

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the directional derivative of $4x^2z + xy^2z$ at $(1, -1, 2)$ in the direction of $2\vec{i} - \vec{j} + 3\vec{k}$. (6)

- (ii) Using Stoke's theorem evaluate $\iint_S \text{curl } \vec{f} \cdot \vec{n} ds$ given $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$. (10)

Or

- (b) (i) Find ∇r^n and hence prove that $\nabla^2 r^n = n(n+1)r^{n-2}$. (6)
- (ii) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes $x=0, x=1, y=0, y=1, z=0$ and $z=1$. (10)

12. (a) (i) Solve : $(D^3 - 2D^2 + 4D - 8)y = e^{2x} + \sin x \cos x$. (8)

- (ii) Solve : $\frac{dx}{dt} + y = \sin t$; $\frac{dy}{dt} + x = \cos t$ given that $x = 2, y = 0$ when $t = 0$. (8)

Or

- (b) (i) Solve : $x^2y'' - 4xy' + 6y = x^2 + \log x$. (8)
- (ii) Solve : $y'' + 4y = \cot 2x$, using the method of variation of parameters. (8)

13. (a) (i) Find Laplace transform of $t^2 e^{-3t} \cos t$ and $\int_0^t \frac{\sin t}{t} dt$. (8)

- (ii) Using convolution theorem evaluate $\int_0^t \sin u \cos(t-u) du$. (8)

Or

- (b) (i) Find the Laplace transform of

$$f(t) = \begin{cases} \frac{4E}{T}t - E; & 0 \leq t \leq \frac{T}{2} \\ 3E - \frac{4E}{T}t; & \frac{T}{2} \leq t \leq T \end{cases} \text{ and } f(t+T) = f(t) \text{ and E is a constant.}$$

(8)

- (ii) Solve using Laplace transform, $x'' - 2x' + x = e^t$ when $x(0) = 2, x'(0) = -1$. (8)

14. (a) (i) If $f(z)$ is an analytic function, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^p = p^2 |f'(z)|^2 |f(z)|^{p-2}$. (8)

(ii) Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines in the w -plane into circles or straight lines in the z -plane. Which circles in the z -plane become straight lines in the w -plane and which straight lines transform into other straight lines? (8)

Or

(b) (i) Determine the analytic function $f(z) = u + iv$, given $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f\left(\frac{\pi}{2}\right) = 0$. (8)

(ii) Find the bilinear transformation which maps the points $-i, 0, i$ into the points $-1, i, 1$ respectively. Into what curve the y -axis is transformed under this transformation? (8)

15. (a) (i) Evaluate $\int_C \frac{\tan \frac{z}{2}}{(z-a)^2} dz$, where $-2 < a < 2$ and C is the boundary of the square whose sides lie along $x = \pm 2$ and $y = \pm 2$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2 + a^2)(x^2 + b^2)}$ using contour integration given $a > b > 0$. (8)

Or

(b) (i) Expand Laurent's series $f(z) = \frac{z}{(z-1)(z-2)}$ valid in $1 < |z| < 2$ and $|z-1| < 1$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta d\theta}{5 - 4 \cos \theta}$. (8)