Reg. No. :

Question Paper Code : 80606

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the unit normal to $xy = z^2$ at $(1, 1, -1)$.

2. Using Green's theorem, evaluate $\int_C (x\,dy - y\,dx)$, where C is the circle $x^2 + y^2 = 1$ in the xy-plane.

3. Find the particular integral of $(D^2 + 2D + 1)\,y = e^{-x}x^2$.

4. Convert the equation $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \log x$ into a differential equation with constant coefficients.

5. State the sufficient conditions for the existence of Laplace transform.

6. Find the inverse Laplace transform of $\frac{s}{(s+2)^2}$.

7. Find the value of m if $u = 2x^2 - my^2 + 3x$ is harmonic.

8. Find the image of the circle $|z| = 3$ under the transformation $w = 2z$.

9. State Cauchy's integral theorem.

10. Find the residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ (8)

(ii) Prove that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + 3xz^2\hat{k}$ is irrotational and find its scalar potential. (8)

Or

(b) (i) Find the directional derivative of $\varphi = 4xz^2 + x^2yz$ at $(1, -2, 1)$ in the direction of $2\hat{i} + 3\hat{j} + 4\hat{k}$. (4)

(ii) Verify Gauss divergence theorem for

$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$, where S is the surface of the cube formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (12)

12. (a) (i) Solve : $(D^2 + 2D + 2)y = e^{-2x} + \cos 2x$. (8)

(ii) Using method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = \sec x$. (8)

Or

(b) (i) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (8)

(ii) Solve the following equations : $\frac{dx}{dt} + 2x + 3y = 0$; $3x + \frac{dy}{dt} + 2y = 2e^{2t}$. (8)

13. (a) (i) Find the Laplace transform of the following functions :

(1) $\frac{e^{-t}\sin t}{t}$

(2) $t^2 \cos t$. (8)

(ii) Using Laplace transform, solve $(D^2 + 3D + 2)y = e^{-3t}$ given $y(0) = 1$ and $y'(0) = -1$. (8)

Or

(b) (i) Using convolution theorem, find $L^{-1}\left\{\frac{s}{(s^2+4)(s^2+9)}\right\}$. (8)

(ii) Find the Laplace transform of the square wave function defined by

$$f(t) = \begin{cases} k, & 0 < t < \frac{a}{2}, \quad f(t+a) = f(t) \\ -k, & \frac{a}{2} < t < a, \end{cases}$$ (8)

14. (a) (i) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function, show that the curves $u(x, y) = c_1$ and $v(x, y) = c_2$ cut orthogonally. (8)

(ii) Find the analytic function $f(z) = u + iv$ whose real part is $u = e^x(x\cos y - y\sin y)$. Find also the conjugate harmonic of u. (8)

Or

(b) (i) Show that the transformation $w = \frac{1}{z}$ transforms in general, circles and straight lines into circles or straight lines. (8)

(ii) Find the bilinear transformation which maps the points $z = 0, 1, -1$ onto the points $w = -1, 0, \infty$. Find also the invariant points of the transformation. (8)

15. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{z\,dz}{(z-1)^2(z+2)}$, where C is the circle $|z-1| = 1$. (8)

(ii) Using Contour integration evaluate $\int_0^\infty \frac{\cos mx\,dx}{x^2+a^2}$. (8)

Or

(b) (i) Find the Laurent's series expansion of $f(z) = \frac{1}{z^2 + 5z + 6}$ valid in the region $1 < |z+1| < 2$. (8)

(ii) Evaluate $\int_C \frac{z\,dz}{(z^2+1)^2}$, where C is the circle $|z-i| = 1$, using Cauchy's residue theorem. (8)