Reg. No. :

Question Paper Code : 53243

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

First Semester

Civil Engineering

MA 6151 – MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

2. Write down the quadratic form corresponding to the matrix $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{pmatrix}$.

3. Examine the nature of the series $1+2+3+4+\ldots+n+\ldots\infty$.

4. State the Leibnitz's rule.

5. Show that the family of straight lines $2y-4x+a=0$ has no envelope, where 'a' is a parameter.

6. Define the following terms : Radius of Curvature, Center of curvature.

7. State two important properties of Jacobians.

8. Write the formula for Taylor's expansion of $f(x, y)$ about the point (a, b) upto second degree terms.

9. Evaluate $\int_0^1\int_0^1\int_0^1 xyz\, dx\, dy\, dz$.

10. Change the order of integration in $\int_{-2}^{1}\int_{x^2+4x}^{3x+2} dy\, dx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find all the eigen values and eigen vectors of the matrix $A=\begin{pmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{pmatrix}$. (10)

(ii) Using Cayley-Hamilton theorem, find the inverse of the matrix $A=\begin{pmatrix}4 & 3 & 1\\ 2 & 1 & -2\\ 1 & 2 & 1\end{pmatrix}$. (6)

Or

(b) Find the orthogonal transformation which transforms the quadratic form $x^2+3y^2+3z^2-2yz$ to canonical form. Also determine the index, signature and nature of the quadratic form. (16)

12. (a) (i) Prove that the Geometric series with common ratio 'r' is convergent if $|r|<1$, divergent if $r\ge 1$ and oscillatory if $r\le -1$. (8)

(ii) Discuss the series for convergence : $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\infty$. (8)

Or

(b) (i) Test the series for convergence : $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\ldots$ by comparison test. (8)

(ii) Test the convergence of the series : $\frac{1}{6}-\frac{2}{11}+\frac{3}{16}-\frac{4}{21}+\frac{5}{26}-\ldots\infty$. (8)

13. (a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (10)

(ii) Show that the radius of curvature of any point (x,y) of the rectangular hyperbola $xy=c^2$ is given by $\rho=\frac{(x^2+y^2)^{3/2}}{2c^2}$. (6)

Or

(b) (i) Find the center and circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(a/4, a/4\right)$. (8)

(ii) Find the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ considering it as the envelope of it normals. (8)

14. (a) (i) If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u, v)}{\partial(x, y)}$. Find also a relation between u and v, if it exists. (8)

(ii) Using Taylor's series, expand $\sin x \sin y$ in powers of x and y upto the terms of third degree. (8)

Or

(b) (i) Find the dimensions of the rectangular box without a top of maximum capacity, whose surface area is 108 sq.cm. (8)

(ii) If $z=x^y+y^x$, then prove that $\frac{\partial^2 z}{\partial x \partial y}=\frac{\partial^2 z}{\partial y \partial x}$. (8)

15. (a) (i) Change the order of integration and evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1} x}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}\, dx\, dy$. (8)

(ii) Find by triple integral, the volume of the tetrahedron bounded by the coordinate planes and the plane $x+y+z=1$. (8)

Or

(b) (i) Evaluate, through the change of variables, the double integral $\iint_R (x+y)^3 e^{-(x-y)} dx\, dy$ where R is the square with vertices (1, 0), (2, 1), (1, 2) and (0, 1) using the transformation $u=x+y$ and $v=x-y$. (8)

(ii) Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$. (8)