UNIT IV

FREQUENCY ANALYSIS OF BJT AND MOSFET AMPLIFIERS

4.1. Hybrid – Π model:

At low frequencies we analyse transistor using h-parameters. But for high frequency analysis the h-parameter model is not suitable for following reasons.

1. The values of h-parameters are not constant at high frequencies. Therefore, it is necessary to analyse transistor at each and every frequency, which is impracticable.

2. At high frequency h-parameters becomes more complex in nature.

4.1.1. Elements in the Hybrid – Π model:

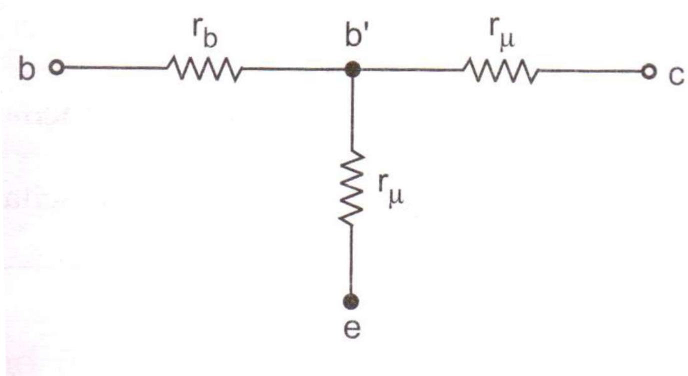


Fig 4.1. Virtual base

i) C_π and C_μ: The diffusion capacitance C_π connected between b' and e represents the excess minority carrier storage in the base. The capacitive effect of normally forward biased base-emitter junction of the transistor is represented by C_π in the Hybrid – Π model. The reverse bias PN junction exhibits a capacitive effect called the transition capacitance. This capacitive effect of normally reverse biased collector base junction of the transistor is represented by C_μ in the Hybrid – Π .

ii) r_b: The bulk resistance between external base terminal and internal node b' s represented as r_b .This resistance is called as base spreading resistance.

iii) r_π: The resistance r_π is that portion of the base emitter which may be thought of as being in series with the collector junction. This establishes a virtual base b' for the junction capacitances to be connected to instead of b.

iv) r_μ: Due to early effect, the varying voltages across the collector to emitter junction results in both results in base-width modulation. A change in the effective base width causes the emitter current to change. This feedback effect between output and input is taken into account by connecting r_μ between b' and c.

v) g_m: Due to the small changes in voltage V_π across the emitter junction, there is excess-minority carrier concentration injected into the base which is proportional to the V_π. Therefore, resulting small signal collector current, with collector shorted to the emitter is also proportional to the emitter is proportional to the V_π. This effect accounts for the current generator g_mV_π . g_m is called transconductance.

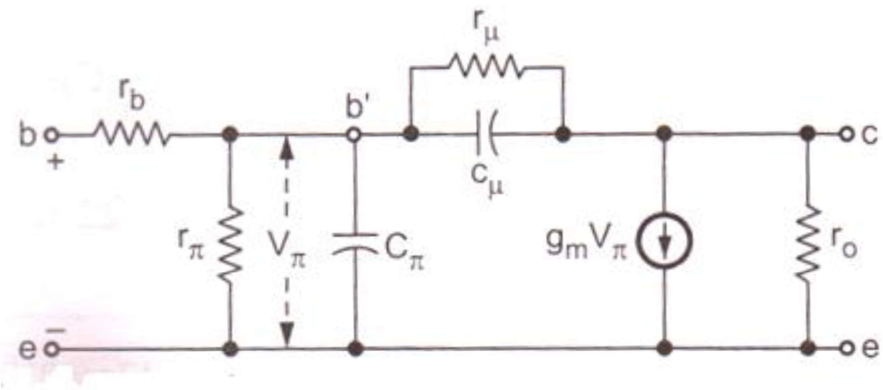


Fig 4.2. Hybrid – Π model for a transistors in the CE configuration

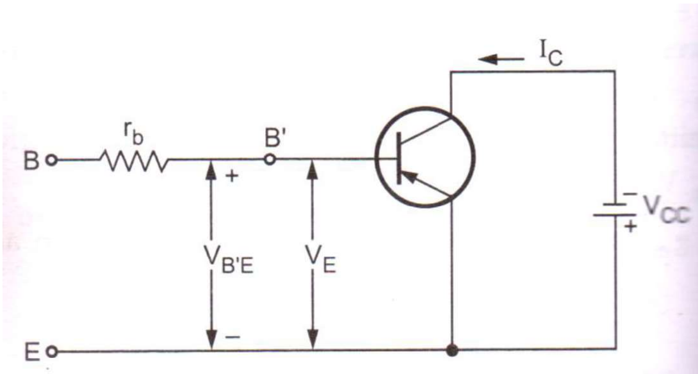


Fig 4.3. Pertaining to the derivation of g_m

i) Transconductance (g_m):

$$g_m = \frac{\partial Ic}{\partial V_{B'E}}\Big| V_{CE}$$

The collector current in active region is given as,

$$I_C = I_{CO} - \alpha I_E$$

And therefore

$$\partial I_C = -\alpha \partial I_E \qquad \because I_{CO} = \text{constant}$$

Sub ∂I_C in g_m,

$$g_m \equiv \propto \frac{\partial I_E}{\partial V_{B'E}} \propto \frac{\partial I_E}{\partial V_E} \qquad \because V_E = V_{B'E}$$

The emitter diode resistance,

$$r_e = \frac{\partial V_E}{\partial I_E}$$

Sub r_e in g_m,

$$g_m = \frac{\alpha}{r_e}$$

The dynamic resistance is given as,

$$r_e = \frac{V_T}{I_E}$$

$$V_T = \frac{KT}{q}$$

where, Boltzmann constant, $K = 1.38 \times 10^{-23}$ J/°k

electronic charge, $q = 1.6 \times 10^{-19}$ C

Sub r_e in g_m

$$g_m = \frac{\propto I_E}{V_T} = \frac{I_{CO} - I_C}{V_T} \qquad \because I_C = I_{CO} - \alpha I_E$$

For npn transistor,

$$g_m = \frac{Ic - Ico}{V_T}$$

$$g_m = \frac{I_c}{V_T} = \frac{I_c q}{K_T} = \frac{I_C}{\frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \times 300}$$

$$g_m = \frac{|I_C| mA}{26mV}$$

ii) Input conductance, $g_{b'e}$:

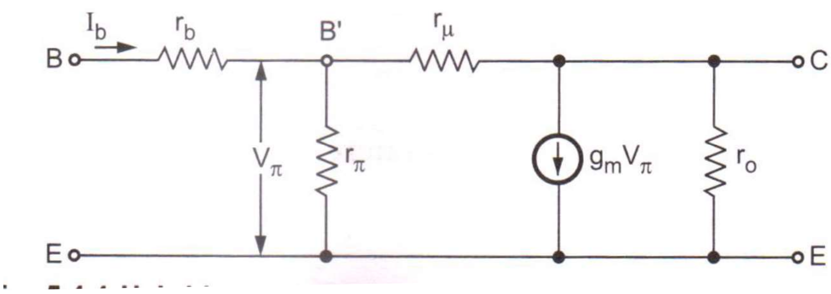


Fig 4.4. Hybrid – Π model for CE configuration at low frequency

The short circuit current gain h_{fe} is defined as,

$$h_{fe} = \frac{I_c}{I_b} \ldots\ldots\ldots\ldots\ldots (1)$$

$$I_c = g_m V_\pi$$

$$= g_m I_b r_\pi \qquad \because V_\pi = I_b r_\pi$$

$$\frac{I_c}{I_b} = g_m r_\pi \ldots\ldots\ldots\ldots\ldots (2)$$

Compare (1) & (2) $\quad hfe = g_m r_\pi$

$$r_\pi = \frac{h_{fe}}{g_m}$$

$$g_{b'e} = \frac{g_m}{h_{fe}} \qquad \therefore \frac{1}{r_\pi} = g_{b'e}$$

$$\text{W.K.T}, gm = \frac{I_c}{V_T}$$

$$r_\pi = \frac{h_{fe}\, V_T}{|Ic|}$$

$$g_{b'e} = \frac{|Ic|}{V_T\, h_{fe}}$$

iii) The feedback conductance , $g_{b'c}$:

Input open circuit,

$$V_i = h_{re} V_{ce}$$

$$V_{ce} = I_1 (r_\mu + r_\pi)$$

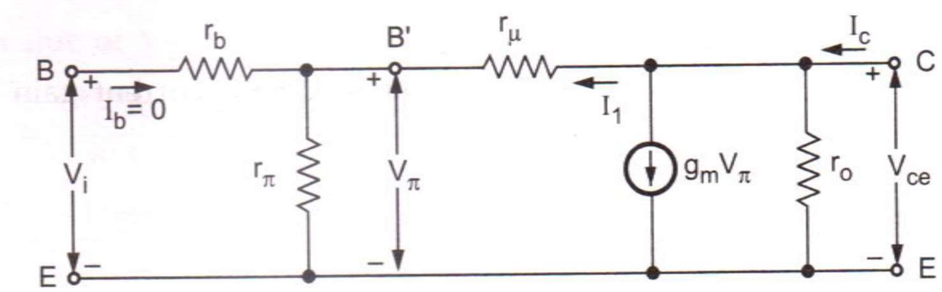


Fig 4.5. Hybrid – Π model for CE configuration

$$I_1 = \frac{V_{ce}}{r_\mu + r_\pi}$$

$$V_\pi = I_1\, r_\pi$$

$$\text{With } I_b = 0, \quad V_i = V_\pi$$

Sub I_1 in V_π,

$$V_i = V_\pi = \frac{V_{ce}}{r_\mu + r_\pi} r_\pi$$

Compare both V_i equation,

$$h_{re} V_{ce} = \frac{r_\pi V_{ce}}{r_\mu + r_\pi}$$

$$h_{re} = \frac{r_\pi}{r_\mu + r_\pi}$$

$$r_\pi = h_{re}\, r_\mu + h_{re}\, r_\pi$$

$$r_\pi - h_{re}\, r_\pi = h_{re}\, r_\mu$$

$$r_\pi\ (1 - h_{re}) = h_{re}\ r_\mu$$

$$r_\mu = \left(\frac{1 - h_{re}}{h_{re}}\right) r_\pi \qquad \because 1 - h_{re} \approx 1$$

$$r_\mu = \frac{r_\pi}{h_{re}}$$

$$g_{b'c} = \frac{h_{re}}{r_\pi} = h_{re}\, g_{b'c}$$

$$g_{b'c} = \frac{h_{re}\ |I_c\ |}{h_{re}\ V_T} \qquad \because g_{b'c} = \frac{|I_c\ |}{h_{re}\ V_T}$$

iv) Base spreading resistance, r_b:

Output shorted,

$$h_{ie} = r_b + r_\pi$$

$$r_b = h_{ie} - r_\pi$$

$$r_b = h_{ie} - \frac{h_{fe} V_T}{|I_C|} \qquad \therefore r_\pi = \frac{h_{fe} V_T}{|I_C|}$$

v) Output conductance , g_{ce}:

The output conductance,

$$hoe = \frac{I_C}{V_{ce}}$$

Applying KCL to the output circuit,

$$I_C = \frac{V_{ce}}{r_o} + gm\left(\frac{r_\pi\ V_{ce}}{r_\pi\ + r_\mu}\right) + \frac{V_{ce}}{r_\mu + r_\pi}$$

$$\frac{I_C}{V_{ce}} = \frac{1}{r_O} + \frac{g_m\ r_\pi}{r_\pi\ + r_\mu} + \frac{1}{r_\mu + r_\pi} \quad \therefore h_{fe} = g_m\ r_\pi$$

$$hoe = \frac{1}{r_o} + \frac{h_{fe}}{r_\pi\ + r_\mu} + \frac{1}{r_\pi\ + r_\mu}$$

$$h_{oe} = \frac{1}{r_o} + \frac{(h_{fe} + 1)}{r_\pi\ + r\mu}$$

$$= \frac{1}{ro} + \frac{h_{fe}}{r_{\pi} + r_{\mu}} \qquad \therefore h_{fe} \gg 1$$

$$hoe = \frac{1}{r_o} + \frac{h_{fe}}{r_{\mu}} \qquad \therefore r_{\mu} \gg r_{\pi}$$

$$h_{oe} = g_{ce} + g_{b'c} h_{fe}$$

$$\frac{1}{r_{ce}} = gce = hoe - g_{b'c} h_{fe}$$

4.2. General shape of frequency response of amplifiers:

Let us consider an audio frequency amplifier which operates over audio frequency range extending from 20Hz to 20KHz. The audio frequency amplifier are used in radio receivers, to address large public meeting, annual social gathering of college, for various announcements to be made for passenger on railway platforms, etc.

An amplifier should ideally provide the same amplification for all frequencies. The degree to which this is done is usually indicated by a curve know as frequency response curve of the amplifier.

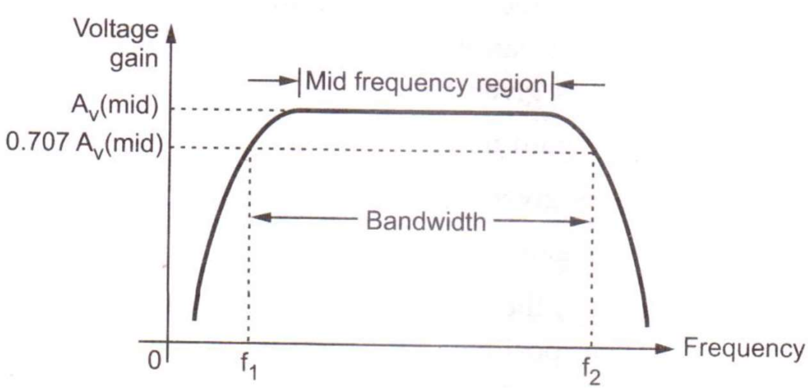


Fig 4.6. A typical frequency response of an amplifier

To plot this curve, input voltage to the amplifier is kept constant & frequency of input signal is continuously varied. It is seen from the frequency response curve of an audio frequency amplifier that the gain of the amplifier remains fairly constant in the mid freq range, while the gain varies with frequency in low & high freq regions on the curve. The decrease in voltage gain with frequency is called roll off.

Bandwidth of the amplifier is defined as the difference between f_2 and f_1.

$$B.W = f_2 - f_1$$

4.3. Significance of octaves &decodes:

1. The octaves & decodes are the measure of change in frequency.
2. A ten times change in frequency is called decode.
3. An octaves corresponds to a doubling or halving of the frequency.
4. For example an increase in frequency from 100Hz to 200Hz is an octave. Likewise, a decrease in freq from 100Hz to 50Hz is also an octave.

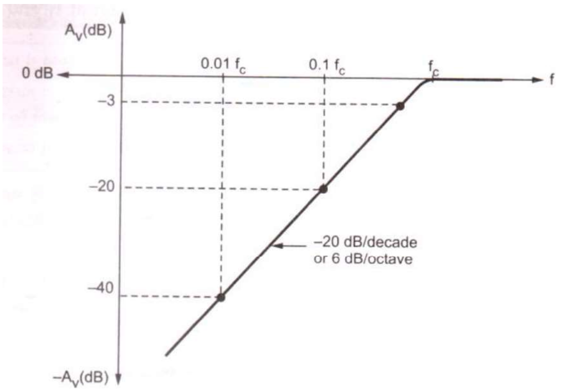


Fig 4.7. Frequency response showing significance of decade and octave

The voltage gain of the amplifier outside the midband is approximately given as,

$$A = \frac{A_{mid}}{\sqrt{1 + (f_1/f)^2}\sqrt{1 + (f/f_2)^2}}$$

i) Midband : $A = A_{mid}$

ii) Below Midband:

$$A = \frac{A_{mid}}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}$$

iii) Above Midband:

$$A = \frac{A_{mid}}{\sqrt{1 + \left(\frac{f}{f_2}\right)^2}}$$

4.4. Effect of various capacitors on frequency response:

4.4.1. Effect of coupling capacitors:

1. Reactance of a capacitors, $X_C = \frac{1}{2\pi f_c}$

2. At medium &high frequencies , the factor f makes X_C very small, so that all coupling capacitor behave as short circuits.

3. At low frequencies, X_C increases.This increase in X_C drops the signal voltage across the capacitor & reduce the circuit gain.

4.4.2. Effect of bypass capacitors:

1. At lower frequencies ,the bypass capacitors C_E is not a short. so the emitter is not ac ground.

2. X_C in parallel with R_E (R_S in case of FET) creates an impedance.

3. The signal voltage drops across this impedance reducing the circuit gain.

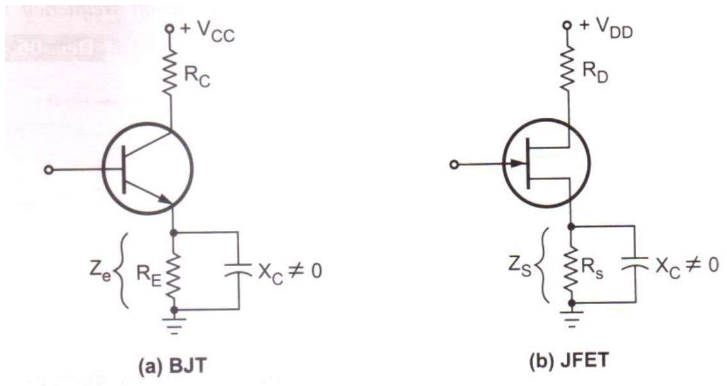


Fig 4.8. At low frequencies emitter (source in case of FET) is not at ac ground

4.4.3. Effect of internal transistor capacitors:

1. At high frequencies, the coupling and bypass capacitors act as short circuit & do not affect the amplifier frequency response.

2. However ,at high frequencies, the internal capacitances, commonly known as junction capacitances, reducing the circuit gain.

3. At higher frequencies, the reactances of the junction capacitances are low .As frequency increases, the reactances of junction capacitances fall.

4. When these reactances become small enough, they provide shunting effect as they are in parallel with junction .This reduces the circuit gain & hence the output voltage.

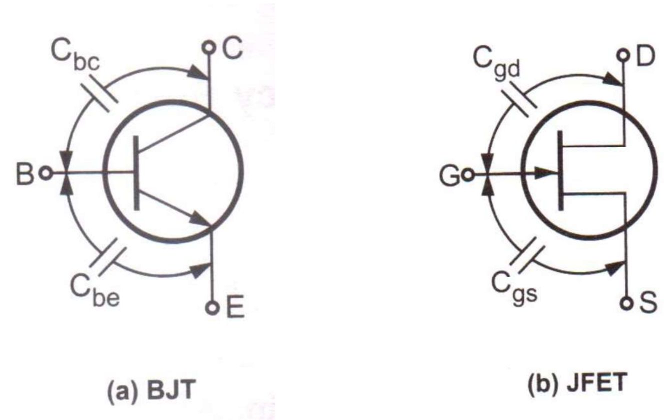


Fig 4.9. Internal transistor capacitances

4.5. Miller theorem:

In transistor amplifier it is necessary to split the capacitance between input & the output. This can be achieved using Miller's theorem.

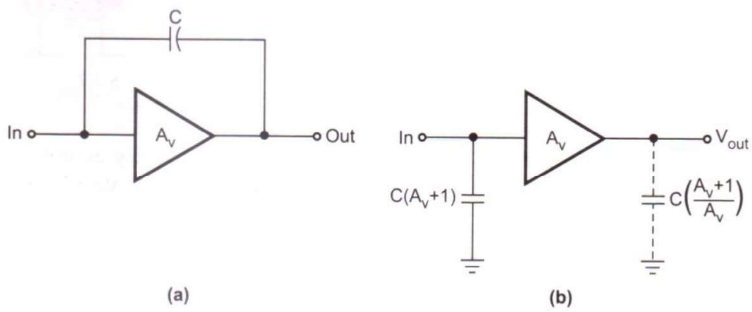


Fig 4.10. Splitting of capacitor using Miller's theorem

4.6. Low Frequency analysis of BJT:

The amplifier has three RC networks that affect its gain as the frequency is reduced below midrange these are,

i) RC networks formed by the input coupling capacitor C_1 & the input impedance of the amplifier.

ii) RC networks formed by the output coupling capacitor C_2, the resistance looking in at the collector & the load resistance.

iii) RC networks formed by the emitter bypass capacitor C_E & the resistance looking in at the emitter.

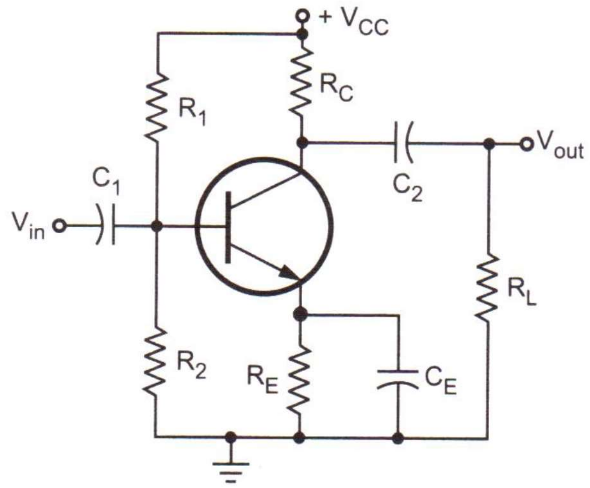


Fig 4.11. Typical RC coupled CE amplifier

4.6.1. Input RC network:

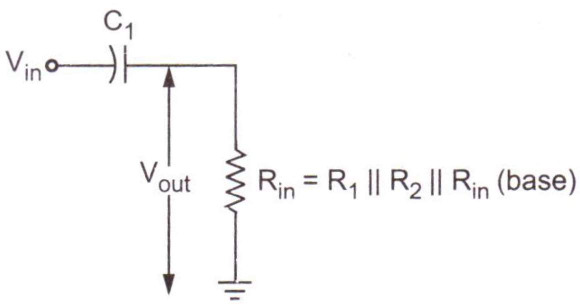


Fig 4.12. Input RC network formed by C_1

Applying voltage divider theorem,

$$V_{out} = \left(\frac{R_{in}}{\sqrt{R_{in}{}^{2+}X_{c1}{}^{2}}}\right) V_{in}$$

$$\text{Vout} = 0.707 \text{ Vin}$$

$$0.707 \text{ Vin} = \left(\frac{R_{in}}{\sqrt{R_{in}{}^{2+}X_{c1}{}^{2}}}\right) \text{Vin}$$

$$\frac{R_{in}}{\sqrt{R_{in}^{2}+X_{c1}^{2}}} = 0.707 = 1/\sqrt{2}$$

The reduction in overall gain is given by,

$$A_v = 20 \log \left(\frac{V_{out}}{V_{in}}\right)$$

$$=20 \log (0.707) = -3dB$$

The frequency f_c at this condition is called lower critical frequency & is given by,

$$f_c = \frac{1}{2\pi R_{in} C_1}$$

$$\text{where } R_{in} = R_1 \parallel R_2 \parallel hie$$

$$f_c = \frac{1}{2\pi(R_1 \parallel R_2 \parallel h_{ie})C_1}$$

If the resistance of the input source is taken into account,

$$f_c = \frac{1}{2\pi(R_S + R_{in})C_1}$$

The phase angle in an input RC circuit is expressed as,

$$\theta = \tan^{-1}\left(\frac{X_{C1}}{R_{in}}\right)$$

4.6.2. Output RC network:

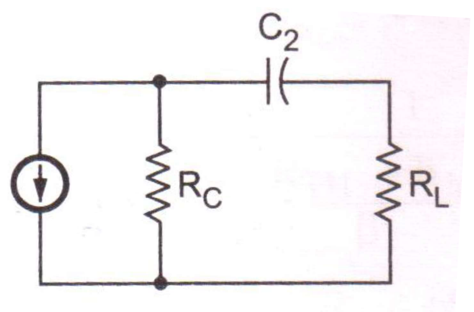


Fig 4.13. Current source

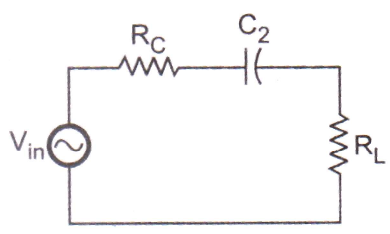


Fig 4.14. Current source replaced by voltage source

The critical frequency source for this RC network is given by,

$$f_c = \frac{1}{2\pi(R_C + R_L)C_2}$$

The phase angle in the output RC circuit is expressed as,

$$\theta = \tan^{-1}\left(\frac{X_{c2}}{R_C + R_L}\right)$$

4.6.3. Bypass Network:

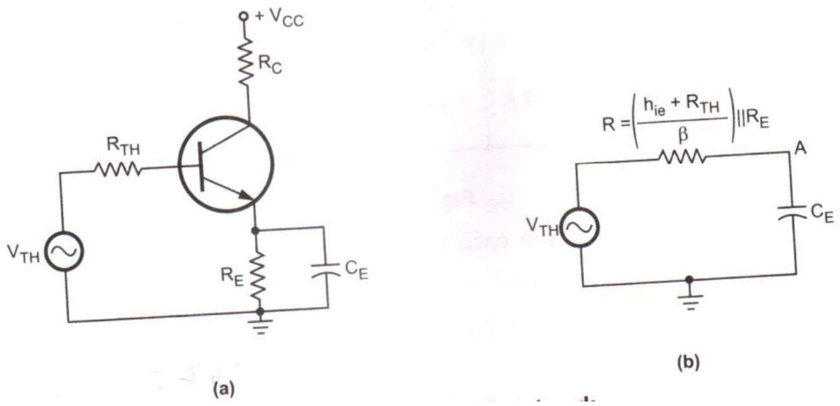


Fig 4.15. Bypass RC Network

The critical frequency for the bypass network is,

$$f_c = \frac{1}{2\pi R C_E}$$

$$f_c = \frac{1}{2\pi\left[\left(\frac{h_{ie} + R_{TH}}{\beta}\right) || R_E\right] C_E}$$

4.7. CE short circuit current gain using hybrid π model:

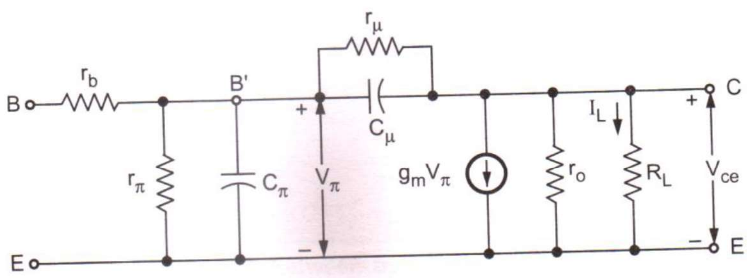


Fig 4.16. Bypass RC Network

Output short circuited r_o becomes zero, r_π, & C_π & C_μ appear in parallel. As $r_\mu \gg r_\pi$, r_μ is neglected with these approximation we get simplified hybrid π model for short circuit CE transistor.

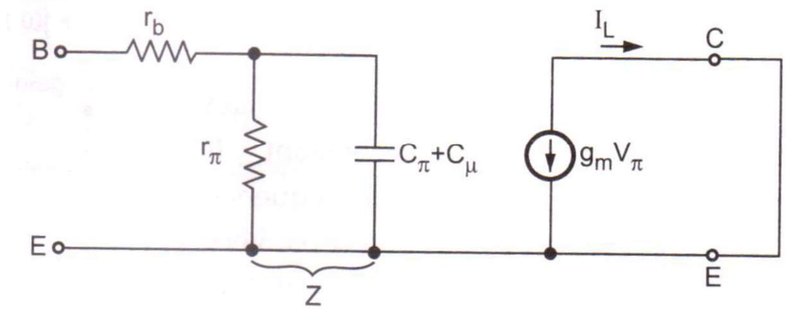


Fig 4.17. Simplified hybrid –π model for short circuit CE transistor

Parallel combination of r_π & $(c_\pi + c_\mu)$ is given as,

$$Z = \frac{r_\pi \times \frac{1}{j\omega(c_\pi + c_\mu)}}{r_\pi + \frac{1}{j\omega(c_\pi + c_\mu)}}$$

$$= \frac{r_\pi \times \frac{1}{j\omega(c_\pi + c_\mu)}}{\frac{j\omega\, r_\pi(c_\pi + c_\mu) + 1}{j\omega(c_\pi + c_\mu)}}$$

$$Z = \frac{r_\pi}{1 + j\omega\, r_\pi(c_\pi + c_\mu)}$$

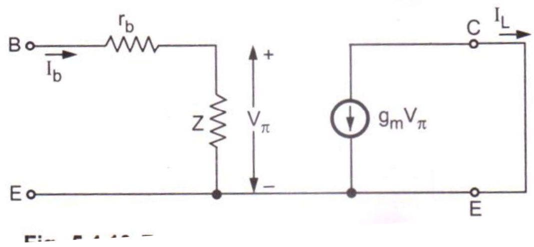


Fig 4.18. Further simplified hybrid –π model

$$V_\pi = I_b Z$$

$$Z = \frac{V_\pi}{I_b}$$

$$A_i = \frac{I_L}{I_b}$$

$$= \frac{-g_m V_\pi}{I_b} \qquad \because Z = \frac{V_\pi}{I_b}$$

$$= -g_m Z$$

$$A_i = \frac{-g_m r_\pi}{1 + j\omega\, r_\pi (c_\pi + c_\mu)} \quad \therefore h_{fe} = g_m r_\pi$$

$$A_i = \frac{-h_{fe}}{1 + j\omega\, r_\pi (c_\pi + c_\mu)}$$

1. The current is not constant .If depends on frequency.
2. When frequency increases, A_i reduces current gain.

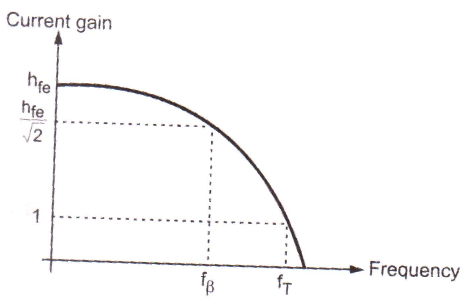


Fig 4.19. Frequency response of CE short circuit

$$f_\beta = \frac{1}{2\pi\, r_\pi (c_\pi + c_\mu)}$$

Sub f_β in A_i ,

$$Ai = \frac{-h_{fe}}{1 + j\frac{f}{f_\beta}}$$

$$|Ai| = \frac{h_{fe}}{\sqrt{1 + (\frac{f}{f_\beta})^2}}$$

At $f = f_\beta$

$$|Ai| = \frac{h_{fe}}{\sqrt{2}}$$

i) f_β (cut off frequency):

It is the frequency at which the transistor's short circuit CE current gain drops by 3dB or $\frac{1}{\sqrt{2}}$ times from its value at low frequency. It is given by,

$$f_\beta = \frac{1}{2\pi\, r_\pi (c_\pi + c_\mu)}$$

$$f_\beta = \frac{g_{b'e}}{2\pi(c_\pi + c_\mu)}$$

$$f_\beta = \frac{g_m}{2\pi\, h_{fe}\,(c\pi + c\mu)} \quad \because g_{b'e} = \frac{1}{r_\pi} = \frac{g_m}{h_{fe}}$$

ii) $f_\propto$ (cut off frequency):

It is the frequency at which the transistor's short circuit CB current gain drops by 3 dB or $1/\sqrt{2}$ times from its value at low frequency. The expression for $f_\propto$ can be derived is the similar manner as for f_β.

The current gain for CB configuration is given as,

$$f_\propto = \frac{1}{2\pi\, r_\pi (1 + h_{fb}) c_\pi}$$

$$f_\propto = \frac{1 + h_{fe}}{2\pi\, r_\pi\, c_\pi} \approx \frac{h_{fe}}{2\pi\, r_\pi\, c_\pi}$$

$$|A_i| = \frac{h_{fb}}{\sqrt{(1 + \left(\frac{f}{f_\propto}\right)^2}}$$

$A_t\, f = f_\propto$

$$|Ai| = \frac{h_{fb}}{\sqrt{2}}$$

iii) F_T : It is the frequency at which short circuit cE current gain becomes unity.

A_t $f = f_T$,$|A_i| = 1$

$$1 = \frac{hfe}{\sqrt{1+\left(\frac{f_T}{f_\beta}\right)^2}}$$

$$h_{fe} = \sqrt{1+\left(\frac{f_T}{f_\beta}\right)^2}$$

$$hfe = \frac{f_T}{f_\beta} \qquad \because \frac{f_T}{f_\beta} \gg 1$$

$$f_T = f_\beta \, h_{fe}$$

Sub f_β in f_T,

$$f_T = \frac{h_{fe} \times g_m}{h_{fe} \, 2\pi \, (C_\pi + C_\mu)}$$

$$f_T = \frac{g_m}{2\pi \, (C_\pi + C_\mu)}$$

Since $C_\pi \gg C_\mu$,

$$f_T = \frac{g_m}{2\pi \, C_\pi}$$

4.8. Internal capacitance of MOSFET & high frequency model:

High frequency response of MOSFET affects due to internal capacitances. There are two types of internal capacitances in the MOSFET.

i) Gate capacitance: It is a parallel plate capacitance formed by a gate electrode with the channel, with the oxide layer acts as a capacitor dielectric .It is denoted as C_{ox}.

ii) Junction capacitance: (Source-body & drain-body depletion layer capacitances)

These capacitances are due to the reverse biased pn junctions formed by the n^+ drain & the p type substrate & the n^+ drain region and the p-type substrate. These are denoted as source diffusion capacitance & drain diffusion capacitance, respectively.

4.8.1. Gate capacitances:

There are three gate capacitances: C_{gs}, C_{gd} & C_{gb}.

In triode region the channel as uniform depth & hence we have,

$$C_{gs} = C_{gd} = \frac{1}{2} WL\, C_{ox}$$

In saturation region ,

$$C_{gs} = \frac{2}{3} WL\, C_{ox}$$

$$C_{gd} = 0$$

In cut off region,

$$C_{gs} = C_{gd} = 0$$

Gate-body model capacitance is given by,

$$C_{gb} = WL\, C_{ox} \quad \text{(cut off)}$$

The overlap capacitance,

$$C_{ov} = WL_{ov} C_{ox}$$

Where, L_{ov} - ovrlap length & is typically 0.05L to 0.1L

4.8.2. Junction capacitances:

Source diffusion capacitance,

$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \frac{V_{SB}}{V_o}}}$$

Where, C_{sbo} -value of C_{sb} at zero body –source bias.

V_{SB} -mangnitude of the reverse bias voltage.

V_o -junction built in voltage, typically 0.6v to 0.8v.

The drain diffusion capacitance,

$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \frac{V_{DB}}{V_o}}}$$

4.8.3. Unity gain frequency:

The f_T is the frequency at which the short circuit current gain of the CS MOSFET amplifier becomes unity. Output terminals are shorted.

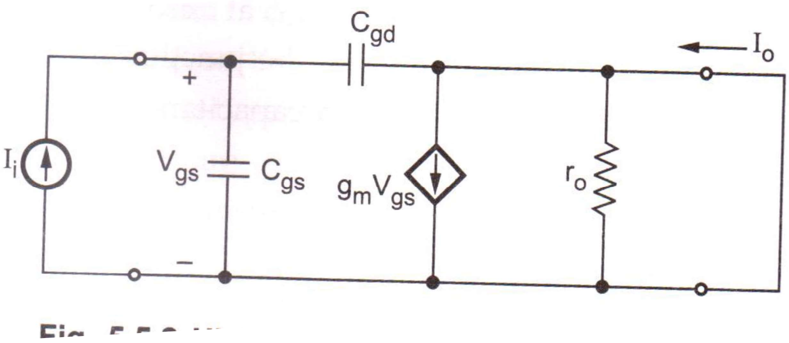


Fig 4.20. High frequency model to determine short-circuit current gain

$$I_o \approx g_m V_{gs}$$

V_{gs} in terms of I_1 ,

$$V_{gs} = \frac{I_1}{S\,(C_{gs} + C_{gd})}$$

$$I_i = V_{gs}\, S\,(C_{gs} + C_{gd})$$

$$|Ai| = \frac{I_o}{I_i} = \frac{g_m V_{gs}}{V_{gs}\, S\,(C_{gs} + C_{gd})} = \frac{g_m}{S\,(C_{gs} + C_{gd})}$$

$$f_T \rightarrow |Ai| = 1$$

$$\omega_T = \frac{g_m}{(C_{gs} + C_{gd})}$$

$$f_T = \frac{g_m}{2\pi\,(C_{gs} + C_{gd})}$$

4.9. Frequency response of CS amplifier:

$$V_o = I_L R_L' = g_m V_{gs} R_L{}^1$$

Where, $R_L' = r_0 \,||R_D||R_L$

By Miller's theorem,

$$C_{eq} = (1 + A_v)C$$

$$C_{eq} = (1 + A_v)C_{gd}$$

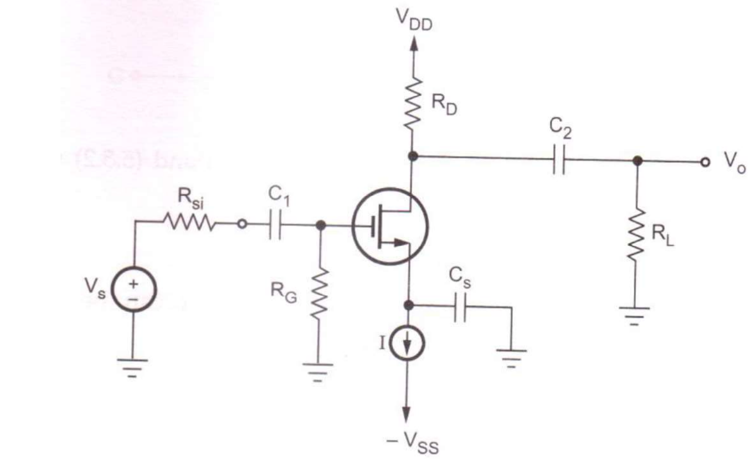


Fig 4.21. CS MOSFET amplifier

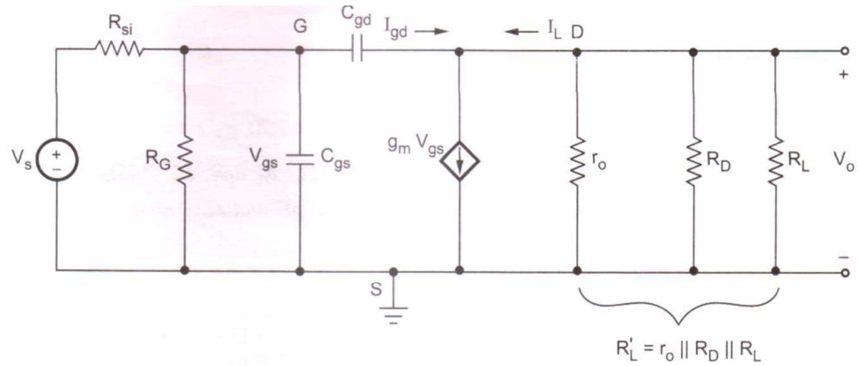


Fig 4.22. Equivalent circuit for CS MOSFET amplifier

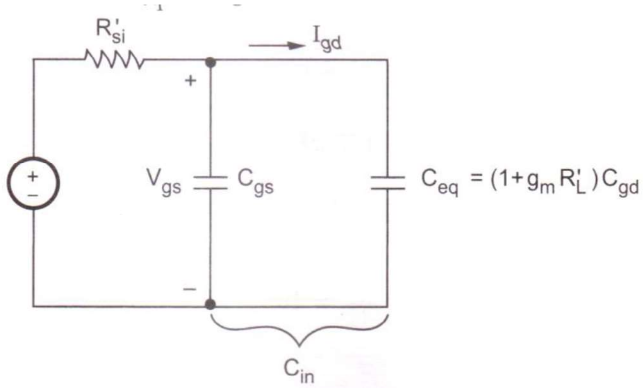


Fig 4.23. Input node

$$A_v = \frac{V_o}{V_i} = \frac{g_m V_{gs} R_L'}{V_{gs}} = g_m R_L'$$

$$C_{eq} = (1 + g_m R_L')C_{gd}$$

The total capacitance C_{in},

$$C_{in} = C_{gs} + C_{eq}$$

$$= cgs + (1 + g_m R_L')C_{gd}$$

The total resistance,

$$R_{si}' = R_{si} || R_G$$

$$\tau = RC = R_{si}' C_{in}$$

$$\omega_H = \omega_o = \frac{1}{\tau}$$

$$= \frac{1}{R_{si}' C_{in}}$$

$$f_H = \frac{1}{2\pi R_{si}' C_{in}}$$

4.10. Low frequency response:

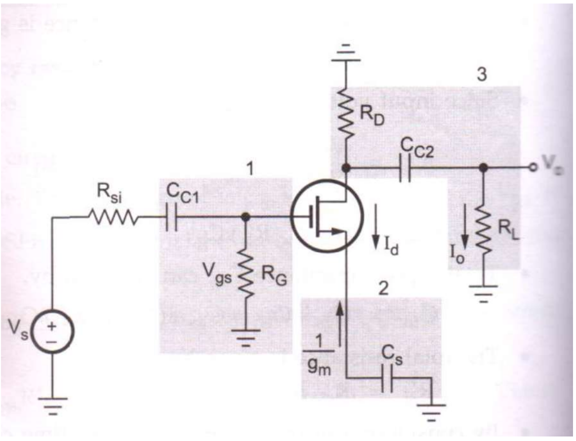


Fig 4.24. Low frequency response of CS MOSFET amplifier circuit

The corner frequencies due to there RC networks can be given by,

$$f_1 = \frac{1}{2\pi\, C_{c1}\,(Rsi + RG)}$$

$$f_1' = \frac{g_m}{2\pi\, C_s}$$

$$f_2'' = \frac{1}{(2\pi\, C_{c2}\,(R_D + R_L)}$$

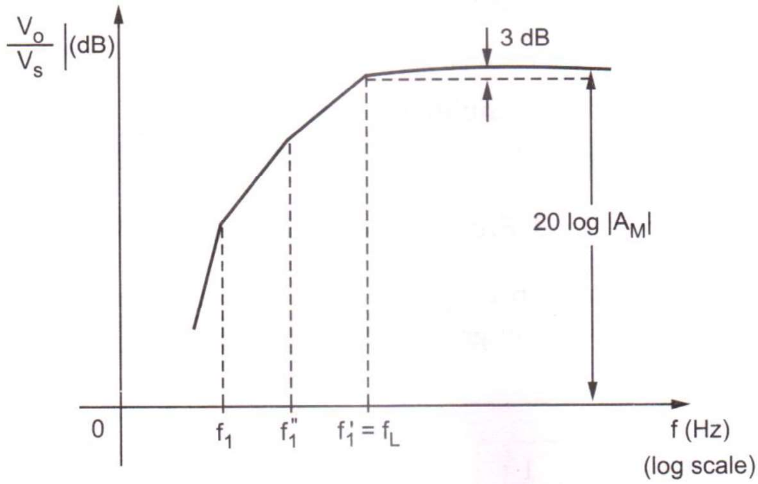


Fig 4.25. Low frequency response of CS MOSFET amplifier

4.11. Bandwidth of Multistage Amplifier:

The Bandwidth of Multistage Amplifier is always less than that of the bandwidth of single stage amplifier.

4.11.1. Overall lower cut off frequency of multistage amplifier:

Let us consider the lower 3db frequency of n identical cascaded stages as $f_L(n)$. It is the frequency for which the overall gain falls to $\frac{1}{\sqrt{2}}(3dB)$ of its midband value.

$$\left[\frac{1}{\sqrt{1+\left(\frac{f_L}{f_L(n)}\right)^2}}\right]^n = \frac{1}{\sqrt{2}}$$

$$\sqrt{2} = \left[\sqrt{1+\left(\frac{f_L}{f_L(n)}\right)^2}\right]^n$$

Squaring on both sides,

$$2 = \left[\sqrt{1 + \left(\frac{f_L}{f_{L(n)}}\right)^2}\right]^n$$

Taking n[th] root on both sides,

$$2^{\frac{1}{n}} = 1 + \left(\frac{f_L}{f_{L(n)}}\right)^2$$

$$2^{\frac{1}{n}} - 1 = \left(\frac{f_L}{f_{L(n)}}\right)^2$$

Taking square root on both sides,

$$\sqrt{2^{\frac{1}{n}} - 1} = \frac{f_L}{f_{L(n)}}$$

$$f_{L(n)} = \frac{f_L}{\sqrt{2^{\frac{1}{n}} - 1}}$$

$$\sqrt{2^{\frac{1}{n}} - 1} = \frac{fL}{fL(n)}$$

$$f_{L(n)} = \frac{f_L}{\sqrt{2^{\frac{1}{n}} - 1}}$$

where $f_L(n)$ – Lower 3dB frequency of identical cascaded stages.

f_L –Lower 3dB frequency of single stage.

n –Number of stages.

4.11.2. Overall higher cut off frequency of multistage amplifier:

Let us consider the upper 3dB frequency of n identical stages as $f_H(n)$.It is the frequency for which the overall gain falls to $\frac{1}{\sqrt{2}}$ (3dB) of its midband value.

$$\left[\frac{1}{\sqrt{1+\left(\frac{f_H(n)}{f_H}\right)^2}}\right]^n = \frac{1}{\sqrt{2}}$$

$$\sqrt{2} = \left[\sqrt{1+\left(\frac{f_H(n)}{f_H}\right)^2}\right]^n$$

Squaring on both sides,

$$2 = \left[\sqrt{1+\left(\frac{f_H(n)}{f_H}\right)^2}\right]^n$$

Taking n^{th} root on both sides,

$$2^{\frac{1}{n}} = 1+\left(\frac{f_H(n)}{f_H}\right)^2$$

$$2^{\frac{1}{n}} - 1 = \left(\frac{f_H(n)}{f_H}\right)^2$$

Taking square root on both sides,

$$\sqrt{2^{\frac{1}{n}} - 1} = \frac{f_H(n)}{f_H}$$

$$f_{H(n)} = f_H.\sqrt{2^{\frac{1}{n}} - 1}$$

In multistage amplifier $f_L(n)$ is always greater than f_L and $f_H(n)$ is always less than f_H. Therefore, we can say that bandwidth of multistage amplifier is always less than single stage amplifier.

If stages are not identical f_H can be given as,

$$\frac{1}{f_H} = 1.1\sqrt{\sqrt{\frac{1}{{f_1}^2}} + \frac{1}{{f_2}^2} + \frac{1}{{f_3}^2} + \cdots + \frac{1}{{f_n}^2}}$$

SOLVED EXAMPLES:

1. For an amplifier, midband gain=100 and lower cut off frequency is 1KHz. Find the gain of an amplifier at frequency=20KHz.

Solution: We know that, Bellow midband:

$$A = \frac{A_{mid}}{\sqrt{1 + (f/f_2)^2}}$$

$$A = \frac{100}{\sqrt{1 + \left(\frac{1000}{20}\right)^2}} = 2$$

2. For an amplifier, 3db gain is 200 and higher cut off frequency is 20KHz. Find the gain of an amplifier at frequency=100KHz.

Solution :

We know that, A_{mid}=3db gain$\times \sqrt{2} = 200 \times \sqrt{2} = 282.84$

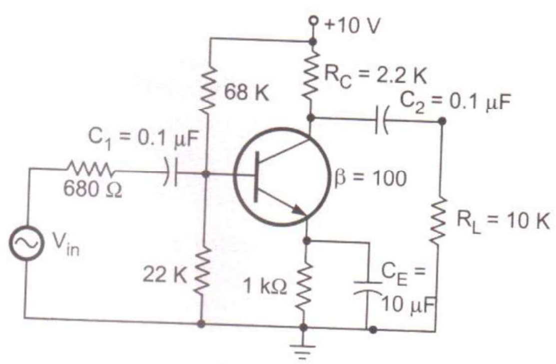
Above midband:

$$A = \frac{A_{mid}}{\sqrt{1 + (f/f_2)^2}}$$

$$A = \frac{282.84}{\sqrt{1 + \left(\frac{100 \times 10^3}{20 \times 10^3}\right)^2}}$$

$$= 55.46$$

3. Determine the low frequency response of the amplifier circuit shown in fig.



Solution: It is necessary to analyze each network to determine the critical frequency of the amplifier

a) Input RC network

$$f_c(\text{input}) = \frac{1}{2\pi[R_S + (R_1 \| R_2 \| h_{ie})]C_1}$$

$$= \frac{1}{2\pi[680 + (68K \| 22K \| 1.1K)]0.1 \times 10^{-6}}$$

$$= \frac{1}{2\pi[680 + 1031.7]0.1 \times 10^{-6}} = 929.8\text{Hz}$$

b) Output RC network

$$f_C(\text{output}) = \frac{1}{2\pi(R_C + R_L)C_2} = \frac{1}{2\pi(2.2K + 10K)0.1 \times 10^{-6}} = 130.45\text{Hz}$$

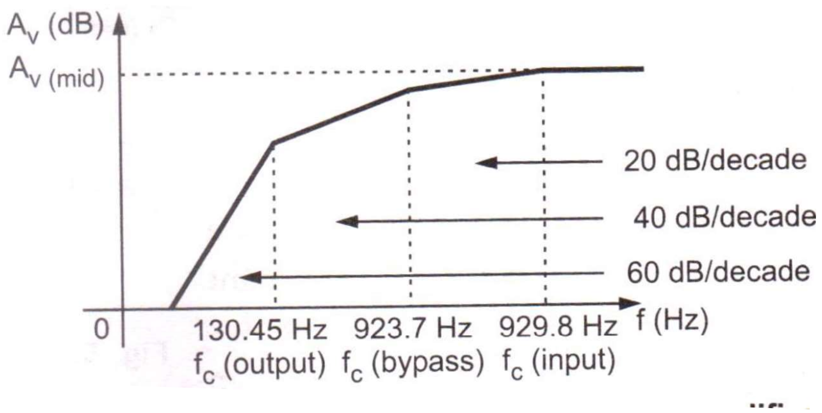
c) Bypass RC network

$$f_C(\text{bypass}) = \frac{1}{2\pi[\left(\frac{R_{TH} + h_{ie}}{\beta}\right) \| R_E]C_E}$$

$$R_{TH} = R_1 \| R_2 \| R_S = 68K \| 22K \| 680 = 653.28\Omega$$

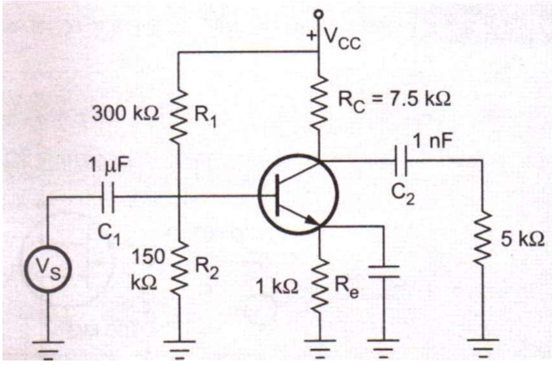
$$f_C(\text{bypass}) = \frac{1}{2\pi[\left(\frac{6.53.28 + 1100}{100}\right) \| 1K]10 \times 10^{-6}} = \frac{1}{2\pi[17.23]10 \times 10^{-6}}$$

$$= 923.7\text{Hz}$$



4. Calculate the cut off frequency due to C_1 and C_2 in the circuit shown in fig.

1)h_{fe}=300 2)h_{ie}=32k



Solution:

1) cut off frequency due to C_1

$$fc = \frac{1}{2\pi R_{in} C_1} \qquad Rin = R1||R2||hie$$

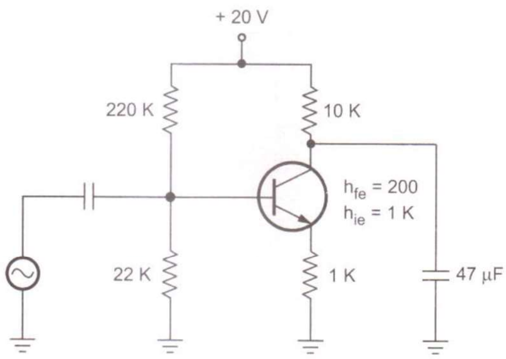
$$= \frac{1}{2\pi[(R_1||R_2||h_{ie})]C_1}$$

$$= \frac{1}{2\pi[(300\times 10^3\ ||150\times 10^3\ ||32\times 10^3\ )]\ (1\times 10^{-6})} = 6.565\text{Hz}$$

2) cut off frequency due to C_2

$$f_C = \frac{1}{2\pi(R_C + R_L)C_2} = \frac{1}{2\pi(7.5\times 60^3 + 5\times 10^3)0.1\times 10^{-9}} = 12732.4\text{Hz}$$

5. Determine the cut off frequency due to the bypass capacitor in the fig.



Solution:

$$f_{C(bypass)} = \frac{1}{2\pi[\left(\frac{R_{TH} + h_{ie}}{\beta}\right) \| R_E] C_E}$$

$$R_{TH} = R_1 \| R_2 \| R_S = 220K \| 22K \| 0 = 0$$

$$f_{C(bypass)} = \frac{1}{2\pi[\left(\frac{1000}{200}\right) \| 1 \times 10^3] 47 \times 10^{-6}} = \frac{1}{2\pi[4.97] 47 \times 10^{-6}}$$

$$= 681Hz$$

6. At I_C=1mA and V_{CE}=10V, a certain transistor data shows C_μ= C_μ=3pF h_{fe}=200 and ω_T=-500M rad/sec. Calculate g_m,r_π,C_π= C_π and ω_β

Solution:

$$\text{i) } g_m = \frac{I_C}{V_T} = \frac{1mA}{26mV} = 38.46mA/V$$

$$\text{ii) } r_\pi = \frac{h_{fe}}{g_m} = \frac{200}{36.46 \times 10^3} = 5.20K\Omega$$

$$\text{iii) } (C\pi + C\mu) = \frac{g_m}{2\pi f_T} = \frac{g_m}{\omega_T} = \frac{38.46 \times 10^{-3}}{500 \times 10^6}$$

$$(C\pi + C\mu) = 76.92pF$$

$$C\pi = C\pi = 76.92pF - 3pF = 73.92pF$$

iv) We know that,

$$f_T = h_{fe} f_\beta$$

$$2\pi f_T = 2\pi h_{fe} f_\beta$$

$$\omega_T = h_{fe} \omega_\beta$$

$$\omega_\beta = \frac{\omega_T}{h_{fe}} = 2.5M \frac{rad}{sec}$$

7. Short circuit CE current gain of transistor is 25 at a frequency of 2MHz if f_β=200KHz calculate 1)f_T 2)h_{fe} 3) find $|A_i|$ *at frequency* of 10MHz and 100MHz.

Solution:

$$\text{i) } f_T = |A_i| \times f = 25 \times 2 \times 10^6$$

$$= 50\text{MHz}$$

$$\text{ii) } h_{fe} = \frac{f_T}{f_\beta} = \frac{50\text{MHz}}{200\text{KHz}} = 250\text{KHz}$$

$$\text{iii) } \quad |A_i| = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f}{f_\beta}\right)^2}}$$

At $f = 10\text{MHz}$

$$|A_i| = \frac{250}{\sqrt{1 + (\frac{10 \times 10^6}{200 \times 10^3})^2}} = 5$$

At $f = 100\text{MHz}$

$$|A_i| = \frac{250}{1 + (\frac{100 \times 10^6}{200 \times 10^3})^2} = 0.5$$

8. A high frequency amplifier uses a transistor which is driven from a source with R_S=0. Calculate value of f_H,if R_L=0 and R_L=1KΩ. Assume typical values of hybrid-π parameters.

Solution:

1) f_H

For R_L=0

$$f_H = \frac{1}{2\pi r_\pi |C_\pi + C_\mu|}$$

Typical values: $r_\pi = 1\text{k}, C_\pi = 100\text{pF}, C_\mu = 3\text{pF}$

$$f_H = \frac{1}{2\pi \times 1 \times 10^3 (100 \times 10^{-12} + 3 \times 10^{-12})} = 1.545\text{MHz}$$

For R_L=1K

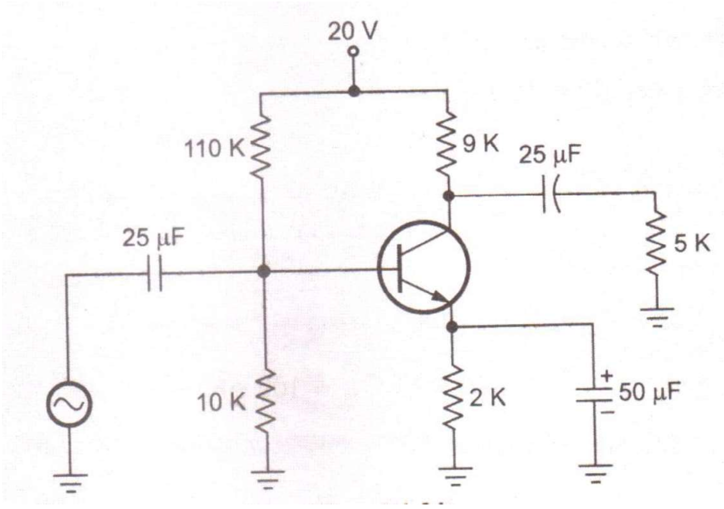
$$f_H = \frac{1}{2\pi r_\pi [C_\pi + C_\mu (1 + g_m R_L)]}$$

Typical values: $r_\pi = 1\text{k}, C_\pi = 100\text{pF}, C_\mu = 3\text{pF}, g_m = 50\text{mA/V}$

$$f_H = \frac{1}{2\pi \times 1 \times 10^3[100 \times 10^{-12} + 3 \times 10^{-12}(1 + 50 \times 10^{-3} \times 1 \times 10^3)]}$$

$$= 0.629\text{MHz}$$

9. Determine the bandwidth of the amplifier shown.



$r_b = 100\Omega$, $r_\pi = 1.1K$, $C_\pi = 3pF$, C_μ=100pF, h_{fe}=225

Solution:

We know that,

$$h_{fe} = r_b + r_\pi = 100\Omega + 1.1k = 1.2k$$

$$f_{c(input)} = \frac{1}{2\pi[R_S + (R_1||R_2||h_{ie})]C_1}$$

$$= \frac{1}{2\pi[0 + (110K||10K||1.2K)]25 \times 10^{-6}} = 6\text{Hz}$$

$$f_{C(output)} = \frac{1}{2\pi(R_C + R_L)C_2} = \frac{1}{2\pi(9K + 5k)25 \times 10^{-6}} = 0.454\text{Hz}$$

$$f_{C(bypass)} = \frac{1}{2\pi[\left(\frac{R_{TH} + h_{ie}}{\beta}\right)||R_E]C_E}$$

$$R_{TH} = R_1||R_2||R_S = 110K||10K||0 = 0\Omega$$

$$f_{C(bypass)} = \frac{1}{2\pi[\left(\frac{1.2\times10^3}{225}\right)\|2\times10^3]50\times10^{-6}} = 598Hz$$

f_L is the smallest of the three i.e. 0.454Hz

$$f_H = \frac{1}{2\pi r_\pi[C_\pi + C_\mu(1+g_m R_L)]}$$

$C\pi = 3pF, C\mu = 100pF$

$$g_m = \frac{h_{fe}}{r_\pi} = \frac{225}{1.1\times10^3} = 204.5mA/V$$

$$f_H = \frac{1}{2\pi\times1.1\times10^3[3\times10^{-12} + 100\times10^{-12}(1+204.5\times10^{-3}\times5\times10^3)]}$$

$$= 1.4136KHz$$

$$\text{Bandwidth} = fH - fL = 1.4136\times10^3 - 0.454 = 1.4131\ KHz$$

10. For n-channel MOSFET, L=1.0µm, L_{ov}=0.05µm, W=10µm, C_{ox}=3.45× 10^{-3}F/m^2, I_D=200µA and K_n'=150µA/V^2. Find the f_T if MOSFET is operating in the triode region.

Solution:

$$C_{ox} = 3.45\times10^{-3}F/m^2 = 3.45\times10^{-15}F/\mu m2$$

$$C_{gd} = C_{gs} = \frac{1}{2}WLC_{OX} + C_{OV} = \frac{1}{2}WLC_OX + WL_{OV}C_{OX}$$

$$= \left(\frac{1}{2}\times10\times1\times3.45\times10^{-15}\right) + 10\times0.05\times3.45\times10^{-15}$$

$$= 17.25\times10^{-15} + 1.725\times10^{-15} = 18.975\times10^{-15}F$$

$$= 18.975fF$$

$$g_m = \sqrt{2K_n'}\sqrt{W/L}\sqrt{I_D}$$

$$= \sqrt{2\times150\times10^{-6}}\sqrt{10/1}\sqrt{200\times10^{-6}} = 0.7746\ mA/V$$

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = \frac{0.7746\times10^{-3}}{2\pi\times(18.975+18.975)\times10^{-15}} = 3.2485GHz$$

11. For CS MOSFET amplifier, R_{si}=120kΩ, R_G=4.7MΩ, R_D=10K, R_L=15K, g_m=1.2mA/V, r_o=150KΩ, C_{gs}=0.3pF. Find the midband gain A_M and the upper 3dB frequency, f_H.

Solution:

$$A_M = -\frac{R_G}{R_G + R_{si}} g_m R_L^{'}$$

$$\text{where } R_L^{'} = r_o \| R_D \| R_L = 150K\|10K\|15K = 5.77K\Omega$$

$$A_M = -\frac{4.7 \times 10^6}{4.7 \times 10^6 + 120 \times 10^3} \times 1.2 \times 10^{-3} \times 5.77 \times 10^3$$

$$= -6.75$$

$$C_{eq} = (1 + g_m R_L^{'})C_{gd}$$

$$= (1 + 1.2 \times 10^{-3} \times 5.77 \times 10^3) \times 0.3 \times 10^{-12} = 2.377\text{pF}$$

$$C_{in} = C_{eq} + C_{gs} = 2.377 + 1 = 3.377\text{pF}$$

$$f_H = \frac{1}{2\pi R_{si}^{'} C_{in}} \quad \text{where } R_{si}^{'} = R_{si} \| R_G = 120K\|4.7M = 117K$$

$$= \frac{1}{2\pi \times 117 \times 10^3 \times 3.377 \times 10^{-12}} = 402.8\text{KHz}$$

12. For a CS MOSFET amplifier, C_{C1}=C_S=C_{C2}=1µF, R_G=12MΩ, R_{si}=180KΩ, g_m=1.2mA/V, R_D=10K and R_L=15K. Calculate A_M, f_l, f_l', f_l" and f_L.

Solution:

$$A_M = -\left(\frac{R_G}{R_G + R_{si}}\right) g_m (R_D \| R_L) \quad \text{where } R_D \| R_L = 10K\|15K = 6K$$

$$= -\left(\frac{12M}{12M + 180K}\right) 1.2 \times 6 = -7.09$$

$$f_1 = \frac{1}{2\pi \times 1 \times 10^{-6}(180 \times 10^3 + 12 \times 10^6)} = 0.013\text{Hz}$$

$$f_1^{'} = \frac{g_m}{2\pi C_S} = \frac{1.2 \times 10^{-3}}{2\pi \times 1 \times 10^{-6}} = 190.98\text{Hz}$$

$$f_1^{''} = \frac{1}{2\pi \times C_{C2}(R_D + R_L)} = \frac{1}{2\pi \times 1 \times 10^{-6}(25 \times 10^3)} = 6.366\text{Hz}$$

$$\text{Here}, f_1'' > f_1 \text{ and hence}$$

$$fL = f_1' = 190.98\text{Hz}$$

TWO MARK QUESTIONS AND ANSWERS

1. What is roll off?

The frequency response is nearly ideal over a wide range of mid frequency. Only at low and high frequency ends, gain deviates from ideal characteristics. The decrease in voltage gain with frequency is called as roll off.

2. What is bandwidth of an amplifier?

Bandwidth of the amplifier is defined as the difference between upper cut-off and lower cut-off frequencies.

Bandwidth = f_2 - f_1

The frequency f_2 lies in high frequency region while the frequency f_1 lies in low frequency region.

3. What is the significance of octaves and decades in frequency response?

The octaves and decades are the measures of change in frequency. A ten times change in frequency is called a decade. On the other hand, octave corresponds to doubling or halving of the frequency. For example, increase in frequency from 100Hz to 200Hz is an ctave. Likewise, a decrease i frequency from 100 kHz to 50kHz is also an octave.

4. State Miller theorem using resistor and capacitor.

For the analysis purpose ,in transistor amplifiers, it is necessary to split the capacitance between input(base or gate) and the output(collector or drain).The capacitance may be C_{bc} (in case of BJT) or C_{gd} (in case of FET). This can be achieved using Miller's theorem.

Miller's theorem: i) For resistor- $\frac{Z}{1-K}, \frac{ZK}{K-1}$

ii) For capacitor- $C(A_v + 1), C\left(\frac{A_v+1}{A_v}\right)$

5.Derive the expression for midband gain.

In the midband, the voltage gain of the amplifier is approximately maximum. It is designated as midband gain or A_{mid}.

$$A = \frac{A_{mid}}{\sqrt{1 + (f_1/f)^2}\sqrt{1 + (f/f_2)^2}}$$