Reg. No.

Question Paper Code : 57499

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Second Semester

Civil Engineering

MA 6251 – MATHEMATICS – II

(Common all branches except Marine Engineering)

(Regulation 2013)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Evaluate $\nabla^2 \log r$.

2. State Stokes' theorem.

3. Solve $(D^2 + D + 1)y = 0$

4. If $1 \pm 2i$, $1 \pm 2i$ are the roots of the auxiliary equation corresponding to a fourth order homogenous linear differential equation $F(D)y = 0$, find its solution.

5. State convolution theorem on laplace transforms.

6. Evaluate $L^{-1}\left(\frac{s}{s^2 + 4s + 5}\right)$.

7. Give an example of a function where u and v are harmonic but u + iv is not analytic.

8. Find the critical points of the map $w^2 = (z - \alpha)(z - \beta)$.

9. Expand $f(z) = \frac{1}{z^2}$ as a Taylor series about the point $z = 2$.

10. Evaluate the residue of $f(z) = \tan z$ at its singularities.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) If $\nabla \phi = 2xyz^3 \vec{i} + x^2z^3 \vec{j} + 3x^2yz^2 \vec{k}$ find $\phi(x, y, z)$ given that $\phi(1, -2, 2) = 4$. **(8)**

(ii) Using Green's theorem in a plane evaluate

$\int_C [x^2(1 + y)dx + (x^3 + y^3)dy]$ where C is the square formed by $x = \pm 1$ and $y = \pm 1$. **(8)**

OR

(b) (i) Find 'a' and 'b' so that the surfaces $ax^3 - by^2z = (a + 3)x^2$ and $4x^2y - z^3 = 11$ cut orthogonally at $(2, -1, -3)$ **(8)**

(ii) Prove that Curl Curl $\vec{F}$ = grad div $\vec{F} - \nabla^2\vec{F}$. **(8)**

12. (a) (i) Solve $(D^2 + 2D + 1)y = xe^{-x} \cos x$. **(8)**

(ii) Solve the equation $(x^2D^2 - xD - 2)\, y = x^2 \log x$. **(8)**

OR

(b) (i) Solve the following simultaneous equations $\frac{dx}{dt} - y = t;\ \frac{dy}{dt} + x = t^2$. **(8)**

(ii) Solve the equation $y'' + y = \tan x$ using the method of variation of parameters. **(8)**

13. (a) (i) Evaluate :

(1) $L(t^2 e^{-t} \cos t)$

(2) $L^{-1}\left[e^{-2s} \frac{1}{(s^2 + s + 1)^2}\right]$ **(4) + (4)**

(ii) Find the inverse Laplace transform of $\frac{s}{(s^2+a^2)(s^2+b^2)}$ using convolution theorem. **(8)**

OR

(b) (i) Find the Laplace transform of f(t) defined by

$$f(t) = \begin{cases} E & \text{if } 0 < t < a/2 \\ -E & \text{if } a/2 < t < a \end{cases} \text{ where } f(t+a) = f(t).$$ **(8)**

(ii) Using Laplace transforms technique solve $y'' + y' = t^2 + 2t$, given $y = 4$, $y' = -2$ when $t = 0$. **(8)**

14. (a) (i) If $f(z) = u + iv$ is an analytic function in $z = x + iy$ then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|u|^2 = 2|f'(z)|^2.$$ **(8)**

(ii) Prove that $w = \frac{z}{z+a}$ where $a \neq 0$ is analytic whereas $w = \frac{\bar{z}}{\bar{z}+a}$ is not analytic. **(8)**

OR

(b) (i) Can $v = \tan^{-1}\left(\frac{y}{x}\right)$ be the imaginary part of an analytic function ? If so construct an analytic function $f(z) = u + iv$, taking v as the imaginary part and hence find u. **(8)**

(ii) Find the bilinear transformation that transforms the points $z = 1, i, -1$ of the z-plane into the points $w = 2, i, -2$ of the w-plane. **(8)**

15. (a) (i) Evaluate using Cauchy's integral formula : $\int_C \frac{(z+1)}{(z-3)(z-1)}\,dz$ where C is the circle $|z| = 2$. **(8)**

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 12\cos\theta}$ by using contour integration. **(8)**

OR

(b) (i) Expand as a Laurent's series the function $f(z) = \frac{z}{(z^2 - 3z + 2)}$ in the regions

(1) $|z| < 1$

(2) $1 < |z| < 2$

(3) $|z| > 2$ (8)

(ii) Evaluate $\int_0^{\infty} \frac{x \sin mx}{x^2 + a^2} dx$ where $a > 0, m > 0$. (8)