blanced at 2 kHz with following components in each arm.

Arm AB = 10 k Ω

Arm BC = 100μF in series with 100 k Ω

Arm AD = 50 k Ω

Find the unknown impedance r ± jX in the arm DC, if the detector is between BD.

Solution : The bridge is shown in the Fig.4.79.

$Z_1 = 10k\Omega$

$Z_2 = 50\ k\Omega$

$Z_3 = 100\ k - j\left(\frac{1}{\omega C_3}\right) = 100\ X\ 10^3 - j\left(\frac{1}{2\pi X 2 X 10^3\ X 100 X 10^6}\right)$

$= 100X10^3 - j\ 0.7957\ \Omega$

$Z_4 = Z_X$

From the basic balance equation,

$Z_1Z_4 = Z_2Z_3$

$Z_4 = \frac{Z_2Z_3}{Z_1} = \frac{50\ X\ 10^3\ [100\ X\ 10^3 - j\ 0.7957]}{10\ X 10^3}$

$= 5\ [100\ X\ 10^3\ - j\ 0.7957] = 500X10^3\ - j$

$3.9788\ \Omega$

$= R_X - j\, X_c$

$R_X \quad = 500X10^3\Omega = 500k\Omega$

And $\quad X_c \quad = 3.9788\,\Omega$

$= \frac{1}{2\pi f c_X}$

$C_X \quad = \frac{1}{2\pi X 2 X 10^3 \quad X 3.9788} = 20\mu F$

Example 11 : The four arms of a bridge are :

Arm ab : an imperfect capacitor C_1 with an equivalent series resistance of r_1

Arm bc : a non induction resistance R_3

Arm cd : a not inductive resistance R_4

Arm da :an imperfect capacitor C_2 with an equivalent series resistance of r_2 in series with a resistance R_2.

A supply of 450 H_z is given between terminal a and the detector is connected between b and d .At balance $R_2 = 4.8\ \Omega$, $R_3 = 200\ \Omega$, $R_4 = 2850\ \Omega$, C_2 0.5 μF,$r_2 = 0.4\ \Omega$. Calculate the value of C_1 and also of the dissipating factor of this capacitor.

Solution : The bridge is as shown in the Fig.4.80.

$Z_1 \quad = \quad r_1 - j\frac{1}{\omega C_1}\,\Omega$

$Z_2 \quad = \quad (R_2 + r_2) - j\dfrac{1}{\omega C_1} = (4.8 + 0.4) - j\dfrac{1}{2\pi X 450\ X\ 0.5\ X\ 10^{-6}}$

$= 5.2 - j\ 707.3553\ \Omega\Omega$

$= 707.3744 \angle -89.5788^{\circ}\ \Omega$

$Z_3 \quad = \quad 200 + j\ 0\Omega = 200 \angle 0^{\circ}\ \Omega$

$Z_4 \quad 2850 + j\ 0\ \Omega = 2850 \angle 0^{\circ}\ \Omega$

At a bridge balance, no current flows through the detector.

$I_1 = \dfrac{E}{Z_1 + Z_3}$ and $I_2 = \dfrac{E}{Z_2 + Z_4}$

Now $\quad I_1 Z_1 = I_2 Z_2$ for null deflection of detector

$$\frac{EZ_1}{Z_1 + Z_3} = \frac{EZ_2}{Z_2 + Z_4}$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$2850\left[r_1 - j\ \frac{1}{\omega C_1}\right] = 200 \angle 0^{\circ}\ X\ 707.3744 \angle -89.5788^{\circ}$$

$$r_1 - j\frac{1}{\omega C_1} = 49.6403 \angle -89.5788^{\circ} = 0.3649 - j\ 49.6389\ \Omega$$

Comparing both sides,

$r_1 = 0.3649\ \Omega$ and $\frac{1}{\omega c_1} = 49.6389$

$$C_1 = \frac{1}{2\pi \times 450 \times 49.6389} = 7.125\mu F$$

Dissipating factor = $\omega r_1 C_1 = 2\pi \times 450 \times 0.3649 \times 7.125 \times 10^{-6}$

$= 0.007351$

Example 12 : An A.C bridge circuit for measurement of effective inductance and capacitance of an iron cored coil is as follows : Arm AB : the unknown impedance, Arm BC : a pure resistance of 10Ω, Arm CB : a loss free capacitance of 1 μF and Arm AD : a capacitance of 0.135 μF in series with 842 Ω resistance. Obtain the balance equation of the bridge and determine the unknown parameter in the arm AB.

Solution : From the given information the bridge is as shown in the Fig .4.81

The general balance equation is,

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_1 = \frac{Z_2 Z_3}{Z_4}$$

Let the frequency is ω rad / sec.

$Z_2 = \left[842 - j\,\frac{1}{\omega 0.135\times 10^{-6}}\right]$

$= 842 - j\,\frac{7.4074\times 10^{6}}{\omega}\,\Omega$

$Z_3 = 10 + j\,0 = 10\angle 0^{\circ}\,\Omega$

$Z_4 = 0 - j\,\frac{1}{\omega\times 1\times 10^{6}} = -j\,\frac{10^{6}}{\omega} = \frac{10^{6}}{\omega}\angle -90^{\circ}\,\Omega$

$Z_1 = \frac{\left[842 - j\,\frac{7.4074\times 10^{6}}{\omega}\right][10]}{\frac{10^{6}}{\omega}\angle -90^{\circ}}$

$= \left[8420 - j\,\frac{7.4076\times 10^{7}}{\omega}\right]\frac{1}{10^{6}}\angle +90^{\circ}$

$= \left[8420 - j\,\frac{7.4076\times 10^{7}}{\omega}\right]\left[+j\,\frac{\omega}{10^{6}}\right]$

$= j\,\omega\,\frac{8420}{10^{6}} + \frac{7.4076\times 10^{7}}{10^{6}}$

$= 74.076 + j\,\omega\,8420 \times 10^{-6}$

Comparing Z_1 with $R_1 + j\omega L_1$

$R_1 = 74.076\,\Omega$ and $L_1 = 8420\,\mu H = 8.42$ mH

These are the unknown parameters of arm AB.

Example 13 : The arms of an a.c Maxwell's Bridge are arranged as follows. AB and BC are non-reactive resistor of 100 Ω each. DA is a standard variable inductor L_1 of resistance 32.7 Ω and CD comprises a variable resistance R in series with a coil of unknown impedance. Balance was obtained with L_1 = 47.8 mH and R = 1.36 Ω Find the resistance and inductance of coil.

Solution : The Maxwell's bridge is as shown in the Fig. 4.82.

The balance is obtained when L_2 = 47.8 mH and R_2 = 1.36 Ω.

At balance,

$$100(r_1 + j\omega L_1) = 100[(R_2 + r_2) + j\omega L_2]$$

Equating real and imaginary terms, we get,

$L_1 = L_2$ and $R_2 + r_2 = r_1$ or $r_2 = r_1 = R_1$

@ inductance of coil in branch CD is $L_2 = L_1$ = 4.7 mH

Resistance of coil in branch CD is given by

$$R_2 = r_1 - R_1 = 32.7 - 1.36$$

$$= 31.34\ \Omega.$$

Example 14 : For a wheatstone bridge, the ratio arms are each 100 Ω and bridge is balance with standard arm adjusted to 230 Ω. If each of the ratio arms have accuracy of ± 0.02 % and standard resistance has ± 0.01 % accuracy guaranteed, what are the limiting values of unknown resistance?

Solution : From given bridge,$R_1 = R_2$ = 100 Ω, R_3 = 230 Ω

$$R_4 = \frac{R_1 R_3}{R_2} = \frac{100 \times 230}{100} = 230\ \Omega$$

Relative limiting error is,

$$\frac{\delta R_4}{R_4} = \frac{\delta R_1}{R_1} + \frac{\delta R_3}{R_3} + \frac{\delta R_2}{R_2}$$

$$= (\pm 0.02 \pm 0.01 \pm 0.02)\% = \pm 0.05\ \%$$

So limiting values of unknown resistance are,

$$= 230 \pm 0.05\ \% = 230 \pm \left[\frac{0.05}{100} \times 230\right] = 230 \pm 0.115\ \Omega$$

$$= 229.885\ \Omega \text{ to } 230.115\ \Omega$$

Example 15: Find the equivalent parallel resistance and capacitance that causes a Wein bridge to null if the following components are given :

$R_1 = 8$ kΩ, $C_1 = 6\mu F$

$R_2 = 30$ kΩ, f = 205 kH$_Z$

$R_3 = 1$ kΩ

Solution : Let us consider wien bridge as shown in the Fig.4.83

From bridge balance condition,

$$\omega^2 = \frac{1}{R_1 \quad C_1 \ R_4 \quad R_4}$$

$$C_4 = \frac{1}{\omega^2 \; R_1 C_1 \; R_4}$$

Now $\frac{R_2}{R} = \frac{R_1}{R_4} + \frac{C_4}{C_1}$

Substituting value of C_4 in above equation

$$\frac{R_2}{R_3} = \frac{R_1}{R_4} + \frac{1}{\omega^2 R_1 \;\; C_1^4 R_4}$$

$$\frac{30 \times 10^3}{1 \times 10^3} = \frac{8 \times 10^3}{R^4} + \frac{1}{(2\times\pi\times2.5\times10^3\,)2 \times8\times10^3 \;\; \times(6\times10^{-6)} \;\; 2\times R_4}$$

$$30 = \frac{8\times10^3}{R^4} + \frac{0.01407}{R_4}$$

$R_4 = 266.667\,\Omega$

Putting R_4 in equation (1), we get,

$$C_4 = \frac{1}{(2\times\pi\times2.5\times10^3)2 \;\; \times8\times10^3 \times10^{-6} \times266.667}$$

$C_4 = 0.3166$ nF

Example 16: The four arm bridge ABCD, supplied with a sinusoid voltage, have the following values :

AB = 330 ohm resistance in parallel with 0.2μF capacitor

BC = 400 ohm resistance

CD = 800 ohm resistance; DA resistance R in series with a 1.5 μF capacitor. Determine the i) Value of R and ii) supply frequency at with the bridge will be balanced.

Solution:

$$Z_1 = R_1 - j\frac{1}{\omega C_1} = R_1 - j\,\frac{1}{\omega\times1.5\times10^{-6}}$$

$= R_1 - j \frac{666.667 \times 10^3}{\omega}$

$Z_1 = R_2 = 800\ \Omega$

$Z_4 = R_2 = 400\ \Omega$

$Z_3 = R_3 \text{ ll } C_3$

$Y_3 = \frac{1}{Z_3} = \frac{1}{R_3} + j\,\omega C_3$

$\frac{1}{330} + j\,0.2 \times 10^{-6}\omega$

According to balance condition,

$800 = \left[R_{1-j\frac{666.\times 10^3}{\omega}}\right]$

$\times 400 \times [3.0303 \times 10^{-3} + j\,0.2 \times 10^{6}\omega]$

$2 = 3.0303 \times 10^{-3} R_1 - j\frac{2020.203}{\omega} + j\ 0.2 \times 10^{-6} R_1\omega +$ 0.13334

$2 = [3.0303 \times 10^{-3} R_1 + 0.3334] + j \left[0.2 \times 10^{-6} R_{1\ \omega - 20} \frac{2020.203}{\omega}\right]$

Equating real parts,

$2 = 3.0303 \times 10^{-3} R_1 + 0.13334$

$R_1 = 616\ \Omega = R$ (unknown)

Equating imaginary parts,

$0.2 \times 10^{-6} \times R_{1\ \omega -} \frac{2020.203}{\omega} = 0$

$$\omega^2 = \frac{2020.203}{0.2 \times 10^{-6} \times 616} = 16397735.28$$

$$\omega = 4.049411 \text{ krad / sec} = 2\pi f$$

$f = 644.4838$ Hz

TWO MARKS

1. Name the different essential torques in indicating instruments?
   a. Deflecting torque
   b. Controlling torque
   c. Damping torque
2. Define deflection sensitivity of CRT?

   The deflection sensitivity of a CRT is defined as the deflection of the screen per unit deflection voltage.

3. List out the main parts of CRT?

   i. Electron gun
   ii. Deflection system
   iii. Fluorescent screen
   iv. Glass tube or envelope
   v. Base

4. What are the different types of amplifiers used for CRO's?

   1. Vertical amplifier
   2. Horizontal amplifier

REVIEW QUESTIONS

Part A

1. List the advantage of digital voltmeter.
2. What are the advantages of digital instruments over analog instruments?